NOISE-ADAPTIVE ANISOTROPIC DIFFUSION FILTERING OF MRI IMAGES RECONSTRUCTED BY SENSE (SENSITIVITY ENCODING) METHOD

Alexei A. Samsonov and Chris R. Johnson

Scientific Computing and Imaging Institute, University of Utah, Salt Lake City, Utah, USA
Emails: *samsonov@cs.utah.edu* and *crj@cs.utah.edu*

ABSTRACT

SENSE (SENSitivity Encoding) imaging provides significant acquisition speedups in MRI. The main drawback of the method is that it generates images that have increased and spatially non-uniform noise levels and, hence, will often require retrospective filtering. In this paper, we show that standard anisotropic diffusion filtering, while being an effective technique for edge-preserving denoising of images with uniform noise levels, is often non-optimal for SENSE-reconstructed data. We have developed a modification of this filter for SENSE images using a robust statistical analysis of the anisotropic diffusion process. The new method utilizes the image noise matrix that is available from the SENSE reconstruction to automatically adjust filtering parameters with local noise levels. The effectiveness of the method and its advantage over standard anisotropic diffusion filtering for SENSE images were demonstrated with phantom and patient MRI data.

1. INTRODUCTION

Partial parallel MRI methods that use hybrid gradient and sensitivity encoding have attracted great attention in the MRI community over the past several years [1-2]. The potential speedup from sensitivity encoding may be several times in comparison to conventional MRI scan times. As a result, higher temporal and/or spatial resolutions are possible in the same imaging time. The SENSE (SENSitivity Encoding) method [1] represents the most general approach for image reconstruction from sensitivity-encoded data. Along with reconstruction, the SENSE theory provides strategies for optimization of coil geometries and allows for the prediction of the noise distribution in the reconstructed images. However, the increased speedup of SENSE does not come for free: the reconstructed images have increased noise levels due to reduced acquisition times. Furthermore, the noise distribution is non-uniform throughout the image due to the unfolding properties of the configuration of the receiver coils and noise amplification by the sensitivity profile demodulation during the reconstruction. Thus, the visual inspection, as well as post processing of the SENSE images, could be greatly impeded. As such, the practical significance of the resulting speedup could be challenged.

One way to improve the quality of SENSE images is to use retrospective filtering. Anisotropic diffusion filtering [3] was demonstrated to be an efficient method for edge-preserving denoising of MRI data [4]. Previous work on anisotropic diffusion was based on the implicit assumption that the noise level is uniform throughout the image. While this assumption holds for fully encoded MRI data, it is no longer valid for SENSE images. As a result, standard anisotropic diffusion filtering shows poor performance for SENSE-reconstructed data: it produces significant edge blurring in low noise areas and/or preserves and enhances the noise in the high noise areas. In this paper, we present a new method for edge-preserving denoising of SENSE images. The method utilizes the image noise matrix from the SENSE reconstruction and performs noise-adaptive anisotropic diffusion filtering. We demonstrate that our method outperforms standard anisotropic diffusion and significantly improves the quality of SENSE-reconstructed images.

2. THEORY AND METHODS

2.1. SENSE Theory Overview

In sensitivity encoding imaging, several surface coils with inhomogeneous sensitivity profiles are positioned around the imaging object to simultaneously acquire the data undersampled in *k*-space (the Fourier domain representation of the physical space). Such undersampling, while providing a speedup by reducing the number of encodings, decreases the imaging FOV and leads to aliasing in the image space. The main idea behind SENSE is to use the known coil sensitivity maps to unfold the image. The maps are usually obtained through the separate reference scan of the imaged object [1].

For brevity, we provide details of SENSE theory for the case of Cartesian sampling of *k*-space. In image space, the contributions of the underlying image I into the point (x, y) of the resulting aliased image M_i from i-th coil is weighted by the corresponding local values of the coil sensitivity profile:

$$M_i(x,y) = \sum_{k=1}^{N_P} S_i(x, y+k\cdot d_y) I(x, y+k\cdot d_y) + n(x,y), \quad (1)$$

where d_y is the imaging FOV in the phase encode direction y, decreased by a reduction factor R, $n(x, y)$ is the complex zero-mean Gaussian noise, N_P is the number of image pixels aliased into (x, y) and S_i (i=1…N_C) are the coil sensitivity profiles. Equation (1) for all coils can be assembled to form a single matrix equation:

$$\mathbf{M} = \mathbf{S}\cdot\mathbf{I} + \mathbf{N}, \quad (2)$$

where $\mathbf{S}$ is *the sensitivity matrix* of size $N_C \times N_P$, $\mathbf{I}$ is a column vector of pixels to unfold from point (x, y), $\mathbf{M}$ is a column vector of aliased image values at point (x, y) for all the coils and $\mathbf{N}$ is the corresponding vector of noise contributions [1]. For each point *(x, y)* of the aliased image, the partial unfolding step can be accomplished by taking the pseudo-inverse of (2):

$$\mathbf{I} = (\mathbf{S^H \Psi^{-1} S})^{-1} \mathbf{S^H \Psi^{-1} M} \,, \tag{3}$$

where $\mathbf{S^H}$ is the Hermitian conjugate of **S** and $\mathbf{\Psi}$ is the receiver noise matrix describing noise levels and correlations in the acquisition channels. The receiver noise matrix is determined prior to the scan from pure noise samples and used in the reconstruction for purposes of signal-to-noise ratio (SNR) optimization [1].

SENSE theory introduces the image noise matrix describing levels and correlations of noise in the reconstructed image. For Cartesian sampling, the partial image noise matrix **X** for each unfolding step is [1]:

$$\mathbf{X} = \frac{1}{n_k} (\mathbf{S^H \Psi^{-1} S})^{-1} \,, \tag{4}$$

where n_k is the number of sampling positions in *k*-space. The diagonal entries of **X** give the relative noise variances of the corresponding unfolded pixels and off-diagonal elements represent noise correlations among them.

2.2. Anisotropic Diffusion Filtering

Anisotropic diffusion filtering in its classic form [3] can be described as a diffusion process in image space:

$$\frac{\partial I(\bar{r},t)}{\partial t} = \nabla \left(g(\|\nabla I\|, k) \cdot \nabla I(\bar{r},t) \right), \tag{5}$$

where $g(\|\nabla I\|, k)$ is a monotonically decreasing diffusion function, depending on the local gradient value and conductance parameter *k*. Particular choices of *g* and *k* cause the diffusion process to slow down or almost stop at edges leading to the edge-preserving denoising of the images.

For analysis purposes, we employed a recently proposed robust statistical interpretation of the anisotropic diffusion filter [5]. In this approach, the filtering problem is formulated as the robust estimation of a piecewise constant image from noisy data where significant gradients, such as edges, are considered as outliers. The estimation is accomplished by solving the following problem:

$$\min_I \int_\Omega \rho(\|\nabla I\|, \sigma) d\Omega \,, \tag{6}$$

where ρ is a robust error norm and σ is a scale parameter reflecting the values of the possible outliers (edges). The formulation is equivalent to the Perona-Malik anisotropic diffusion using a gradient descent minimization with diffusion function [5]

$$g(x,\sigma) = \rho'_x(x,\sigma)/x \,. \tag{7}$$

2.3. Determination of Noise Maps

The partial image noise matrix **X** (4) that is available on each partial unfolding step (3) can be used to obtain the spatial distribution of relative noise levels in the reconstructed image:

$$N_R = \sqrt{diag(\mathbf{X})} \,. \tag{8}$$

The relative noise map N_R reflects the true values of the standard deviation of pixel noise only when the receiver noise matrix $\mathbf{\Psi}$ contains true noise values for the given scan. In practice, $\mathbf{\Psi}$ is determined in a separate scan, often with different parameters and, hence, could represent only relative values of noise in the receiver channels. In our approach, we estimate the true noise by analyzing background values of the reconstructed magnitude image. First, part of the background area $\mathbf{\Omega}$ and corresponding relative noise map $N_{R,\Omega}$ (8) are reconstructed by the SENSE method. The limitation on the choice of $\mathbf{\Omega}$ is that it should not contain correlated pixels obtained by (4); in the Cartesian case this implies that the linear size in the aliasing direction should be no more than the decreased FOV. Then, every magnitude value from $\mathbf{\Omega}$ is normalized by a corresponding value of the noise map $N_{R,\Omega}$ and standard deviation σ_{BG} of the normalized samples is determined. The true noise map for the image can be found as

$$N_T = \frac{\sigma_{BG} \cdot N_R}{\sqrt{2-\pi/2}} \,, \tag{9}$$

where the denominator accounts for a change of the noise standard deviation in the background due to the magnitude reconstruction [6].

2.4. Noise-Adaptive Anisotropic Diffusion for SENSE Images

SENSE images are free of modulation by coil sensitivity profiles and hence can be well represented by a piecewise constant tissue model. The noise in MRI magnitude images is well described by a Gaussian distribution for SNR of 3 or higher [6]. The distribution of the difference (I_n-I_m) for each pair of neighboring pixels *m* and *n* belonging to the same tissue type of the SENSE image is then zero-mean Gaussian $N(0,\sigma_{m,n})$. In the case of Cartesian sampling, the noise correlation in SENSE image occurs only among distant pixels obtained from each partial unfolding step (4) and for smoothing purposes could be neglected. Under an assumption of uncorrelated noise, the variance of the distribution is

$$\sigma_{m,n}^2 = \sigma_m^2 + \sigma_n^2 \,, \tag{10}$$

where σ_m and σ_n are entries of noise map N_T (9). The difference (I_n-I_m) across an edge may be considered an outlier in the distribution. The "robust scale" for rejection of such outliers (edges) for each pair of the neighboring pixels *m* and *n* may be chosen in terms of the population standard deviation:

$$\sigma_e = \sigma_{m,n} \,. \tag{11}$$

The parameter *k* for a given diffusion function *g* is then found from the condition that the influence of gradients should start decreasing at σ_e. That is, the derivative of the flow function $\psi(x,k) = x \cdot g(x,k)$ should be zero at the "robust scale" σ_e:

$$\omega : \psi'_x(x,k)\big|_{k=\omega \cdot \sigma_e} = 0 \,. \tag{12}$$

Expressions (10)-(12) give rise to a spatially variant conductance parameter

$$k_{m,n} = \omega \cdot \sigma_{m,n} \,. \tag{13}$$

Such a choice of *k* leads to noise-adaptive anisotropic diffusion, where the decision concerning what gradients are to be considered edges depends on the local noise values.

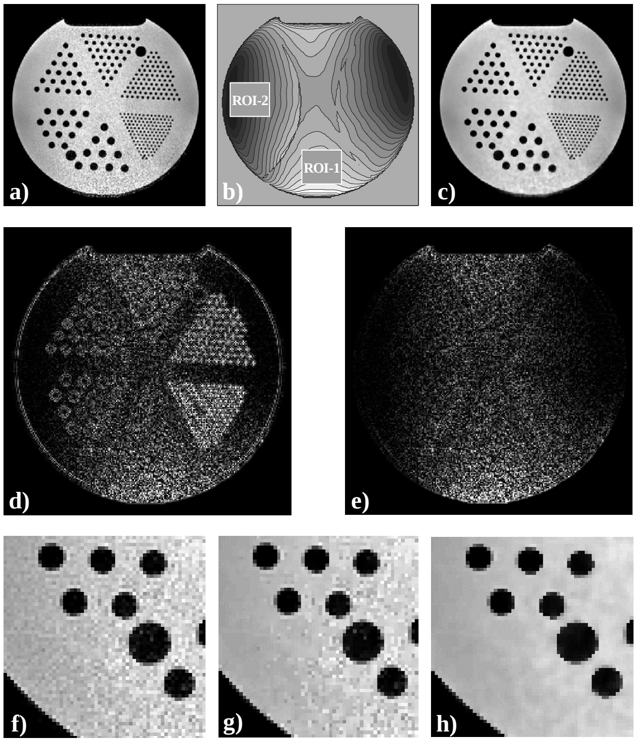

Figure 1. Comparing standard and noise-adaptive anisotropic diffusion filtering of SENSE images. **a)** Image reconstructed by SENSE (N_C=2, R=2), **b)** noise map of the SENSE image (log scale) with regions of interest ROI-1, 2, **c)** image *(a)* filtered by noise-adaptive anisotropic diffusion, **d)** absolute residue image of *(a)* from image filtered by standard anisotropic diffusion with "high-noise" choice of *k*, **e)** absolute residue image of *(a)* from *(c)*, **f)** magnified part of initial image *(a)*, **g)** *(f)*, filtered by standard anisotropic diffusion with "low-noise" choice of *k*, **h)** *(f)*, filtered by noise-adaptive anisotropic diffusion.

The normalization weight ω (12) for the conductance parameter (13) should be determined for each diffusion function. For example, for the exponential diffusion function

$$g(\|\nabla I\|, k) = e^{-(\|\nabla I\| / k)^2}, \tag{14}$$

proposed in [3], ω is $\sqrt{2}$ and the resulting conductance parameter is

$$k_{m,n} = \sqrt{2} \cdot \sqrt{\sigma_m^2 + \sigma_n^2}. \tag{15}$$

Discretization of the diffusion equation (5) with spatially variant conductance values is straightforward and leads to the final expression for noise-adaptive anisotropic diffusion:

$$\begin{aligned} I_m^{t+1} &= I_m^t + \lambda \cdot \sum_{n \in \eta(m)} \nabla I_{m,n}^t \cdot g(\nabla I_{m,n}^t, k_{m,n}) \\ \nabla I_{m,n}^t &= I_n^t - I_m^t, \ k_{m,n} = \omega\sqrt{\sigma_m^2 + \sigma_n^2} \end{aligned}, \tag{16}$$

where $\eta(m)$ is the averaging neighborhood of pixel m and λ is chosen to provide numerical stability of the approximation [3].

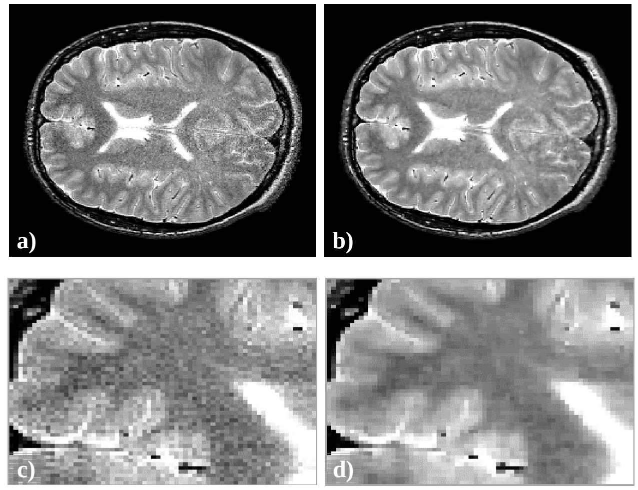

Figure 2. Noise-adaptive filtering of T2-weighted SENSE image (N_C=4, R=2). **a)** Image reconstructed by SENSE method, **b)** image *(a)* after 5 iterations of the proposed method, **c)** magnified part of image *(a)*, **d)** magnified part of image *(b)*

3. RESULTS

Reference and folded images of a quality phantom and a human brain were acquired on a 1.5T MR scanner (GE SIGNA, GE Medical Systems, Milwaukee, WI) using a 4-coil phased array and body coil. The reconstruction was done in the limits of the object mask [1]. The true noise map (9) was obtained using separate reconstructions of small background areas in each case. Both standard and noise-adaptive anisotropic diffusion filtering was done with diffusion function (14).

3.1. Phantom Studies

The goal of the phantom studies was to evaluate the performance of the standard anisotropic diffusion for SENSE images and to compare it with the proposed method. Ten iterations of anisotropic diffusion were applied in both standard and noise-adaptive versions of the filter. Standard anisotropic diffusion was used with two different values of conductance parameter *k*. In the first case, *k* was chosen using average noise estimation from region-of-interest ROI-1 (Fig. 1-b) to provide optimal filtering in the high-noise area ("high-noise" *k*). In the second case, *k* was estimated from average noise values in ROI-2 (Fig. 1-b) ("low-noise" *k*) to achieve adequate filtering of the low-noise area. The difference between initial and filtered images demonstrates that for "high-noise" choice of *k* there is significant blurring of edges of the phantom structures (Fig. 1-d). The same difference for noise-adaptive anisotropic diffusion proposed in this paper does not contain significant correlated features (Fig. 1-e). For "low-noise" choice of *k*, standard anisotropic diffusion leaves unfiltered noise in high-noise areas (Fig. 1-g). At the same time, the proposed method performs well both in low and high noise-areas (Fig. 1-h).

3.2. T2 Brain Image Filtering

Five iterations of the proposed filtering technique were applied to T2-weighted SENSE image of human brain. The results of the filtering are presented in Figure 2. There is significant improvent of the image SNR while most of the tissue structures are still preserved.

4. DISCUSSION

The phantom studies demonstrated that the standard anisotropic diffusion with constant conductance parameter fails to produce adequate edge-preserving filtering of SENSE images. With a "high-noise" choice of conductance parameter *k*, the low-noise areas suffer from undesired edge over-smoothing. On the other hand, when *k* is optimal for filtering of low-noise image parts ("low-noise" choice of *k*), noise values of large amplitudes in high-noise areas are preserved and even enhanced.

The proposed method takes into account the noise distribution in the SENSE images and adjusts the conductance parameter accordingly. Phantom studies and real data filtering demonstrated that the method is capable of edge-preserving filtering of SENSE images with good denoising of the areas with different noise levels.

The SNR of SENSE images goes down as the reduction factors *R* increase. Furthermore, at high and maximal *R* ($R \approx N_C$) and with non-optimized coil properties/positioning, the noise distribution in the reconstructed images is usually strongly non-uniform. The proposed method may be especially useful for non-optimal coil configurations and at high reduction factors.

The noise-adaptation used in the method does not incur significant computational overhead, as the partial image noise matrix (4) is found as part of each partial unfolding step (3). Reconstruction of background areas is not usually performed in SENSE, for it may corrupt SNR of the reconstruction [1]. In our method, some part of the background areas should be reconstructed to provide an automatic choice of the filtering parameters. This reconstruction can be achieved without significant additional overhead.

The results of the method analysis (section 2.4) are in good agreement with the choice of conductance parameter *k* in [4] for anisotropic diffusion filtering of MRI images with uniform noise level. In that work, *k* for diffusion function (14) was empirically chosen to be

$$1.5 \cdot \sigma_{noise} \le k \le 2 \cdot \sigma_{noise} . \qquad (17)$$

For constant noise levels, (15) leads to $k = 2 \cdot \sigma_{noise}$, which is in the range of (17).

In cases when the *k*-space sampling trajectory is not Cartesian, several issues need to be considered. The reconstruction from arbitrary trajectories by direct matrix inversion is very slow due to the large size of the matrices [1]. For iterative approaches for the reconstruction [7], the image noise matrix is not known. However, it is still possible to retrieve the noise map by simulations [8]. The noise correlations among pixels may be more complex in the case of reconstructions from arbitrary trajectories. But even then, the correlations are still long range and could be again neglected.

5. CONCLUSION

We developed a noise-adaptive method for SENSE image denoising using an anisotropic diffusion filtering framework. The method provides a significant increase of image SNR without substantial degradation of spatial resolution of the image. The standard anisotropic diffusion filter, with constant conductance parameter, was shown to produce filtering artifacts for SENSE images. The new method proposed in this paper takes into account the noise distribution in SENSE images and provides robust denoising in all image areas with minimized filtering artifacts. The image noise matrix available from SENSE reconstruction may be useful in adaptation of other filtering methods for SENSE images.

The proposed method is especially important for increasing the utility of SENSE images acquired at high reduction factors and with non-optimal coil geometries. Automatic choice of the filtering parameters and overall method efficiency make it particularly suitable for improving diagnostic quality of SENSE images in clinical environments.

6. ACKNOLEDGEMENTS

This work was supported by NIH BISTI grant 1P20HL68566-01. The authors wish to thank Eugene Kholmovski for his assistance with the imaging experiments and Ross Whitaker for his valuable comments.

7. REFERENCES

[1] K.P. Pruessmann, M. Weiger, M.B. Scheidegger, and P. Boesiger, "SENSE: Sensitivity Encoding for Fast MRI," *Magnetic Resonance in Medicine*, vol. 42, pp. 952–962, 1999

[2] D.K. Sodickson and W.J. Manning, "Simultaneous acquisition of spatial harmonics (SMASH): ultra-fast imaging with radiofrequency coil arrays," *Magnetic Resonance in Medicine*; vol. 38: pp. 591–603, 1997

[3] P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion," *IEEE Trans. Pattern Anal. Machine Intell.*, vol.12, no. 7 pp. 629–639, July 1990.

[4] G. Gerig, O. Kubler, R. Kikinis, and F. A. Jolesz, "Nonlinear anisotropic filtering of MRI data," *IEEE Trans. Med. Imag.*, vol. 11, pp. 221–232, 1992.

[5] M.J. Black, G. Sapiro, D.H. Marimont, and D. Heeger, "Robust Anisotropic Diffusion," *IEEE Trans. Image Processing*, vol. 7, no. 3, pp.421-432, March 1998

[6] W.A. Edelstein, P.A. Bottomley, L.M. Pfeifer, "A Signal-to-Noise Calibration Procedure for NMR imaging systems", Medical Physics, 11(2), pp. 180-185, 1984

[7] K.P. Pruessmann, M. Weiger, P. Bornert, and P. Boesiger, "Advances in Sensitivity Encoding With Arbitrary *k*-Space Trajectories", *Magnetic Resonance in Medicine*, vol. 46, pp. 638–651, 2001

[8] P. Mazurkewitz, U. Katscher, and V. Schulz, "Simulation of SENSE Geometry Factor", First Würzburg Workshop on Parallel MR Imaging Basics and Clinical Applications, Würzburg, Germany, November 7-10, 2001