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A Next Step: Visualizing Errors and Uncertainty

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When was the last time you saw an isosurface with error bars or streamlines with standard deviations or volume visualizations with representations of confidence intervals? With few exceptions, visualization research has ignored the visual representation of errors and uncertainty for 3D visualizations. However, if you look at peer-reviewed science and engineering journals, you will see that the majority of 2D graphs represent error and/or uncertainty within the experimental or simulated data. Why the difference? Clearly, if it's important to represent error and uncertainty in 2D graphs, then it's equally important to represent error and uncertainty in 2D and 3D visualization.

The possible detriment caused by the failure to represent errors and uncertainties in 3D visualizations became clear to us a couple of years ago when neurosurgeons and radiologists used one of our volume renderings of the brain and cerebral vasculature during their surgical planning for a patient. In this situation, accuracy and good understanding would have made a significant difference for the patient. As we explained linear interpolation errors and other possible uncertainties to the surgeons and radiologists, it occurred to me that our visualization was incomplete and we needed to do a better job of visually representing errors and uncertainties.

In a similar incident, at an Advanced Simulation and Computing workshop that we attended, one US Department of Energy national laboratory experimental scientist pointed out that he could compare possible differences and errors between two 3D visualizations only by printing the visualizations out on transparencies, laying them on top of each other, and holding them up to the light. He dubbed this the "view graph norm" and noted that even such simple comparison techniques were not available within most visualization systems.

Certainly, this lack can be partly attributed to the inherent difficulty in defining, characterizing, and controlling comparisons between different data sets and to the corresponding error and uncertainty in the experimental, simulation, and/or visualization process. In addition, we in the visualization community have developed few methods that allow for easy comparison and representation of error and uncertainty in visualization data. However, the main reason most 2D and 3D simulation and experimental data visualizations do not contain representations of error and uncertainty is that the visualization research community has not made such representations a priority. To take visualization research—and its usefulness to researchers in science, engineering, and medicine—to the next level, the visualization research community needs to make visually representing errors and uncertainties the norm rather than the exception.

What's been done so far

Fortunately, a few visualization researchers have started thinking about 3D visual representations of errors and uncertainties, the sources of which can include uncertainty in

- acquisition (instrument measurement error, numerical analysis error, statistical variation),
- the model (both mathematical and geometric),
- transformation (errors introduced from resampling, filtering, quantization, and rescaling), and
- visualization.

(See Taylor and Kuyatt[1] for a useful overview of uncertainty definitions.) Though space precludes a comprehensive discussion of previous work in error and uncertainty visualization, we thought it would be useful to highlight a few examples.

The geographic information systems (GIS) community carried out some of the earliest work on 3D representations of such errors and uncertainties for terrain models, where the effect of uncertainty on subsequent operations is an area of particular concern. Imagine an engineer planning a sewer pipeline based on an inaccurate terrain model and later discovering that the pipeline and its contents must flow uphill, when, had the uncertainty been known, the engineer could have performed an onsite inspection or chosen a better model. Wood and Fisher address the effect of uncertainty on GIS operations such as in the previous example, in addition to errors in spatial distributions.[2] In their work they also explore different interpolation methods and their affects on a final terrain model.

Data is interpolated (or filtered in some way) in almost any visualization. Lodha et al. looked at the uncertainty in different surface interpolates.[3] However, unlike Wood and Fisher, who relied on the observer to

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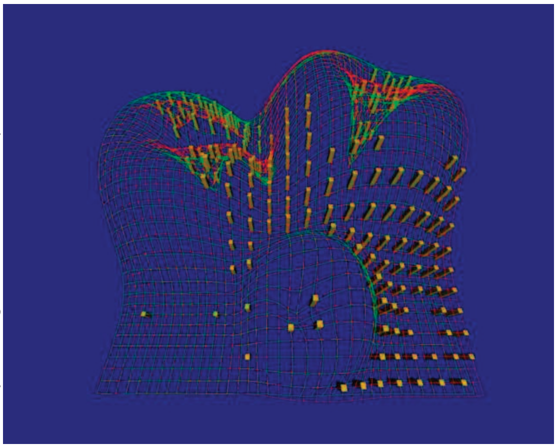
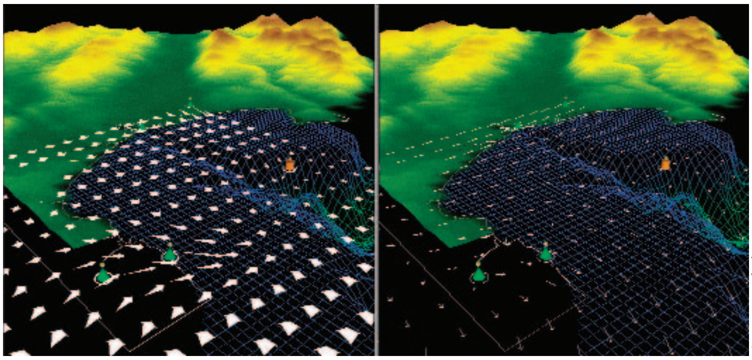


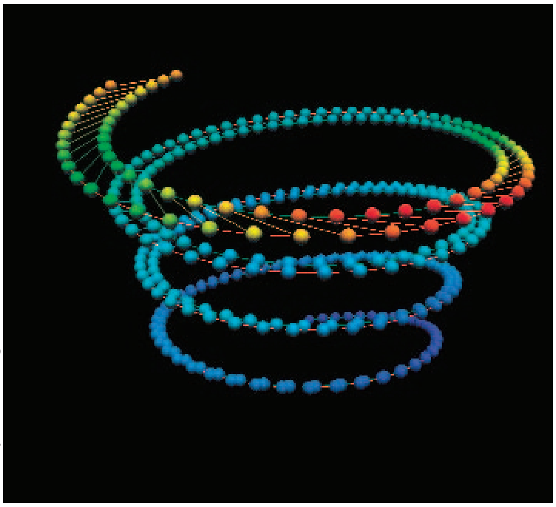
Courtesy Alex Pang/Canadian Human–Computer Comm. Soc.

1 **Visualization of the uncertainty between two surface interpolants using displacement glyphs.**



Courtesy of Alex Pang

2 **Visualization of wind velocity (a) with and (b) without uncertainty using direction uncertainty glyphs and regular arrow glyphs respectively.**



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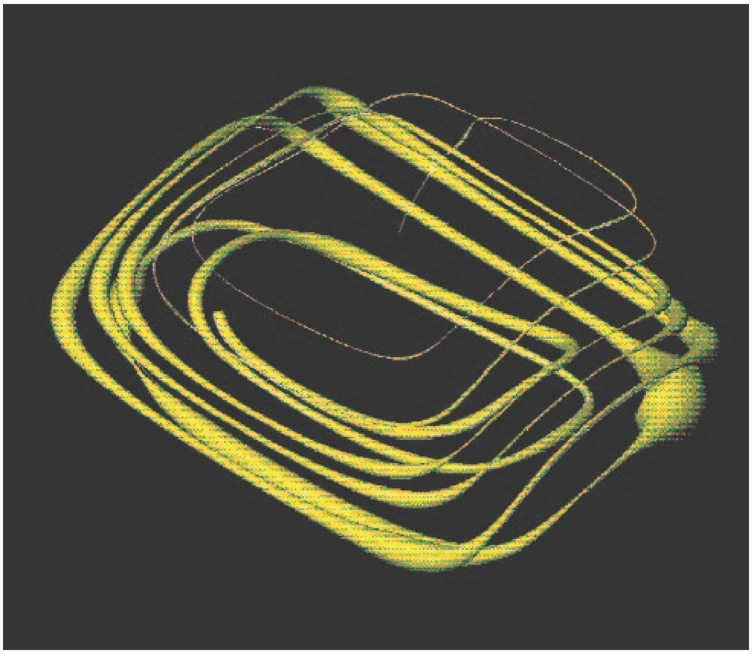
3 **Barbell glyphs showing the uncertainty between two numerical integration algorithms used for streamline calculations.**



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4 **Stream tube showing the uncertainty in a particle's path using different integration step sizes and tolerances.**

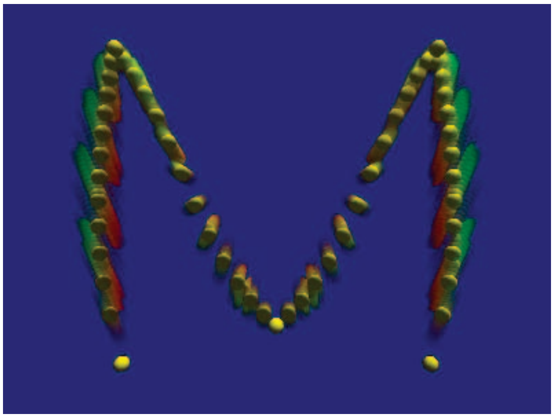
compare the different interpolation methods, Lodha et al. developed techniques for direct comparison of surfaces using a variety of geometric glyphs. Their glyphs can be 2D or 3D and might represent a wide variety of information. For instance, displacement glyphs, which are similar to error bars, can provide a good indication of the differences between surfaces, as illustrated in Figure 1.

Wittenbrink et. al also used glyphs for visualizing uncertainty in vector fields.[4] Their work concentrated on designing glyphs to convey the uncertainty in both orientation and magnitude. Figure 2 shows an example of their work. These types of glyphs work quite well with one exception: when the glyphs overlap the visualization becomes cluttered, making it difficult to understand. This is a common problem in many glyph-based 3D visualizations.

Lodha et al. combined their techniques to create UFlow, a method to view the uncertainty in fluid flow using different numerical integration algorithms and different time steps.[5] As in their work with vector glyphs, they developed several 3D glyphs that ranged from path envelopes and ribbons to batons and barbells to visualize fluid flow differences, as Figure 3 shows. Lopes and Brodlie also looked at this problem.[6] They used strips and tubes to visualize differences, as Figure 4 shows.

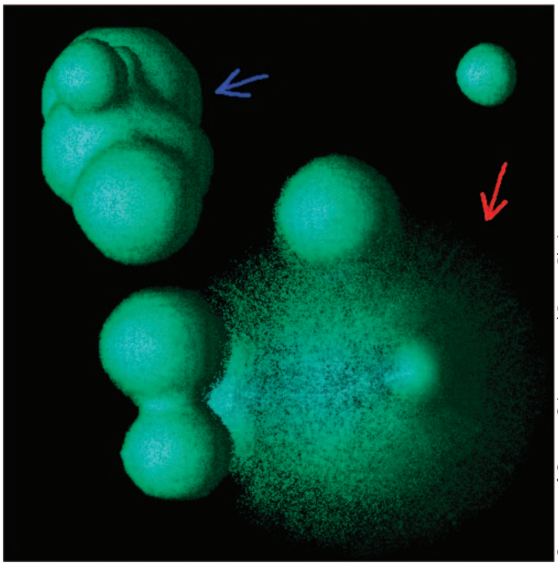
Pang et. al. summarized a variety of techniques suited for uncertainty visualization.[7] These techniques ranged from adding or modifying the model's geometry with, for example, a bump map or altered lighting attributes to using textures. Perhaps the most interesting technique they proposed was the use of blurring, as Figure 5 (next page) shows. Instead of blurring, Grigoryan et al.[8] used point-based primitives to create a fuzzy surface that achieved similar results, as Figure 6 shows. Blurring is a natural cue to the eye that something is amiss. We can easily apply this technique to a variety of different visualization techniques from particle tracing to isosurfacing.

We have found that techniques such as blurring provide excellent tools for visualizing uncertainty because



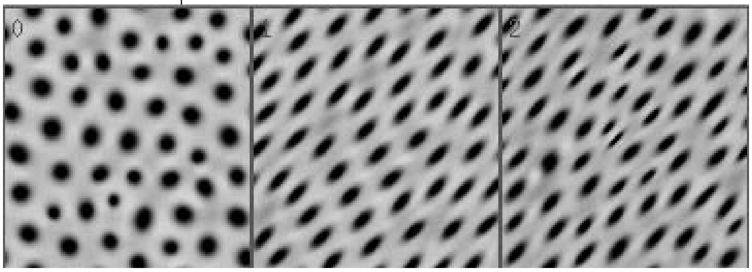
Courtesy of Alex Pang and Springer Verlag

5 **Particle tracing using blurring to show the uncertainty in the path.**



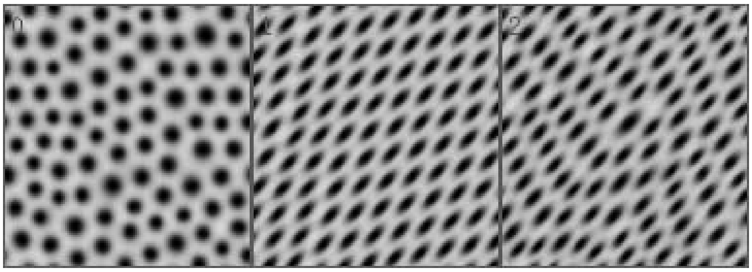
Courtesy of Gevorg Grigoryan and Penny Rheingans

6 **Point-based primitives used to create a fuzzy surface to show uncertainty.**



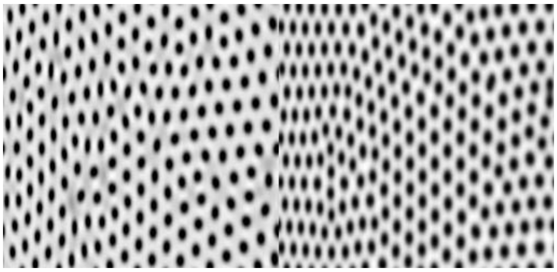
(a) **(b)** **(c)**

7 **Turing model visualization of a vector field with (a) random magnitude, (b) constant orientation, and (c) magnitude and orientation.**



(a) **(b)** **(c)**

8 **Gray–Scott model visualization of a vector field with (a) random magnitude, (b) constant orientation, and (c) magnitude and orientation.**



(a) **(b)**

9 **(a) Turing and (b) Gray–Scott model visualizations of orientation uncertainty. The orientation uncertainty increases from left to right across each image.**

users intuitively associate such visual representations with uncertainty. We are currently researching similar approaches using patterns formed with reaction diffusion systems.[9] The brain naturally follows a spatio-temporal pattern and can easily perceive subtle changes. For example, we can create a pattern of elliptical spots based on a mapping of a vector's orientation and magnitude and the diffusion matrix in a reaction-diffusion system. This matrix—which is anisotropic—exists for each vector in the flow field.

Figures 7 and 8 show the anisotropic diffusion applied to the Turing[10] and Gray–Scott[11,12] reaction-diffusion models for a vector field at 45 degrees with a random variation in the magnitude.

Uncertainty measurements

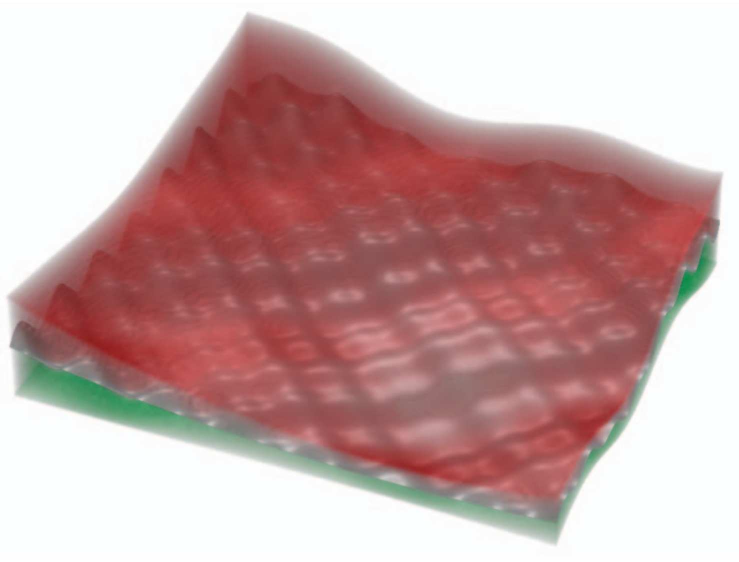
In our previous examples, we fixed the amount of anisotropy in the diffusion matrix. However, by allowing the amount of anisotropy to vary, we produce another variable that we can map. When the amount of anisotropy is small, the spot formed is almost circular, with the ratio of the semi-axes at approximately one. However, when the anisotropy is high, the spot formed is elliptical, deforming at times in such an extreme manner that it almost becomes a thick line. For example, the ratio of the semi-axes could be much greater than one. This creates a visual difference well suited to mapping an orientation uncertainty. When the orientation uncertainty is small, the spot is elliptical, reflecting a precise orientation. When the uncertainty is high, the spot is more circular, reflecting the uncertainty in the orientation. Figure 9 demonstrates this change.

Where to go from here

A primary goal of effective visualization is to provide a complete and accurate visual representation of data and models for users to interrupt. A complete visual representation would include representations of error and uncertainty in addition to standard scientific visualization techniques. Certainly other important criteria exist for an effective visualization, including the often-overlooked perceptual issues regarding the user's psycho-physical ability to effectively understand the images. The Visualization Viewpoints article in the July/August 2003 issue of *IEEE Computer Graphics and Applications* addresses the important subject of user studies as a method to measure a visualization's performance and effectiveness.

We see the need to create a formal, theoretical error and uncertainty visualization framework and to investigate and explore new visual representations for characterizing error and uncertainty. Furthermore, a formal evaluation (testing with user studies) of the visual techniques for comparing experimental and simulation 2D and 3D data while incorporating statistical, numerical, and/or measurement errors is necessary. Such new techniques could include

- the ability to overlay and compare 2D and 3D visualizations and uncertainties (automating the view-graph norm);
- use of new physically based glyphs such as those built around a reaction-diffusion type model;
- modification to data and/or visualization attributes—for example, using bump mapping;
- improvement to psycho-visual metaphors, such as highlighting an area;
- better use of annotation and interactive information overloading;
- better visual representation of, and interaction with, statistical data; and
- use of information visualization methods applied to 3D scientific visualization data.



10 **Isosurface of a synthetic data set (in gray) bracketed with volume-rendered regions (red and green) indicating uncertainty around the isovalue.**

11 **Isosurface of an magnetic resonance imaging data set (in orange) surrounded by a volume-rendered region of low opacity (in green) to indicate uncertainty in surface position.**

One simple example of error and uncertainty visualization techniques that we are investigating involves combining isosurface methods with volume rendering methods. For example, we can represent the average value of a scalar field with an isosurface and then represent the error or uncertainty of the scalar field using volume rendering, as Figures 10 and 11 show.

Conclusions

The development of formal theoretical frameworks (in a similar approach to that of the scientific computing area (see http://www.siam.org/journals/sisc/sp_issue.htm) and the creation of new visual representations of error and uncertainty will be fundamental to a better understanding of 3D experimental and simulation data. Such improved understanding will validate new theoretical models, enable better understanding of data, and facilitate better decision making. We urge the scientific visualization research community to take the next step and make visually representing errors and uncertainties the norm rather than the exception. ■

Acknowledgments

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References

1. B.N. Taylor and C.E. Kuyatt, *Guidelines for Expressing and Evaluating the Uncertainty of NIST Experimental Results*, NIST tech. note 1297, 1994.
2. J.D. Woods and P.F. Fisher, “Assessing Interpolation Accuracy in Elevation Models,” *IEEE Computer Graphics and Applications*, vol. 13, no. 3, May 1993, pp. 48-56.
3. S. Lodha et al., “Visualizing Geometric Uncertainty of Surface Interpolants,” *Proc. Graphics Interfaces*, Canadian Human–Computer Communications Society, 1996, pp. 238-245.
4. C.M. Wittenbrink, A. Pang, and S.K. Lodha, “Glyphs for Visualizing Uncertainty in Vector Fields,” *IEEE Trans. Visualization and Computer Graphics*, vol. 2, no. 3, Sept. 1996, pp. 266-279.
5. S.K. Lodha, A. Pang, and R.E. Sheehan, “UFLOW: Visualizing Uncertainty in Fluid Flow,” *Proc. IEEE Visualization*, IEEE CS Press, 1996, pp. 249-254.
6. A. Lopes and K. Brodlie, “Accuracy in 3D Particle Tracing,” *Mathematical Visualization: Algorithms, Applications and Numerics*, HC Hege and K. Polthier, eds., Springer Verlag, 1998, pp. 329-341.
7. A.T. Pang, C.M. Wittenbrink, and S.K. Lodha, “Approaches to Uncertainty Visualization,” *The Visual Computer*, vol. 13, no. 8, Nov., 1997, pp. 370-390.
8. G. Grigoryan and P. Rheingans, “Probabilistic Surfaces: Point Based Primitives to Show Surface Uncertainty,” *Proc. IEEE Visualization*, IEEE CS Press, 2002, pp. 147-153.
9. A.R. Sanderson and C.R. Johnson, *Display of Vector Fields Using a Reaction-Diffusion Model*, tech. report UUSCI2003-002, SCI Inst., 2003.
10. A.M. Turing, “The Chemical Basis of Morphogenesis,” *Philosophical Trans. Royal Soc. London*, vol. B237, 1952, pp. 37-72.
11. P. Gray and S.K. Scott, “Sustained Oscillations and Other Exotic Patterns of Behaviour in Isothermal Reactions,” *J. Physical Chemistry*, vol. 59, 1985, pp. 22-32.
12. P. Gray and S.K. Scott, *Chemical Oscillations and Instability: Nonlinear Chemical Kinetics*, Oxford Univ. Press, 1990.

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