The Effect of Non-Conformal Finite Element Boundaries on Electrical Monodomain and Bidomain Simulations

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Abstract

The simulation of electrical activity in the heart, such as normal and abnormal ventricular rhythms and ischemia, utilize computational methods that rely on an underlying geometric model, or polygonal mesh of cardiac tissues and boundaries. Because of the complex shape of many biological structures, it is often difficult to create meshes that conform to the boundaries between distinct regions. The resulting meshes can be non-conformal, i.e., they have element faces that do not align with the surface tangents and the elements represent a smooth surface as a jagged boundary. We hypothesize that these jagged, non-conformal meshes produce local concentrations of current that lead to artifacts large enough to distort the resulting potential fields and generate misleading results. In simulations of acute ischemia, these artifacts can alter the location and severity of the epicardial elevations and depressions, which, in turn, can impact clinical diagnosis. In the case of defibrillation, these artifacts can distort the current density computed through thin structures such as the myocardial wall.

1. Introduction

Biomedical computing has seen a rapid growth in the use of image-based models and simulations [1]. One of the more flexible and appropriate choices for many of these simulations is the finite element method (FEM) which requires a geometric representation of the different materials or tissues in the model using polygonal elements. Due to the complex shape of many biological structures, it is often difficult to accurately represent the boundaries that delineate the interface between tissue types. Some tissues share part of their boundaries with multiple other tissues forming what is know as non-manifold surfaces. The non-manifold nature of most biological anatomy is particularly challenging when constructing a polygonal representation of the geometry, often referred to as meshing.

Because these geometries present numerous challenges for meshing algorithms, many meshes do not enforce a hard boundary, in that the elements do not necessarily conform to the boundary they represent, or non-conformal meshes. As seen in Figure 1A , the element faces do not align with surface tangents. The mesh is generated in a arbitrary fashion and then the material is assigned to each element based on their centroid location or other similar methods.

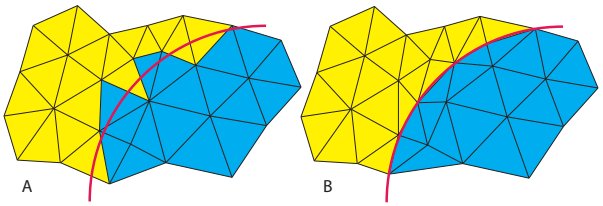

Figure 1. A: Illustrates how a boundary is defined in a non-conformal mesh. B: Illustrates how a boundary is defined in a conformal mesh.

There are a few freely available software packages that generate conformal meshes from segmented image data, BioMesh3D [2], DelPSC [3], and CGAL [4]. However, the advantages of creating conformal meshes with these packages are overshadowed by the greatly increased complexity and computation costs where mesh construction take hours rather than minutes.

The compromise of creating non-conformal meshes to save time and computational resources may or may not have a significant impact on the final result to the simulations. The impact of the meshing type is highly problem specific. This study was primarily interested in modeling electric fields using both a monodomain and bidomain. As such, we have selected a defibrillation simulation and an ischemia simulation to model the monodomain and bidomain respectively. Though the results are not limited to just ischemia and defibrillation, they could extend to most simulations that are based on Laplace's equation.

The primary concern is that non-conformal meshes represent smooth surfaces as jagged boundaries at the inter-

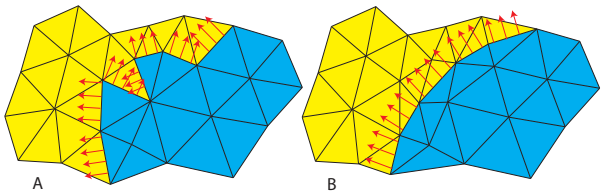

Figure 2. A: Is a representation of how currents may cause voltage elevations and depressions. B: Shows how a smooth surface creates a much more uniform current flow.

faces between conduction changes. This artifact leads to questions of whether or not the jagged surfaces produce concentrations of currents that produce artificial local maximum and minimum potentials as depicted in Figure 3.

2. Methods

2.1. Monodomain

Two models were used to simulate a defibrillation shock that passes through multiple material types. The first was an abstract model of a torso, with various materials of different shapes randomly placed throughout a cylinder. An implantable cardioverter-defibrillator (ICD) wire and can were placed in the model and then conformal and non-conformal meshes were generated. The ICD wire was set to 0 V and the ICD can to 400 V as boundary conditions for Equation 1. Additionally, each unique tissue or shape was assigned a different physiologically realistic conductivity value, σ, to model isotropic conduction. The monodomain was solved in the SCIRun [5] problem solving environment. The non-conformal mesh was then refined in regions of large gradients and solved again to produce more accurate results.

$$\nabla \cdot \sigma \nabla \Phi = 0 \tag{1}$$

The second model was created from MRI data of a torso and a model of a ICD wire and can. A defibrillation model was constructed as done by Triedman et al. [6] including the same conductivity values as used in these previous experiments. However, instead of a structured hexahedral mesh, a conformal tetrahedral mesh was generated using BioMesh3d, as well as a non-conformal tetrahedral mesh. The non-conformal mesh was refined around the ICD wire and can electrodes as is commonly done in defibrillation simulations due to the large voltage gradients in this region. The simulations were used to predict the minimum shock needed to defibrillate the heart based on the critical mass hypothesis [7].

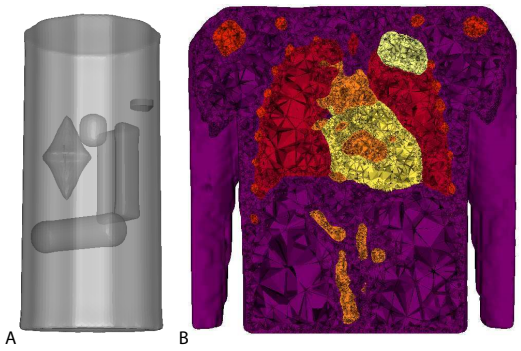

Figure 3. A: Shows the boundaries of an abstract model that was used to study defibrillation. B: Shows the conformal mesh of a realistic torso model to study defibrillation with each color representing a different tissue type.

2.2. Bidomain

A model of acute ischemia was used to evaluate the effects of non-conformal meshing of the Bidomain simulations. Acute ischemia was modeled using Equation 2 as described in previous studies [8].

$$\nabla \cdot (\sigma_i + \sigma_e) \nabla \Phi_e = -\nabla \cdot \sigma_i \nabla \Phi_m \tag{2}$$

Where Φ_e and Φ_m are the extracellular and transmembrane potentials, and σ_e and σ_i are the extracellular and intracellular conductivities. The ischemic region was modeled as 30 mV transmembrane potential difference from health tissue of the heart. A simplified spheroid ischemic zone was modeled both conformally and non-conformally in a realistic heart geometry. A more realistic ischemic zone, derived from experimental measurements, was also used in a similar fashion.

3. Results

3.1. Monodomain

The cylinder defibrillation model before mesh refinement had global error greater than 200%. The mesh was then refined until the global error averaged less than 5%. Locally, at interface boundaries, the mean error was relatively small, less than 5%, while the max error close to the surface was much larger ranging from 10 to 15% as seen in Figure 4.

The realistic torso defibrillation model showed similar results to that of the cylindrical model as shown in Figure 5. When the model was used to predict the minimum shock needed to produce a gradient of 5 V/cm across 95% of the heart [7], the non-conformal mesh predicted a result 9% higher than the conformal mesh.

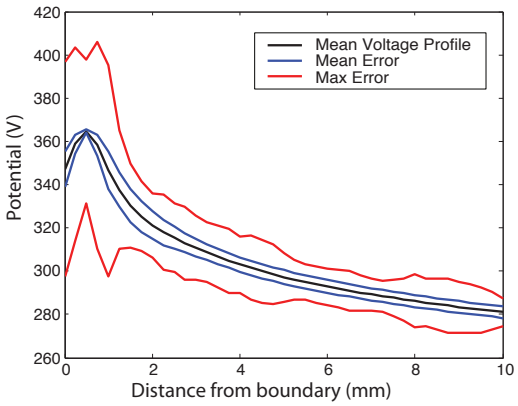

Figure 4. The error or difference from the high resolution conformal mesh as a function of distance from one of the non-conformal surfaces.

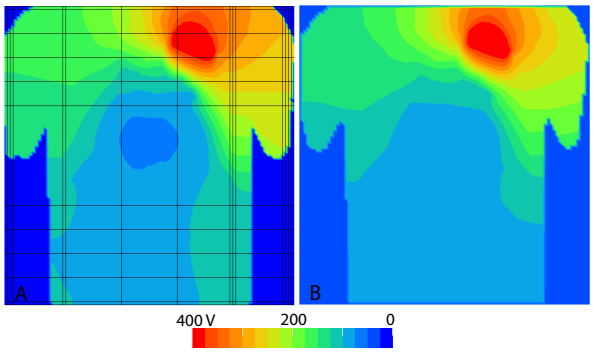

Figure 5. A slice through the center of the two defibrillation results where A is the non-confrmal and B is the conformal.

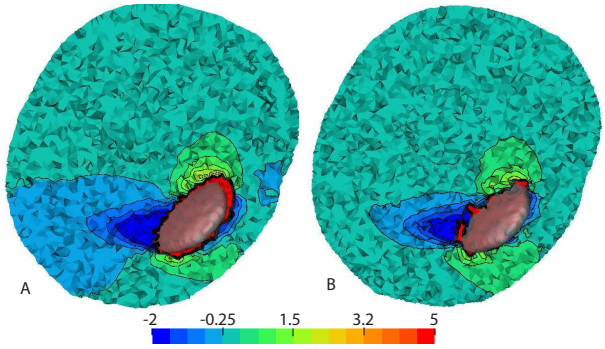

Figure 6. A: The voltage profile of a conformal mesh of an ischemic zone in a bidomain simulation. B: The voltage profile of a non-conformal mesh of an ischemic zone in a bidomain simulation.

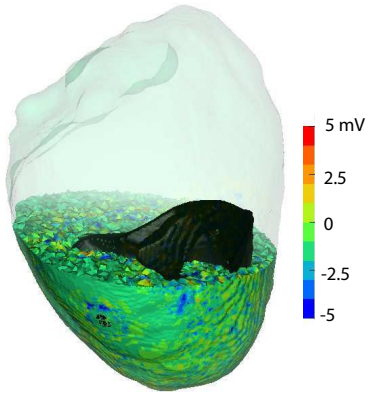

Figure 7. This image shows the difference or what is believed to be error between the results of a bidomain simulation using a non-conformal mesh and a high resolution conformal mesh to simulate acute ischemia.

3.2. Bidomain

The spheroid ischemia model produced gradients near the voltage sources significantly different between the conformal mesh and the non-conformal mesh. At a distance less than 2.5 mm or within three elements, there were differences in gradients as large as a mean of 238%. Further away from the surface, greater than 5 elements or about 5 mm, the difference fell to around 120%. The non-conformal meshes had gradient distributions that were heterogeneous and irregular (Figure 6B), while the conformal meshes showed smoother more regular distributions of gradients along the surface (Figure 6A).

Similar numbers were reported for a more realistic representation of an ischemic zone where close to the ischemic region there were non-homogeneous voltage distributions that produced artificial maxima and minima. In this case the boundary of the ischemic zone was close enough to the epicardial surface to produce errors in locations that may be physiologically important and potentially confound studies looking the formation of maxima and minima on the epicardial surface due to the physiological consequences of ischemia. See Figure 7.

4. Discussion and conclusions

It was apparent from our simulations that the bidomain and the monodomain exhibited the same types of behaviors and can be considered together. The results can be divide into two types of errors: local and global. To determine the error of each simulation, we compare them to the results from high resolution conformal meshes for a surrogate of an exact solution that does not exist.

The global error was particularly evident in the non-torso defibrillation simulation where the non-conformal mesh showed over 200% error before including the mesh refinement around the stimulating electrodes. When mesh refinement was included around the electrodes, the global

error was decreased to a difference of less than 5%, though the surface was only slightly smoother. The biggest difference was that the total volume in the refined mesh of the stimulating electrode was much closer to the total volume of the conformal stimulating electrode. If we used a different approach from refining the mesh, and dilated or constricted the region representing the stimulating electrode until the total volume was similar to that of the conformal mesh, then the total global error was less than 5%. This indicates that the total volume of the sources and sinks are more important in to the global error than the jagged surfaces produced by non-confoming elements.

While in many cases it may be sufficient to refine the mesh around the sources and sinks, it is uncertain how much refinement is necessary to produce the correct volume such that the global error is minimized. Conformal meshing of these regions reduces these uncertainties by accurately representing the total volume of all surfaces defined from the image data. However, areas with large gradients, such as those found next to sources and sinks, often require mesh refinement to accurately model the rapidly changing voltage profiles. Conformal meshing does not directly address the issue of large gradients near the sources and sinks and may still require mesh refinement.

Local errors appear to be a function of a poor representation of the tissue interface, i.e. jagged surfaces. The irregularity of the voltage profiles along the non-conformal meshes of the ischemic regions were in stark contrast to the much more uniform and smooth profiles produced by the conformal meshes seen in Figure 6. The irregular profiles in the ischemia studies showed local minimum and maximum which could produce physiologically significant misinformation. Depressions and elevations are used in ischemia studies to describe the severity and location of the ischemic tissue. Relative depressions are often seen flanking elevations that are centered over ischemic regions. However, in this simulation, the relative depression could just be an artifact from a non-conformal mesh near the surface of the heart. Mesh refinement slightly reduces the maximum error along the surfaces, but did not completely remove them. It is presumed that at some level of mesh refinement, the meshes would produce only minimal errors related to the the high resolution conformal mesh, but we were unable to determine the specific threshold for each of the simulations.

We found that non-conformal meshes have utility in many applications, but because of the difficulty in accurately representing the total volume of voltage sinks and sources and the local errors from jagged surfaces, there are many cases in which they are not appropriate. As expected from a Laplacian solution that essentially blurs the voltage sources, the further away from the non-conformal boundary, the more it resembled the voltage profile of the conformal mesh. The largest errors seemed to extend only a few mm, which represented about 3 element of thickness in our ischemia models. It is therefore reasonable to use non-conformal meshes when the area of interest is far from a non-conformal boundary. However, as in the cases of our realistic ischemia and torso models, there were boundaries very near the area of interest and it skewed our result by about 10%. While conformal meshes may be more difficult and time consuming to construct, these simulations show that they are necessary in a variety of electrophysiology settings and well worth the investment.

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