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The Influence of Stochastic Organ Conductivity in 2D ECG Forward Modeling: A Stochastic Finite Element Study

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Abstract— **Quantification of the sensitivity of the electrocardiographic forward problem to various parameters can effectively direct the generalization of patient specific models without significant loss in accuracy. To this purpose we applied polynomial chaos based stochastic finite elements to assess the effect of variations in the distributions of tissue conductivity in a two-dimensional torso geometry generated from MRI scans and epicardial boundary conditions specified by intra-operatively recorded heart potentials. The polynomial chaos methodology allows sensitivity analysis of this type to be done in a fraction of the time required for a Monte Carlo analysis.**

Keywords— **forward problems; polynomial chaos; stochastic processes; uncertainty quantification**

I. Introduction

The standard 12-lead electrocardiogram has proven valuable as a means of non-invasively inferring qualitative cardiac electrical activity and thus abnormal heart function. In the past few decades investigators have sought more quantitative descriptions of cardiac function from body surface potential measurements. The determination of cardiac activity from externally recorded potentials is an inverse problem that has proven to be rather difficult. This is partly due to the attenuation of the potentials in propagation from the heart to the torso surface. Another source of error in these ill-posed problems is their high sensitivity to perturbations in the geometry of thorax and electrical conductivity of the tissues of the volume conductor [1]–[7]. Errors in this forward solution exacerbate uncertainties in the inverse problem so that understanding the relationship between geometric and conductivity errors and those errors arising in bioelectric modeling is imperative.

Uncertainties in computed body surface potentials can result from many factors, including accuracy of the numerical solver and assumptions made during model generation. We attempt to address the latter, and focus specifically on deviations in body surface potentials resulting from uncertainties in conductivity values utilized in the forward problem solved via the finite element method. Sensitivity to conductance is of particular interest because accurate tissue conductance measurements are difficult to obtain experimentally and vary experimentally and physiologically with factors such as temperature, hydration level, frequency, *etc,* [6], [8]. Indeed, textbook values of conductivity for certain in-vivo tissues can differ by more than 50% [9]–[12].

Model parameters with uncertainties can be viewed as having statistical distributions, which in turn result in stochastic systems whose solutions have statistical characteristics as well. Though one can determine the mean and standard deviation of the stochastic body surface values via Monte Carlo methods, these are often computationally prohibitive for complex systems [13]. More economical approaches exist but most are limited in their utility, for example, the sensitivity method is less robust and depends strongly on the modeling assumptions [14], while the widely used perturbation method is limited to relatively small perturbations and first-order expansions.

Polynomial chaos (PC) is a more effective approach that has been applied to stochastic solid and computational fluid dynamic problems. It is more efficient than Monte Carlo and can easily handle systems exhibiting complex interactions in their stochastic parameters [15]–[18]. We utilized generalized PC in the framework of the finite element method to numerically obtain the stochastic characteristics of torso potentials resulting from the electrical propagation of intra operatively recorded epicardial potentials in a two-dimensional model of a torso cross-section in which conductivities varied stochastically.

II. Methods

A. EEG Forward Modeling

We can pose a forward problem in electrocardiography as follows:

$$\nabla \cdot (\sigma(\boldsymbol{x})\nabla u(\boldsymbol{x})) = 0, \quad \boldsymbol{x} \in \Omega \tag{1}$$

$$u(\boldsymbol{x}) = u_0(\boldsymbol{x}), \quad \boldsymbol{x} \in \Gamma_D \tag{2}$$

$$\vec{n} \cdot \sigma(\boldsymbol{x})\nabla u(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \Gamma_N \tag{3}$$

where u is the potential field on the domain Ω, u_0 is the known epicardial potential function, Γ_D and Γ_N are the epicardial and torso boundaries respectively, and $\vec{n}$ denotes

the outward facing normal. We formulate the problem in the finite element framework with a triangular tessellation on Ω and appropriate test and trial functions for u.

B. Polynomial Chaos Representation of Random Processes

In this section we present the procedure for solving the stochastic elliptic problem via the generalized PC expansion for the deterministic forward problem of electrocardiography in equation (1).

Introducing stochastic conductivity, both u and σ are denoted $u(\boldsymbol{x};\boldsymbol{\xi})$ and $\sigma(\boldsymbol{x};\boldsymbol{\xi})$ for $\boldsymbol{x} \in \Omega$, where $\boldsymbol{\xi}$ is the n-dimensional stochastic variable, $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_n)$. These processes are represented via the generalized PC expansion as follows

$$u(\boldsymbol{x};\boldsymbol{\xi}) = \sum_{i=0}^{P} \hat{u}_i(\boldsymbol{x})\phi_i(\boldsymbol{\xi}) \quad (4)$$

$$\sigma(\boldsymbol{x};\boldsymbol{\xi}) = \sum_{i=0}^{P} \hat{\sigma}_i(\boldsymbol{x})\phi_i(\boldsymbol{\xi}). \quad (5)$$

Substituting into the elliptic equation and projecting the resulting system into the random space spanned by the basis polynomials, ϕ_k, we obtain the following linear system: For $k = 0, \ldots, P$

$$\sum_{i=0}^{P}\sum_{j=0}^{P} C_{i,j,k} \nabla \cdot (\hat{\sigma}_i(\boldsymbol{x})\nabla\hat{u}_j(\boldsymbol{x})) = 0, \ \boldsymbol{x} \in \Omega \quad (6)$$

$$u(\boldsymbol{x};\boldsymbol{\xi}) = u_0(\boldsymbol{x}), \ \boldsymbol{x} \in \Gamma_D \quad (7)$$

$$\vec{n} \cdot \hat{\sigma}_i(\boldsymbol{x})\nabla u(\boldsymbol{x};\boldsymbol{\xi}) = 0, \ \boldsymbol{x} \in \Gamma_N \quad (8)$$

where $C_{i,j,k} = \langle \phi_i(\boldsymbol{\xi}), \phi_j(\boldsymbol{\xi}), \phi_k(\boldsymbol{\xi}) \rangle$ is the inner product in the appropriate measure space. The basis polynomials should be chosen to match the distribution of the conductivity to ensure the best convergence rates [19]. When the deterministic problem is formulated in the finite element framework, one obtains a stiffness matrix and right hand side. For stochastic conductivity the PC linear system is simply a linear combination of stiffness matrices and right hand sides that are generated via the standard finite elements. Figure 1 shows a schematic of the type of system one must solve to obtain the stochastic moments of the solution. Here, the conductivity distribution is assumed to have one random dimension and two moments in the underlying stochastic space. The A_0 matrix and f_0 right hand side are obtained from the finite element procedure on the torso mesh with the mean conductivity values and mean boundary conditions, while the A_1 matrix is obtained using the first moment of the conductivity values and the mean boundary conditions. Further description of the use PC in such bioelectric problems can be found in [20].

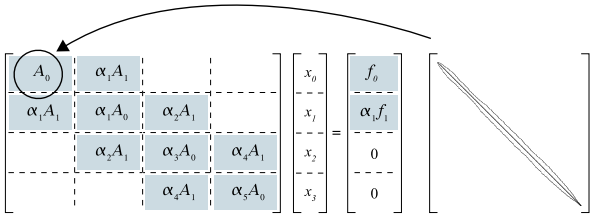

Fig. 1. Diagram of the large linear system resulting from the linear combination of stiffness matrices and right hand sides. $\alpha_1 = C_{1,1,0}$, $\alpha_2 = C_{2,1,1}$, $\alpha_3 = C_{2,2,0}$, $\alpha_4 = C_{3,2,1}$ and $\alpha_5 = C_{3,3,0}$.

TABLE I
CONDUCTIVITY VALUES CORRESPONDING TO TISSUE GROUPS IN THE MODEL

category	conductivity (S/m)	percent area of domain Ω
lungs	0.096	36.37%
muscle	0.300	22.93%
fat	0.045	21.60%
torso cavity	0.239	19.09%

C. Computational Experiment

The domain Ω was approximated by a mesh obtained from a three-dimensional human thorax model utilized in [3] consisting of 14611 nodes and 6893 triangular elements. Reference points were chosen on the mesh to aid in interpretation of the epicardial and torso surface plots. These points are depicted in figure 3, while the organ categories are illustrated in figure 4. Deterministic and mean conductivity values were assigned to each tissue category according to table I, and forward solutions were calculated with quadratic finite elements and seven modes of PC. Epicardial potentials recorded by an electrode sock during open chest surgery on a patient diagnosed with Wolff-Parkinson-White syndrome served as the Dirichlet boundary conditions. These potentials are depicted in figure 2.

We compared PC and Monte Carlo results for a $\pm$ 20% uniform lung conductivity distribution in the fully inhomogeneous model. Mean and standard deviations calculated from

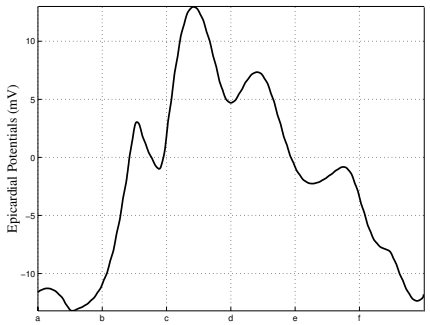

Fig. 2. These intraoperatively recorded epicardial potentials were used as boundary conditions for solving the forward problem. The labels a-f correspond to reference points on the epicardial surface depicted in figure 3.

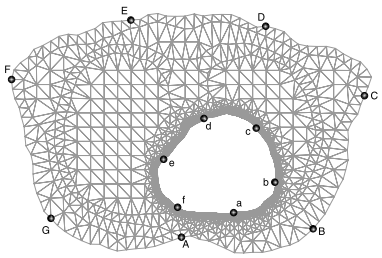

Fig. 3. The reference points on the epicardial and torso surface of the adaptively refined two-dimensional mesh correspond to the labeling of epicardial and torso potential plots.

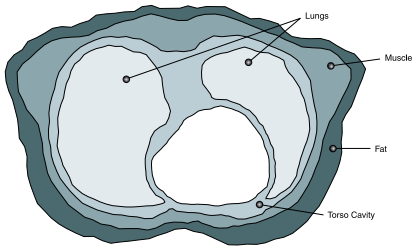

Fig. 4. Conductivity values were assigned according to the different regions of the torso slice. Tissues were grouped into one of the following: lungs, skeletal muscle, subcutaneous fat, and a miscellaneous category; torso cavity.

the simulation of 6,000 trials were the same as those from the seven-stochastic modal PC to within four significant digits. However, the Monte Carlo trials required more than $2,700$ times the CPU time required to compute the same result using PC. This performance discrepancy would be further exacerbated in the case of an even larger number of Monte Carlo trials.

III. Results

In an attempt to characterize the importance of accurate conductivity values in forward cardiac modeling, we calculated the mean and standard deviation of the torso potentials where each organ conductivity was distributed uniformly around its mean value. Figure 5 depicts the mean and standard deviation on the torso exterior for uniform distributions in lung, muscle and fat conductivity of $\pm$ 20% and $\pm$ 50%. Conductance values for each organ that range over a larger interval result in a broader spread of electrical potential values on the exterior of the torso. In both cases, stochastic lung conductivity results in the largest maximum standard deviation in external torso potential, while stochastic fat conductivities exhibit the lowest maximum standard deviation in external torso potentials.

Figure 6 depicts the standard deviation over the entire torso for $\pm$ 50% intervals in the various organ conductivities. From these it is clear that the maximum standard deviation in potential over the entire two-dimensional domain is highest for stochastic lung conductivity and lowest for stochastic fat conductivity.

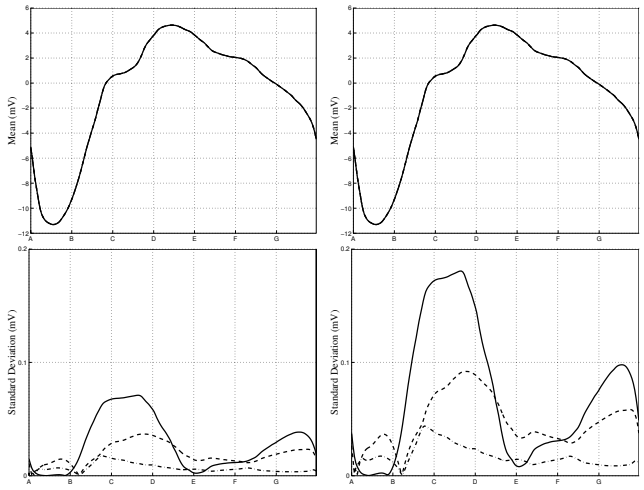

Fig. 5. **The effects of distributed conductivity values for various organ regions upon the potentials along the torso exterior:** The stochastic regions have uniform distribution of $\pm$ 20% (left figures) and $\pm$ 50% (right figures) from the reference conductivity value. The solid line corresponds to stochastic lung, while the dashed line corresponds to stochastic muscle and the dash-dotted line corresponds to stochastic fat. Note that the mean values are overlapping, and the differences between lung, muscle and fat mean voltages are not visually discernable.

IV. Discussion

Our results show that variability in the conductivity values of the lungs produces larger standard deviations in the potentials across the torso. This result is somewhat expected, as the lungs comprise a greater percentage of the volume of the two-dimensional torso slice in consideration, see table I. At 21.60%, fat tissue comprises less area in the model than lung and muscle, and stochastic fat conductivity results in the smallest maximum standard deviation in the potentials across the torso slice. This suggests that accurate determination of the lung conductivity is more important than fat in the case of this particular torso slice. One would expect that the conductivity of tissues comprising the greatest volume in a given model would be most important to determine accurately.

The light areas in figure 6 correspond to the regions of greatest standard deviation and are located in the vicinity of the left shoulder for stochastic lung, muscle, and fat conductivities. Interestingly enough, the epicardial potentials peak between the reference points c and d (refer to figures 2 and 3). In the case of propagation paths unimpeded by regions of stochastic conductivity from the epicardial surface to the surface of the torso, the standard deviation in the potential is fairly low.

Polynomial Chaos provides higher order statistics over the entire domain and is thus a valuable tool for investigating the response of two- and even three-dimensional biological models to stochastic parameters. In addition, PC is not limited by the correctness of the random number generator, and can be applied with little modification of existing finite element solvers. The computational tractability of PC solutions also

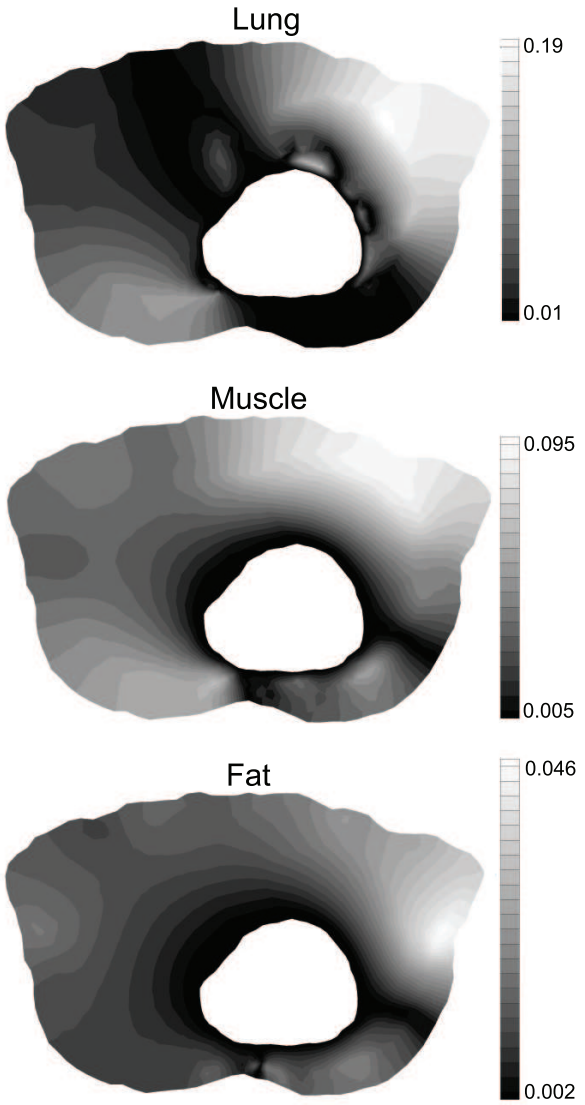

Fig. 6. **Effects of regions of stochastic conductivity upon the entire torso surface:** These contour plots correspond to the standard deviation in electrical potential across the torso surface resulting from stochastic lung, muscle, and fat conductivity with uniform distribution of $\pm$ 50% from the reference values.

allows for investigating the effects of multiple stochastic parameters or even multiple random-dimensional stochastic parameters in a reasonable amount of time. Thus various experiments can be performed in a short amount of time as compared to Monte Carlo methods, enabling more thorough investigation of complex systems.

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