A Sliding Window Approach to Regularization in Electrocardiographic Imaging

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Abstract

Introduction: *The inverse problem of ECGI is ill-posed, so regularization must be applied to constrain the solution. Regularization is typically applied to each individual time point (instantaneous) or to the beat as a whole (global). These techniques often lead to over- or under-regularization. We aimed to develop an inverse formulation that strikes a balance between these two approaches that would realize the benefits of both by implementing a sliding-window regularization.* ***Methods:*** *We formulated sliding-window regularization using the boundary element method with Tikhonov 0 and* 2^{nd} *order regularization. We applied regularization to a varying time window of the body-surface potentials centered around each time sample. We compared reconstructed potentials from the sliding-window, instantaneous, and global regularization techniques to ground truth potentials for 10 heart beats paced from the ventricle in a large-animal model.* ***Results:*** *The sliding-window technique provided smoother transitions of regularization weights than instantaneous regularization while improving spatial correlation over global regularization.* ***Discussion:*** *Although the differences in regularization weights were nuanced, smoother transitions provided by the sliding-window regularization have the ability to eliminate discontinuities in potential seen with instantaneous regularization.*

1. Introduction

Electrocardiographic imaging (ECGI) is a technique used to noninvasively reconstruct parameters of an electrical cardiac source model from measured body-surface potentials that also requires detailed knowledge of torso geometry and the electrical conductivity of all the elements of the torso. [1] The resulting inverse problem is known to be ill-posed and requires regularization in order to constrain the solution within physiological values. [2] ECGI is similar to many reconstruction approaches in that it balances matching the products of a forward model against adhering to physiological constraints, a process known as regularization.

Tikhonov regularization is a common approach in ECGI and works by adding a weighted regularization weight and operator matrix to the least-square solution of the forward model. [2] This operator matrix represents some physiological penalty assumption about the solution, based on *e.g.,* amplitude (Tikhonov 0 order) or spatial smoothness (Tikhonov 2^{nd} order). Regularization can be applied instantaneously or globally, *i.e.,* at every time point in a solution independently, or across the entire signal simultaneously. [3,4] Instantaneous regularization can produce rapid changes in the regularization weight, which, in turn, causes discontinuities in the reconstructed signals. [3] Global regularization, by contrast, applies a single regularization weight to the whole solution, which ensures smoothness but can lead to over-regularization in areas with a high signal-to-noise (SNR) ratio, such as the QRS complex, and under-regularization in areas with a low SNR, such as the ST segment [4].

In this study, we aimed to develop an inverse formulation that allows for the regularization weight to change throughout the signal, while maintaining smoothness in the solution. To achieve this balance, we propose regularizing around a window at each time point in our reconstructed solution and refer to this as 'sliding window regularization'. To evaluate this approach, we tested it on 10 experimentally obtained beats following pacing from the anterior ventricle. We tested a variety of window sizes, and compared the resulting solutions to those obtained using the instantaneous and global regularization techniques.

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2. Methods

ECGI Formulation: Our regularization technique is based on a standard Tikhonov inverse formulation that assumed epicardial potentials as the source model. We can write the resulting minimization as

$$\underset{\mathbf{X}}{\operatorname{argmin}} \, || \, \mathbf{A}\,\mathbf{X} - \mathbf{B} \, ||_F^2 + \lambda || \, \mathbf{R}\,\mathbf{X} \, ||_F^2, \tag{1}$$

where the matrix $\mathbf{A}$ is a $M \times N$ transfer matrix between the epicardium and torso, M is the number of torso measurement points, and N is the number of epicardial measurement points. For this study, $\mathbf{A}$ was calculated using the boundary element method implemented in the open-source SCIRun software. [5] The $N \times T$ matrix $\mathbf{X}$ contains the epicardial potentials and the $M \times T$ matrix $\mathbf{B}$ the corresponding body surface potentials, where T is the number of time instances. The second term of Eq 1 shows how regularization is applied, where $\mathbf{R}$ is the $M \times N$ regularizing operator matrix and λ is the regularization weight.

How λ is chosen depends on the method of regularization. When using instantaneous regularization, there is a separate λ value for each time t. In this context, $\mathbf{B}$ and $\mathbf{X}$ are reduced to b (a $M \times 1$ vector) and x (a $N \times 1$ vector) and a separate inverse solution is computed for each time instant in the original recording. The resulting x vectors are then concatenated into the final $\mathbf{X}$ solution matrix. The global regularization technique uses a single λ value for the whole beat simultaneously, resulting in a single inverse solution matrix. The λ values are typically determined using either instantaneous or Frobenius L-curve methods. [4, 6]

Our sliding-window regularization approach selects a value for λ at each t using a window of time around that individual time instance. Applying the approach requires two steps for each time instant: 1) identify the λ value using a window of time, and 2) calculate an inverse solution using the resulting value of λ. For a given time instant t, we first replace $\mathbf{B}$ and $\mathbf{X}$ in Eq 1 with $\mathbf{B}^*$ (a $M \times W$ matrix) and $\mathbf{X}^*$ (a $N \times W$ matrix) where W is the window size. The values of $\mathbf{B}^*$ are selected as the time window of size W centered on the time instant t. The value of λ is then determined for this window using the Frobenius L-curve method. Once identified, the λ value for this time instant (λ_t) is used to calculate an inverse solution vector for x_n using the vector b_t. Once all x have been found, they are concatenated into the $\mathbf{X}$ solution matrix. The $\mathbf{B}$ matrix is padded by taking the first and last $(W-1)/2$ columns from $\mathbf{B}$, mirroring them, and then placing them at the beginning and end of $\mathbf{B}$.

Dataset: To test this regularization technique, we utilized 10 beats paced from the left ventricle recorded from a modified Langendorff preparation that has been described previously [7, 8]. Briefly, an isolated heart was surrounded by a 256-electrode pericardial cage ($N = 256$) and submerged in an electrolyte filled torso tank with 192 embedded electrodes ($M = 192$). The signals from the tank and cage electrodes were recorded simultaneously at a rate of 1 kHz using a custom recording system throughout a variety of interventions.[7] These recorded signals were filtered and processed as described previously.[9] The recordings from the pericardial cage were treated as the ground truth epicardial potentials in this study.

Analysis: We generated inverse solutions for each of the 10 beats with the sliding window regularization for window sizes of $W = 3, 9, 19, 29,$ and 39 milliseconds, as well as with instantaneous and global regularization using Tikhonov 0 (Tik 0) and Tikhonov 2^{nd} (Tik 2) order regularization. The largest window size (39 ms) was chosen so that when in the middle of the QRS complex, it would encompass the entire QRS complex. The accuracy of reconstruction from each technique was assessed using three metrics: 1) root mean squared error (RMSE), 2) spatial correlation (SC), and 3) temporal correlation (TC) [8]. We also compared the λ values selected for each method.

3. Results

Tikhonov 0: Figure 1 summarizes the results, from which the RMSE with Tik 0 regularization ranged marginally, from only 0.573 to 0.551 mV across all regularization techniques. SC showed decreasing median values as the window size increased. The median SC for the instantaneous and global regularizations were 0.79 and 0.74, respectively. The sliding window regularizations had medians of 0.787 for a 3-ms window, 0.789 for a 9-ms window, 0.785 for a 19-ms window, 0.776 for a 29-ms window, and 0.768 for a 39-ms window. Conversely, the temporal correlation increased as the window size increased. The median temporal correlation values for instantaneous, 3-ms, 9-ms, 19-ms, 29-ms, 39-ms and global regularization were 0.888, 0.888, 0.890, 0.893, 0.895, 0.896, and 0.904, respectively.

Tikhonov 2: The error metrics for Tik 2 regularization followed similar overall patterns but with reduced error and great variation over the different beats. RMSE ranged from 0.531 mV to 0.626 mV across all regularization techniques (Figure 1) with little variation in the median RMSE values across window sizes. The median SC followed a similar trend to the Tik 0, with spatial correlation decreasing as window size increased. However, the difference between the instantaneous regularization (median = 0.810) and the global regularization (median = 0.797) was less pronounced than with Tik 0 regularization. The median temporal correlation stayed approximately constant at 0.929 with varying window sizes.

Regularization Weights: The λ values and RMS values of a sample inverse solution are shown in Figure 2. Visual

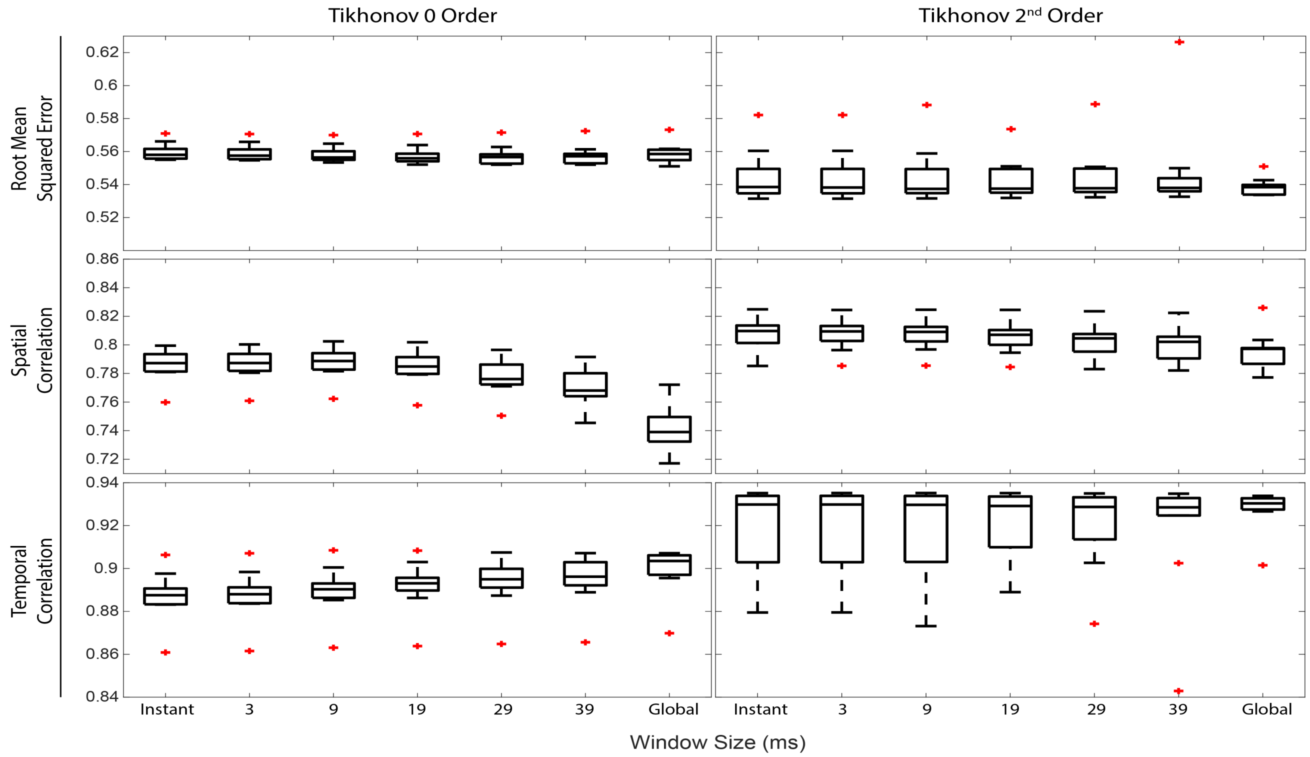

Figure 1. Reconstruction Statistics. Boxplots showing the RMSE (top row), spatial correlation, SC (middle row), and temporal correlation, TC (bottom row) are shown for both Tikhonov 0 (left column) and 2^{nd} (right column) order. The x-axis of each plot goes from instantaneous regularization, through increasing window sizes, to global regularization.

comparison of the regularization weights shows that the sliding window technique had much smoother transitions of weights across the signal. The instantaneous regularization technique showed large jumps at the beginning and end of the beat. With these rapid spikes, the regularization weight jumped from 2.5×10^{-4} to 6.2×10^{-3} for tik 0 and 230 to 5466 for tik 2. The global regularization had a relatively low λ value of 2.53×10^{-4} for Tik 0 and 229.3 for Tik 2. The RMS signals from the different reconstructions showed only small variations.

4. Discussion

In this study, we aimed to develop an inverse formulation that provides a compromise between locally optimal regularization and temporal continuity. We implemented a sliding window approach to Tikhonov regularization and tested it against global and instantaneous regularization approaches using 10 experimentally obtained paced beats from a torso tank experiment [7, 8]. We found that the statistical metrics (RMSE, SC, and TC) showed sliding window regularization to be a compromise between the instantaneous and global regularization techniques. The sliding window regularization solutions had an RMSE comparable to the instantaneous and global solutions. However, the spatial correlation decreased as the window size increased, with the windowed regularization values falling between those for instantaneous and global regularization. The temporal correlation increased as window sized increased, again with the windowed regularization values falling between those of instantaneous and global regularization. A window length of 19 ms marked a sweet spot that achieved acceptable temporal correlation while avoiding the sharp drop in spatial correlation that occurred with longer window duration.

The changes to the λ value throughout the beat were nuanced. Figure 2 shows that the sliding window regularization smoothed over the rapid increases seen in the instantaneous regularization, while maintaining a low regularization weight during the QRS complex. Based on this ability to smoothly change the regularization weight, the sliding window regularization technique offers a compromise between instantaneous and global regularization.

Our sliding window regularization technique had different effects with Tik 0 and Tik 2 regularization. The differences in spatial correlation were larger when increasing window size with Tik 0 regularization than with Tik 2 regularization. We theorize that this difference may be

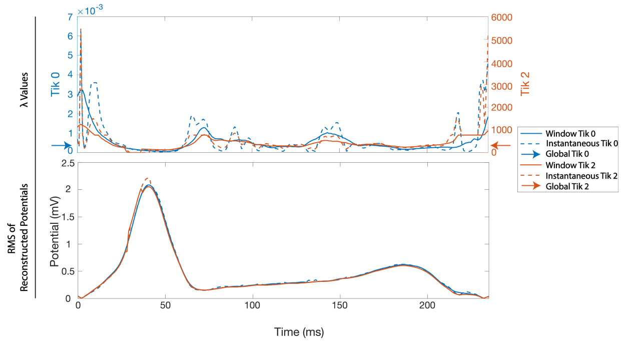

Figure 2. Regularization weights throughout an example beat. The regularization weights (λ values) are shown in the upper plot for the instantaneous and 19-ms sliding window regularization methods with Tikhonov 0 and 2^{nd} order regularization. The lower plot shows the root mean square (RMS) values of the solutions from each regularization technique.

due to the differing constraints applied by Tik 0 and Tik 2. The sliding window regularization applies a temporal constraint on the solution, albeit a simple one. Tik 0 regularization applies a general amplitude constraint, so the temporal constraints of the sliding window regularization and Tik 0 regularization may work together, leading to a more profound effect when compared to Tik 2 regularization. Previous studies that described temporal constraints in ECGI found these constraints often improved the reconstructed solution [3, 10, 11]. In the future, we hope to apply our sliding window technique along with other spatial and temporal regularization techniques to identify possible causes for this discrepancy.

Overall, the sliding window regularization technique provided a middle ground to instantaneous and global regularization. Although the differences were nuanced, it did show smoother transitions of regularization weight across the signal. The sliding window regularization also appeared to have a larger effect on Tik 0 than Tik 2, providing an avenue for future research to investigate its effects on models that use spatial or temporal constraints.

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