

Statistics of Longitudinal Shape Data

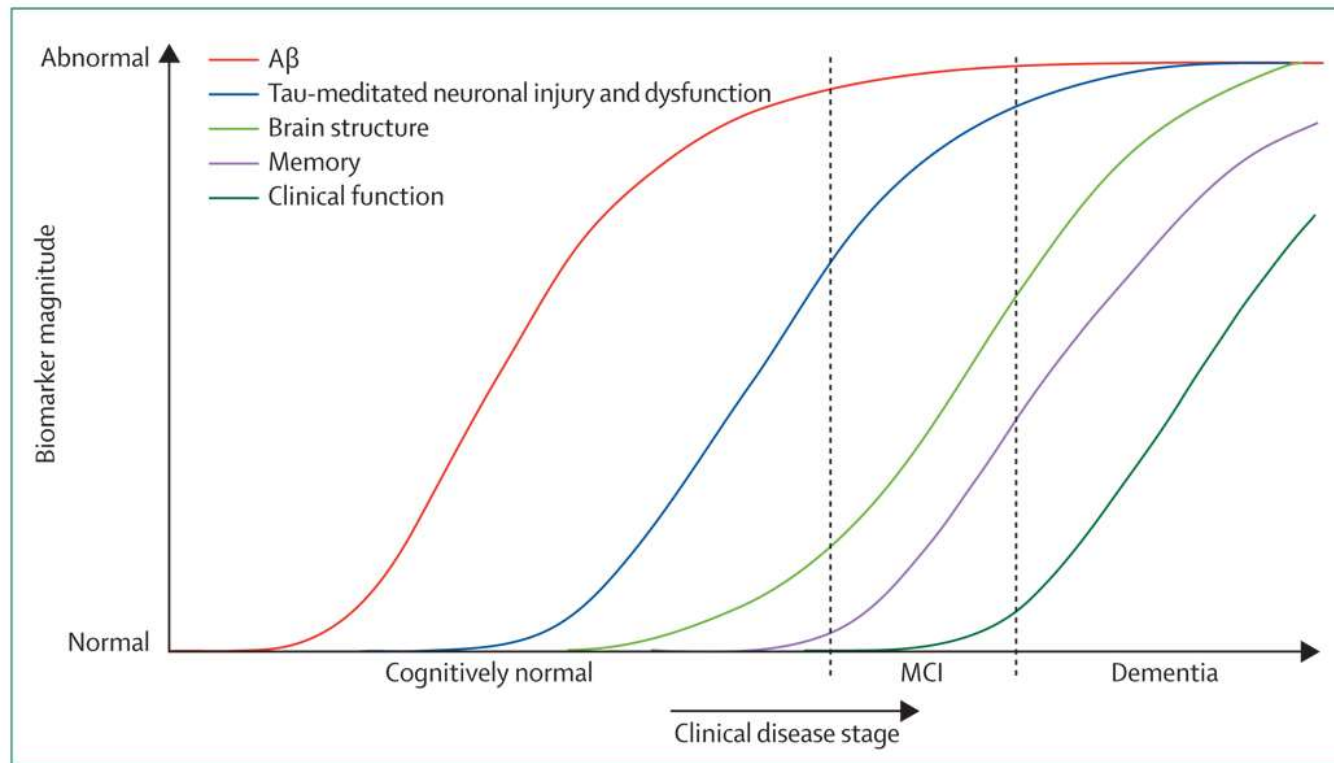
Stanley Durrleman

Institut du Cerveau et de la Moëlle (ICM)

Pitié Salpêtrière Hospital, Paris



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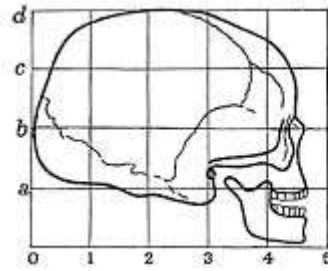


- Need to study the dynamics of anatomical alterations for:
 - To monitor disease progression
 - Detect subjects at risk
 - Classify subjects according to patterns of anatomical alterations
- Challenges:
 - Shape data: image, 3D surface meshes, point sets, etc.
 - Infer dynamics from few time points
 - Average inter-subject differences → normative scenario

Jack et al. Lancet Neurol'10

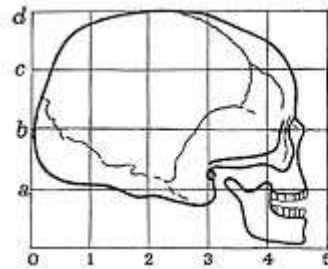
Forms and Deformations

Forms and deformation



Complex *differences in shape can be described by simple space deformation*

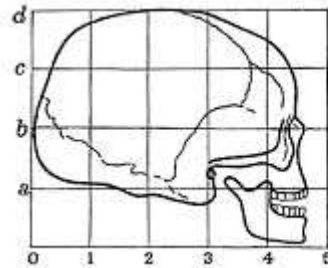
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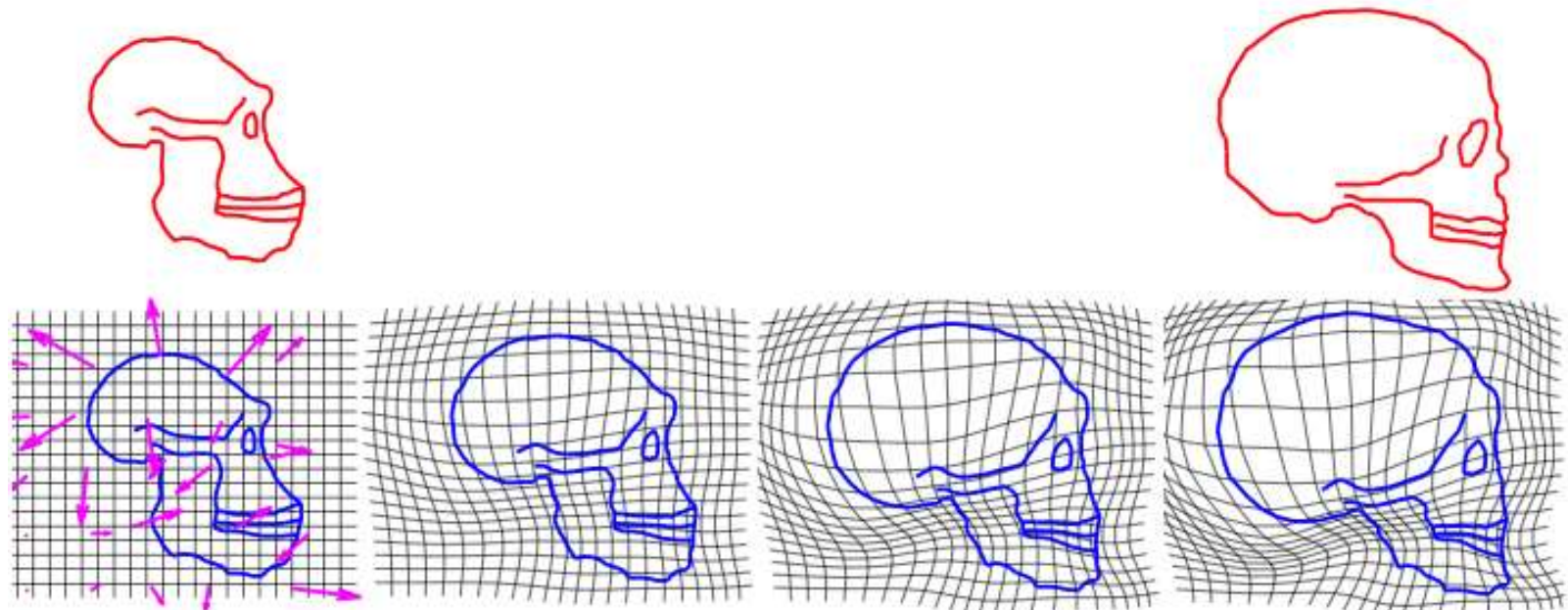
Grenander (1993), Miller, Younes, Trouvé, etc.: the emergence of the field of Computational Anatomy

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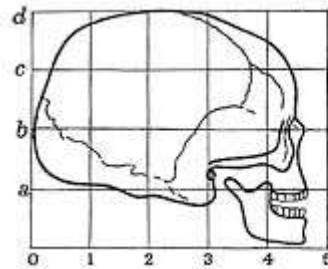
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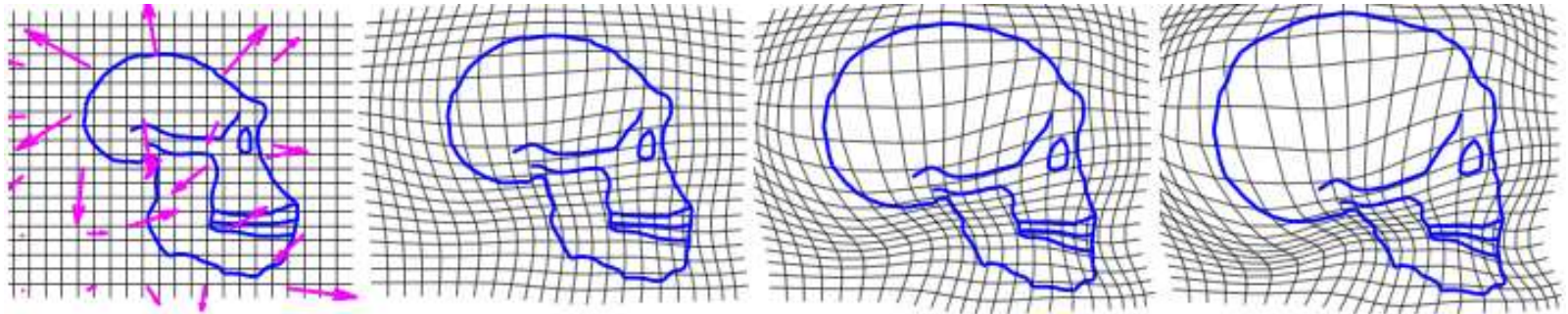
Durrleman et al. MedIA'09

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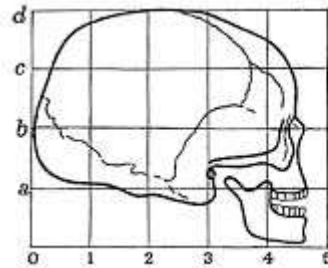


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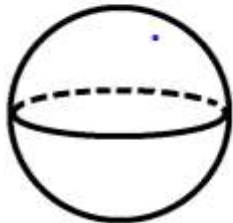
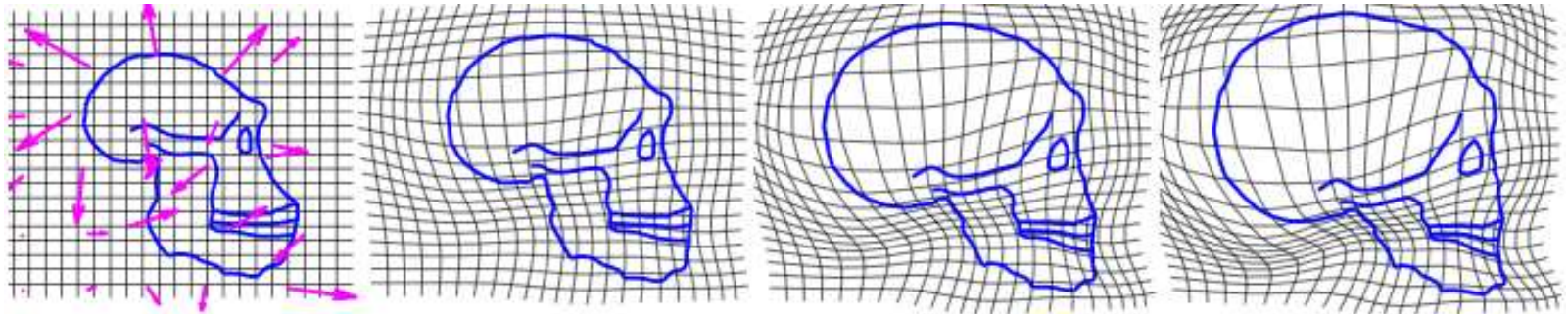


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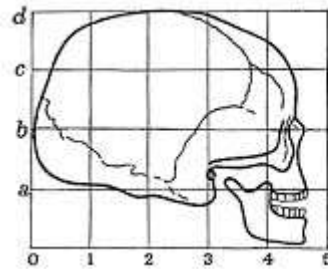


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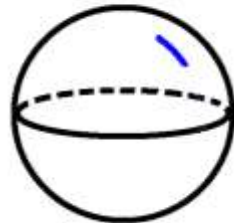
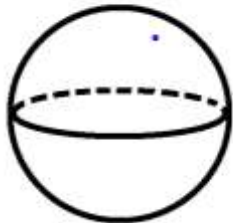
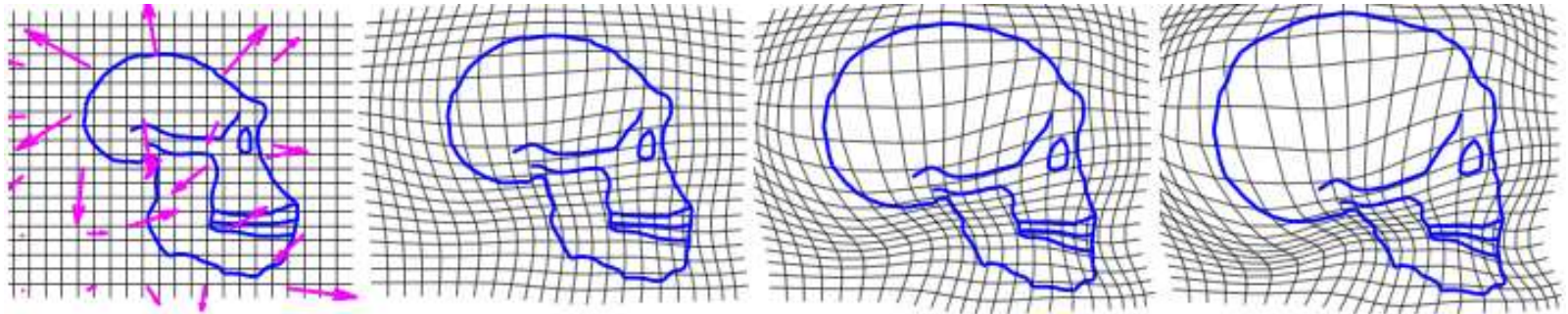


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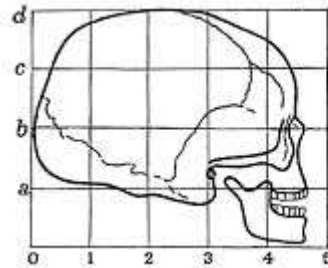


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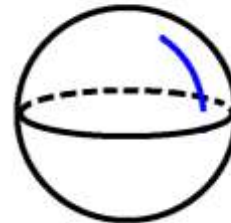
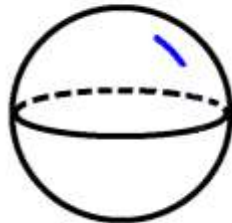
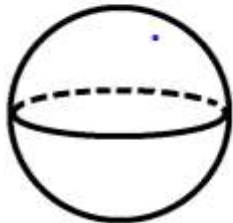
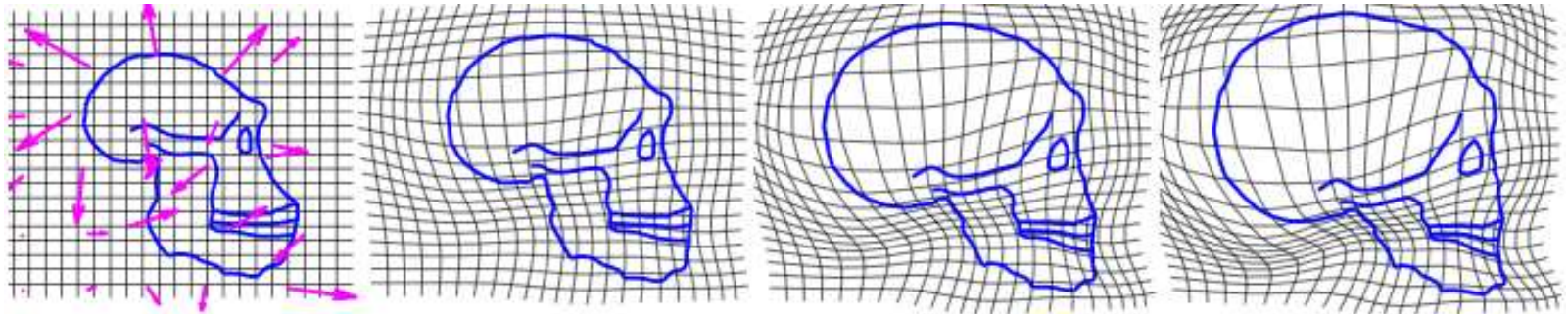


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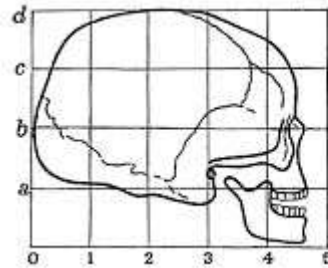


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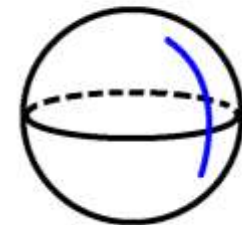
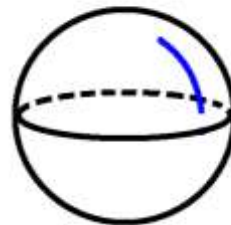
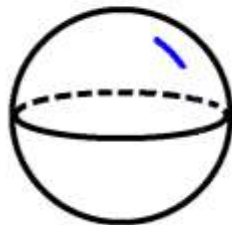
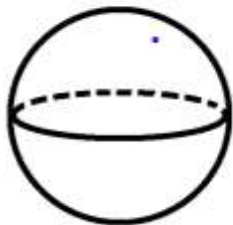
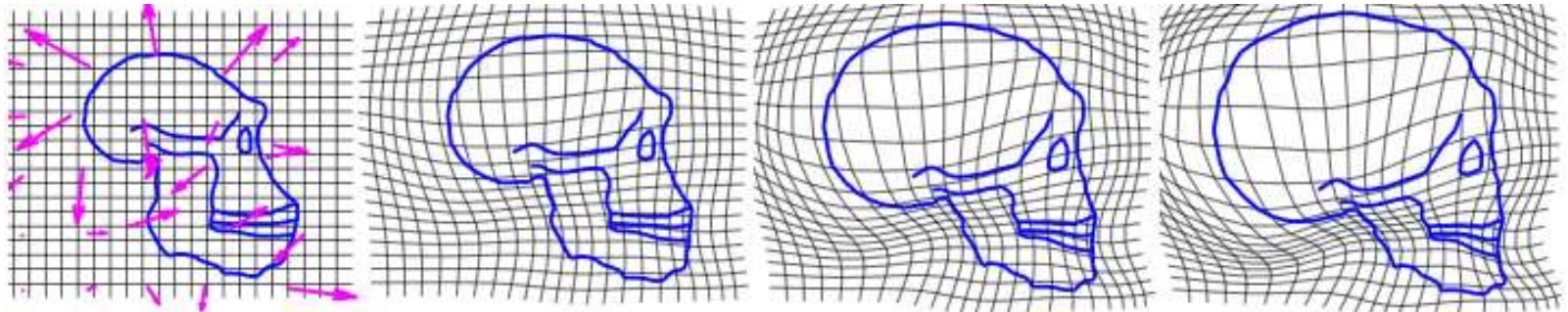


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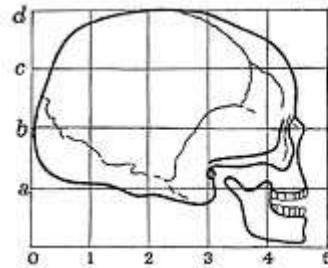


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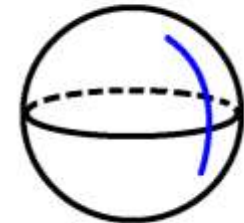
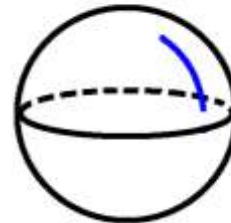
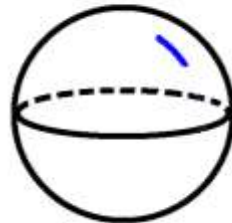
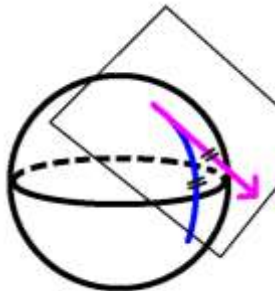
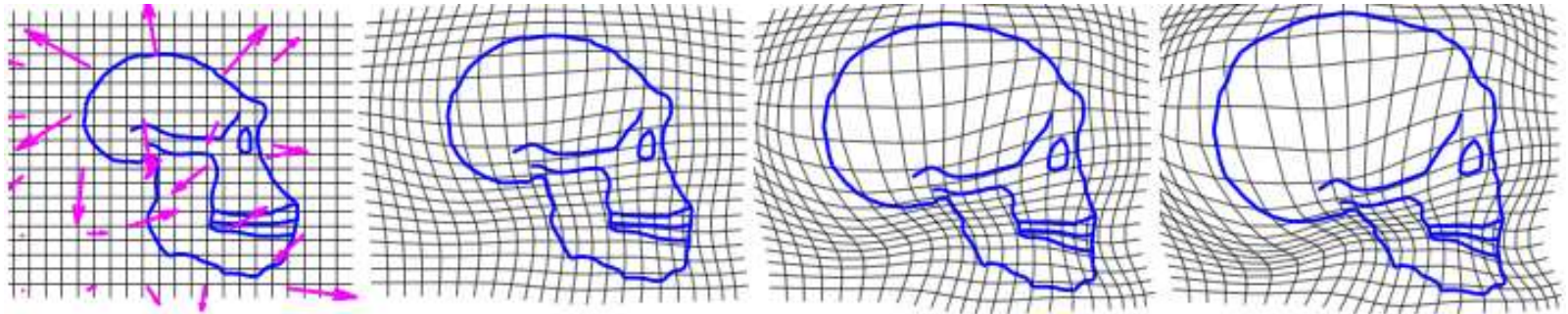


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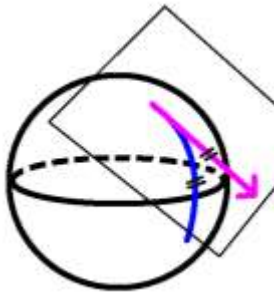
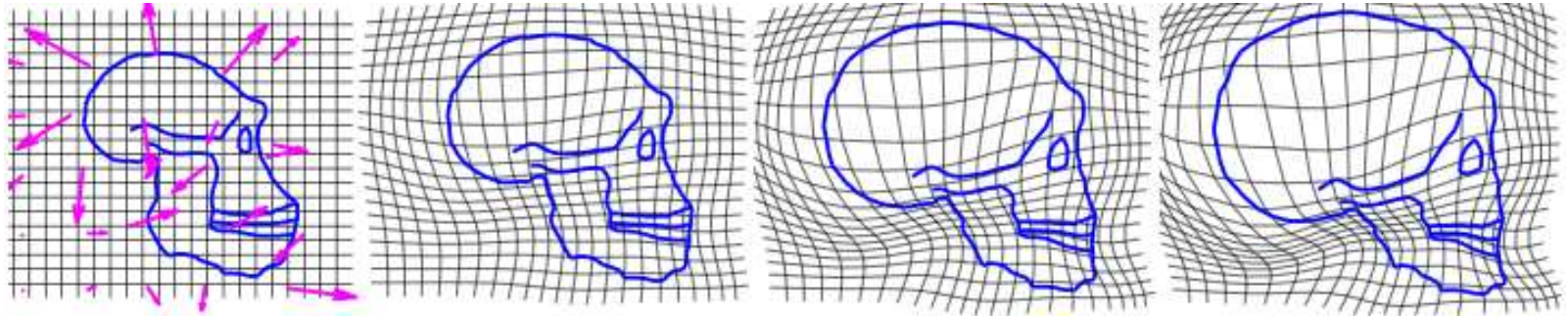


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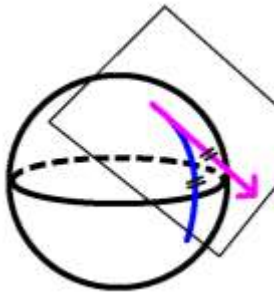
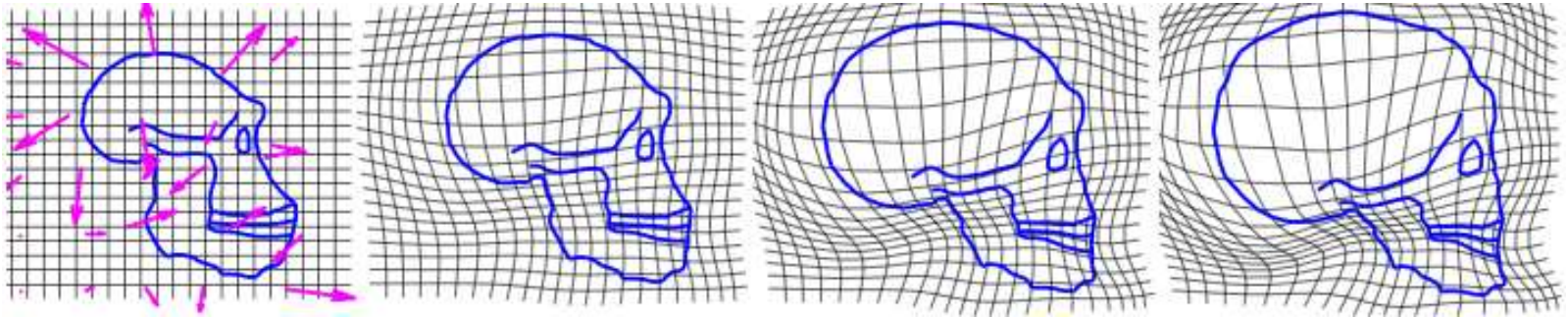
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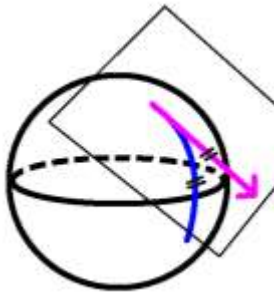
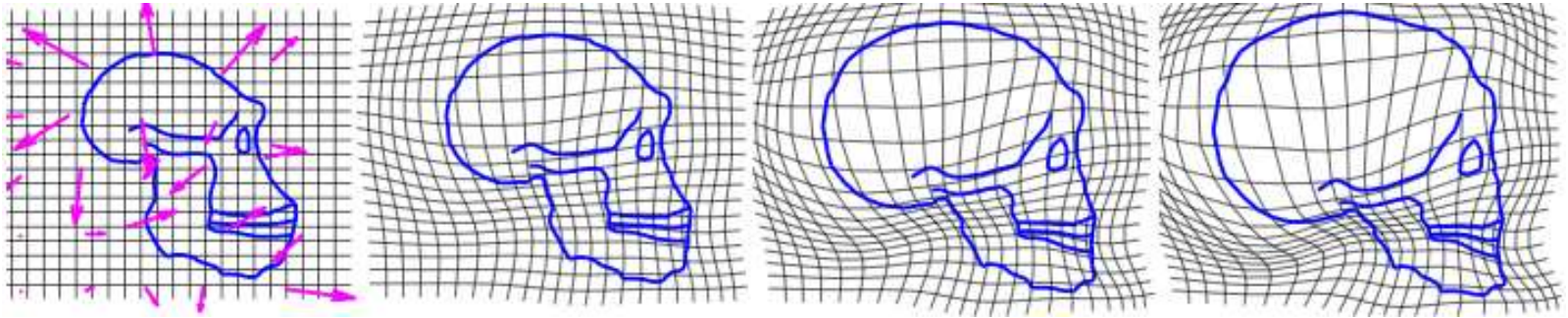
Forms and deformation



$$\frac{\partial \phi(t, x)}{\partial t} = v(t, \phi(t, x)) \Rightarrow \dot{X}(t) = v(t, X(t))$$

$$\phi(0, x) = x$$

Forms and deformation

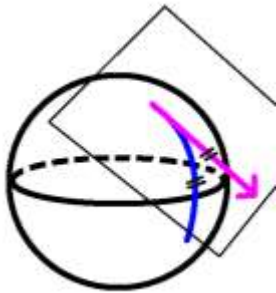
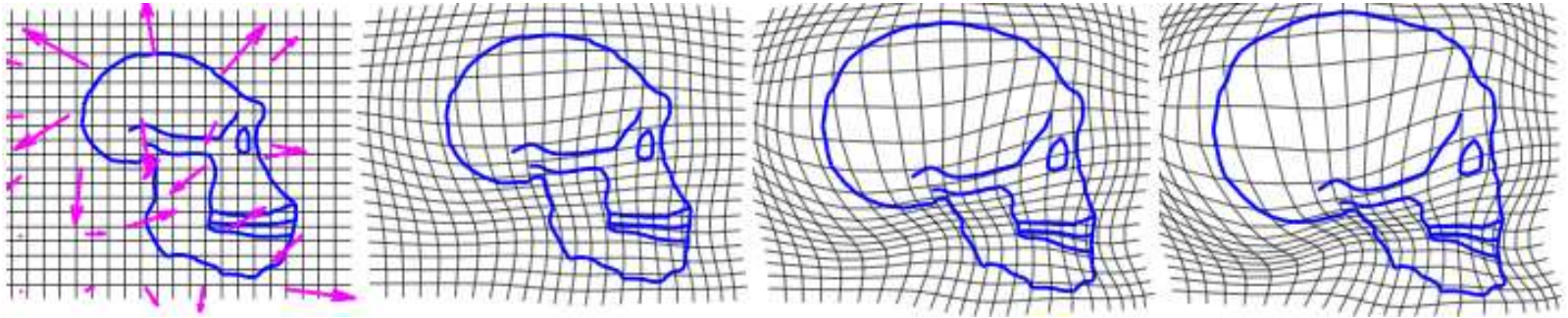


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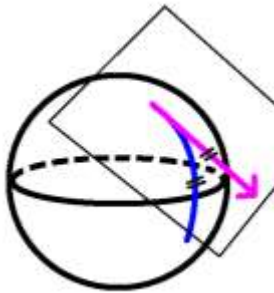
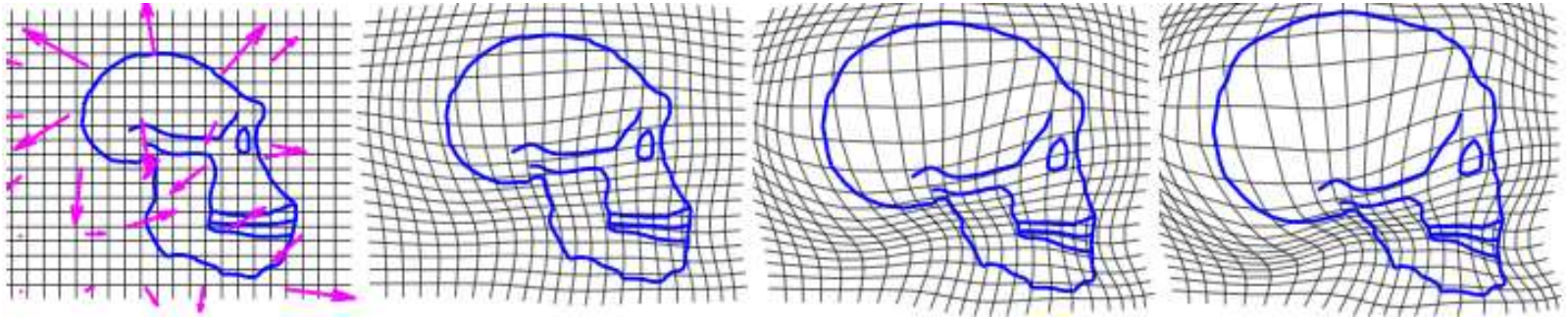
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$$\text{Extremal paths of } H = \int_0^1 |v(t, \cdot)|_K^2 dt$$

$$\begin{cases} \dot{c}_k(t) = v(t, c_k(t)) = \sum_p K(c_k(t), c_p(t)) \alpha_p(t) \\ \dot{\alpha}_k(t) = - \sum_p \alpha_k(t)^T \alpha_p(t) \nabla_1 K(c_k(t), c_p(t)) \end{cases}$$

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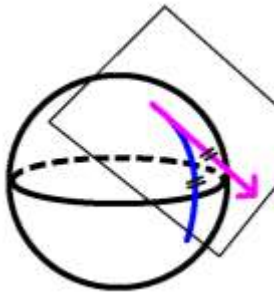
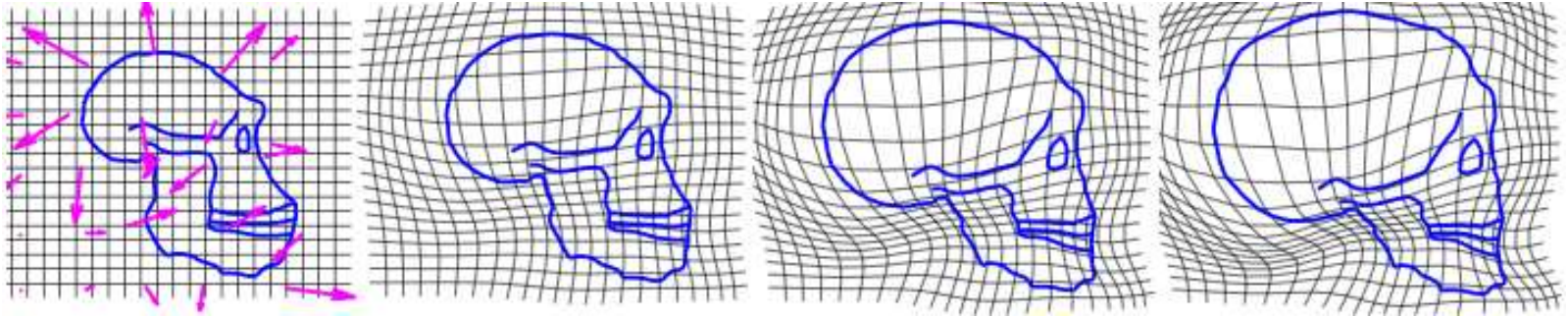
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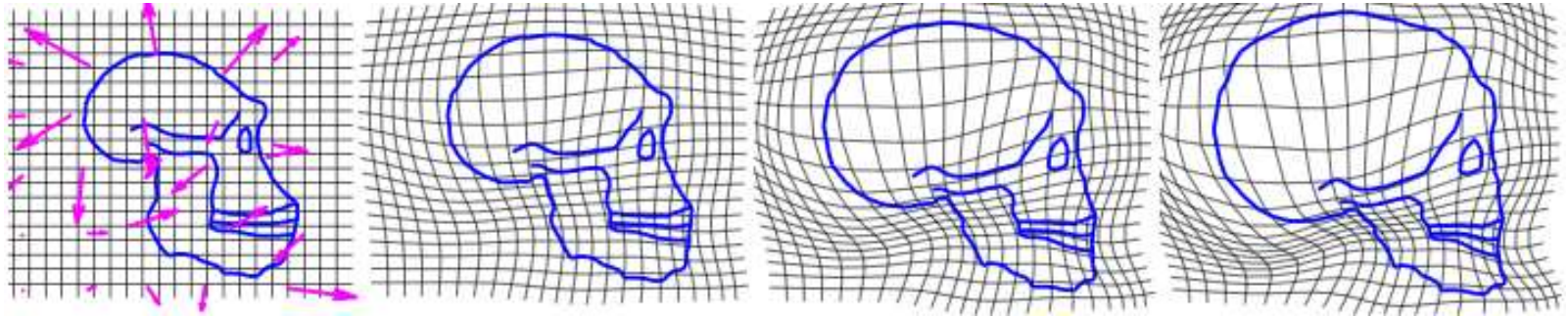
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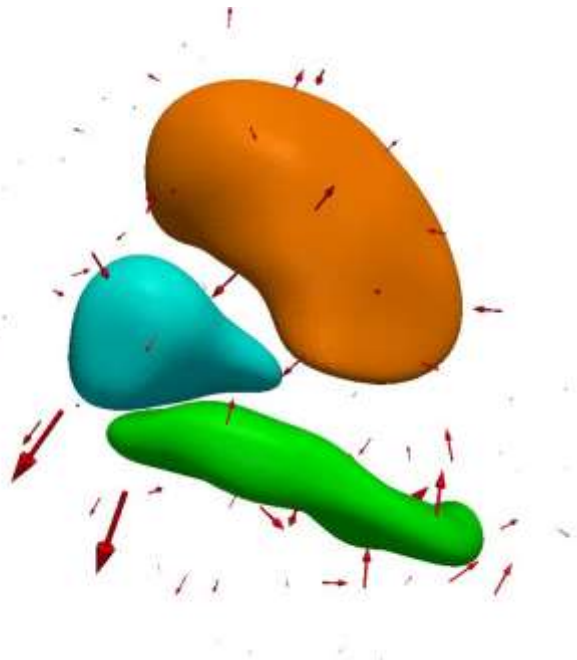
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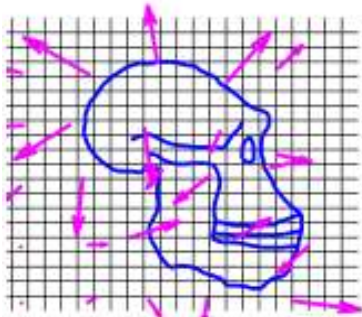
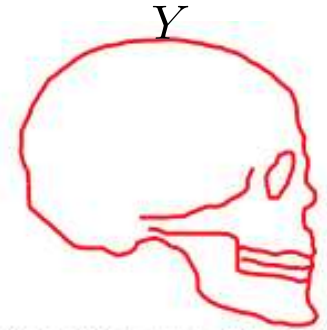
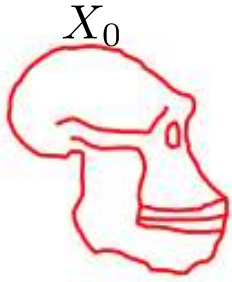
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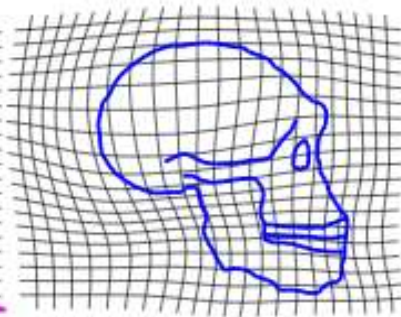
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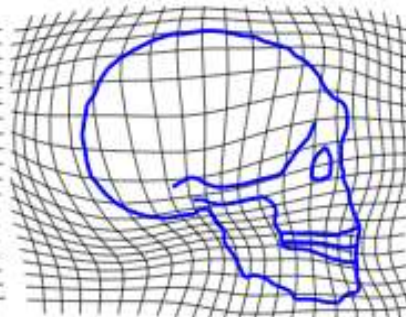
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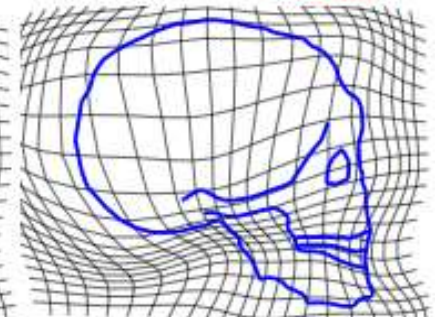
$X(0)$



$X(0.33)$

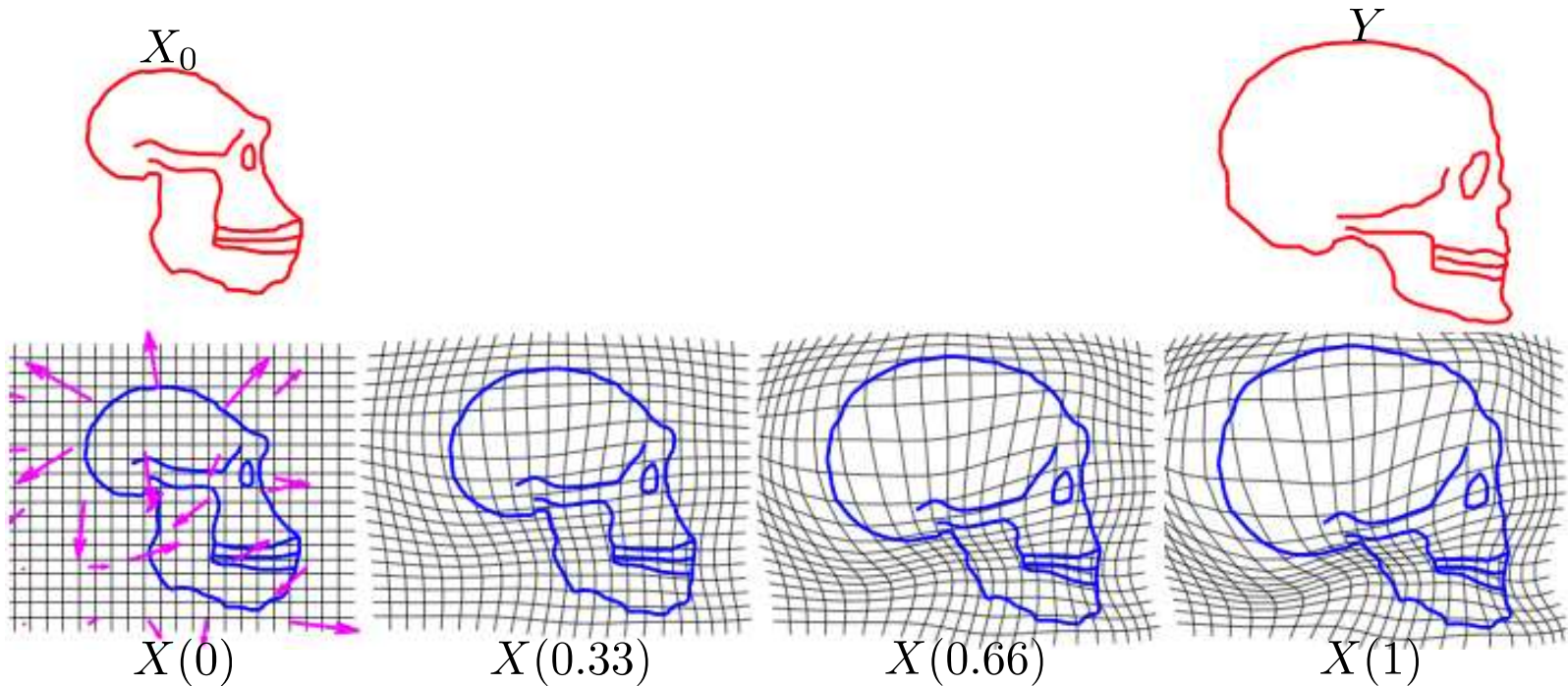


$X(0.66)$



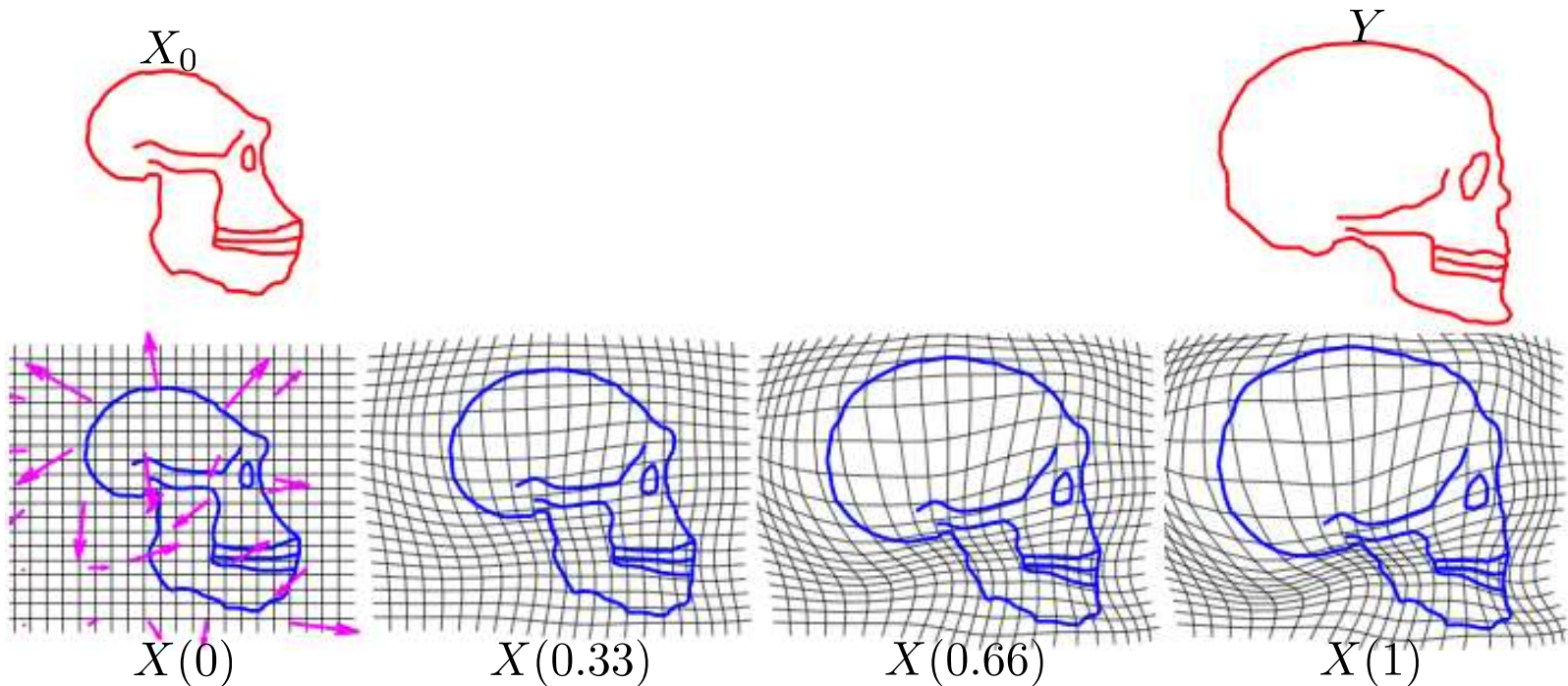
$X(1)$

Forms and deformation



$$E(S(t)) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + \int_0^1 \|v(t, \cdot)\|_K^2$$

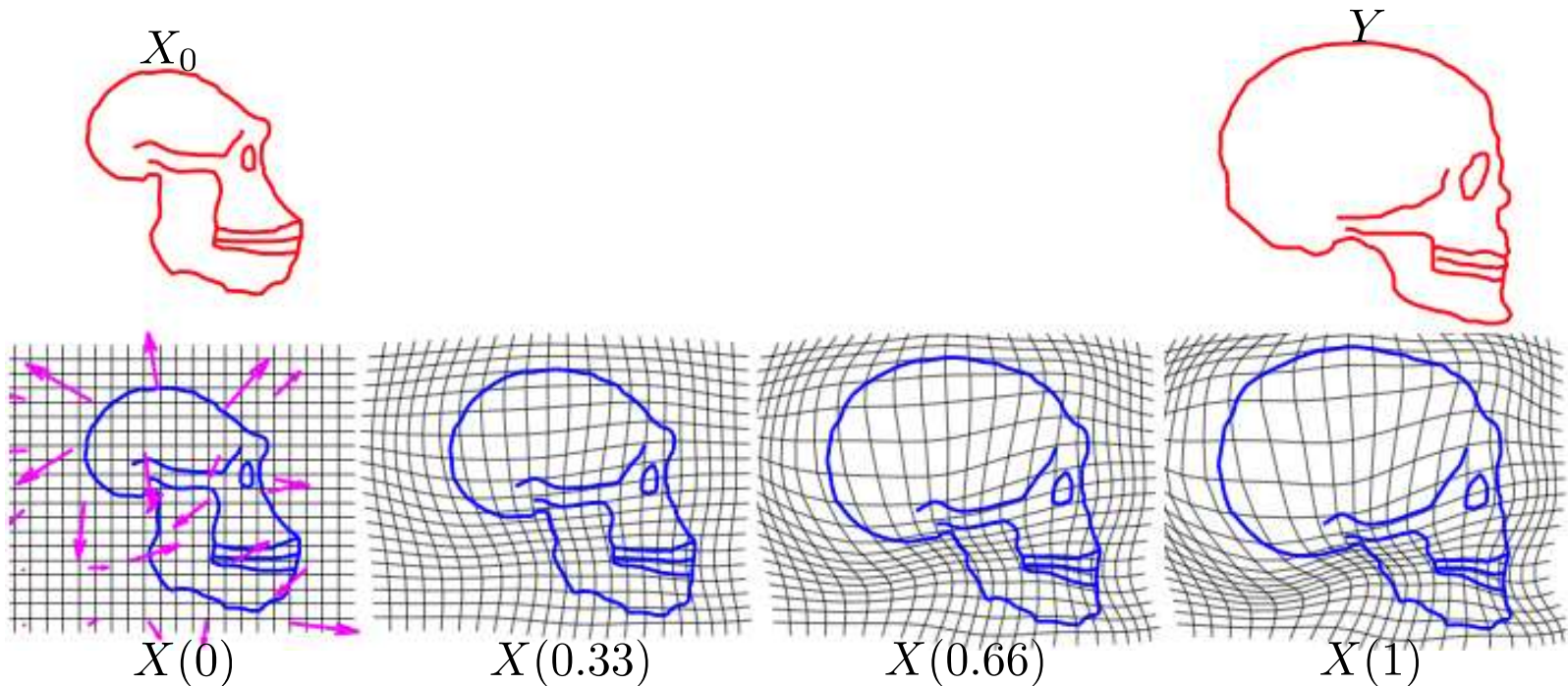
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Forms and deformation

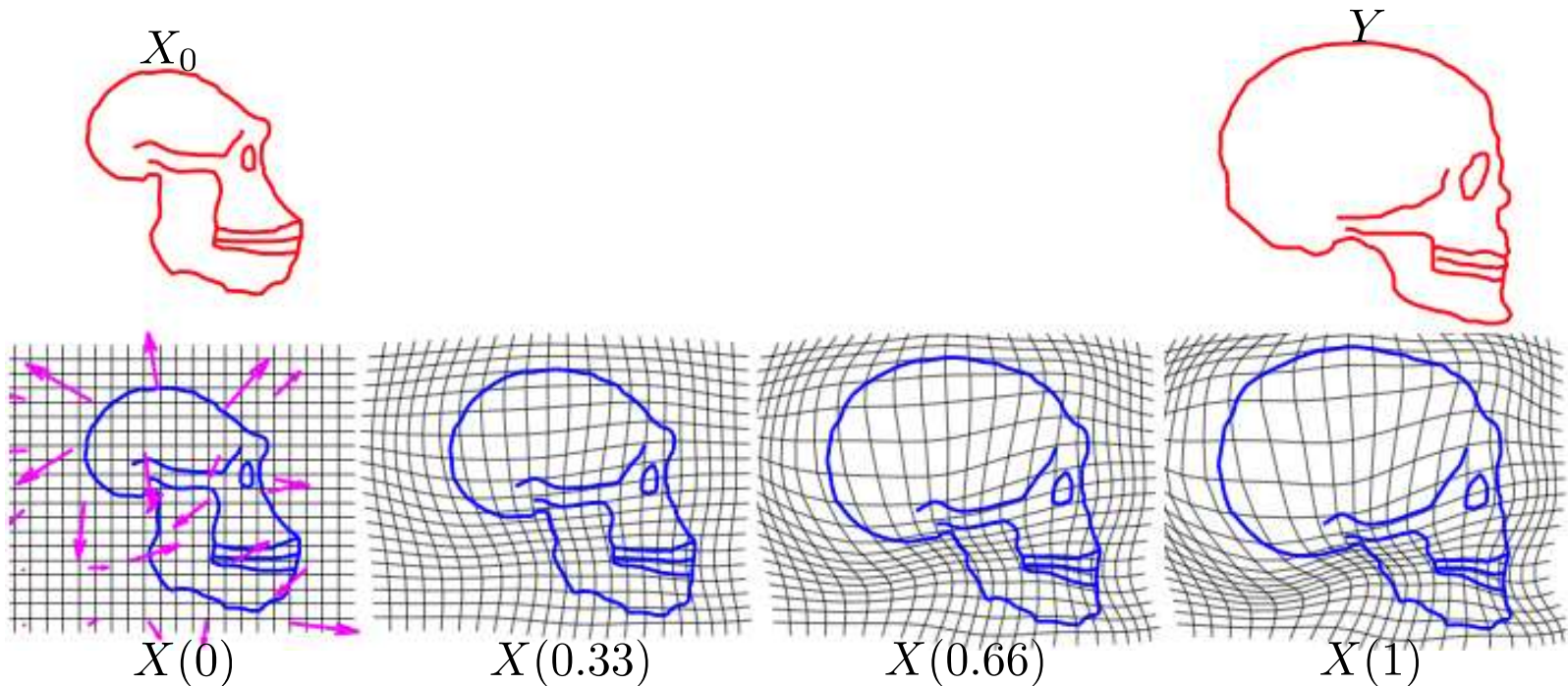


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The solution is a geodesic path!

Forms and deformation

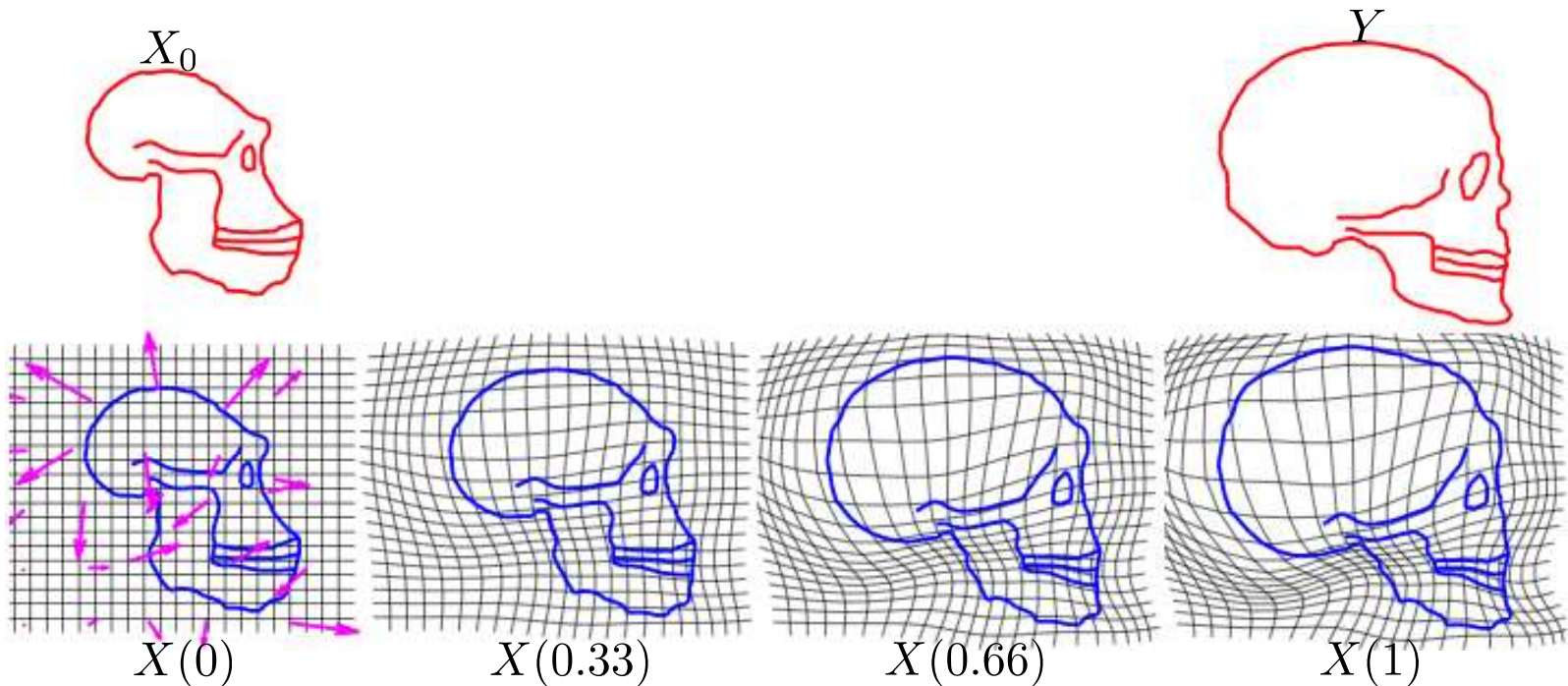


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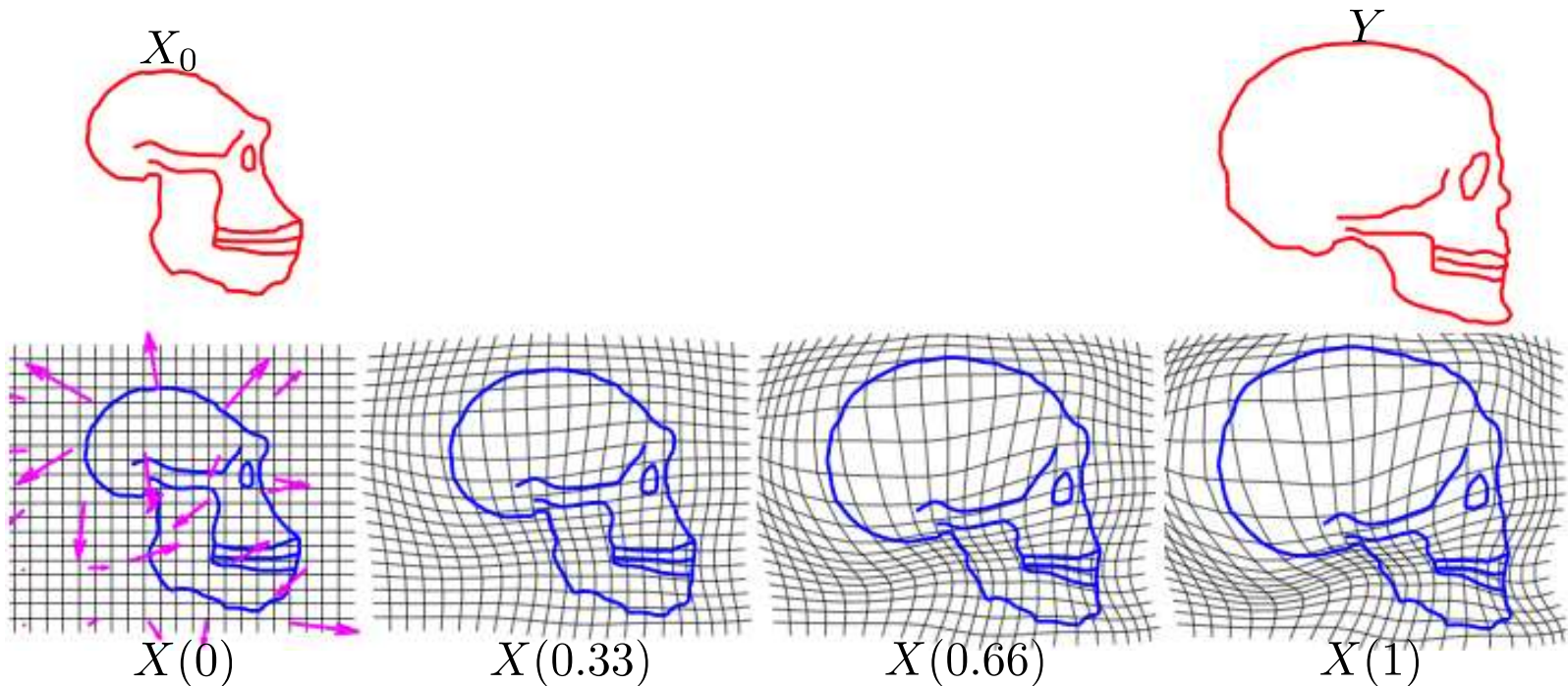
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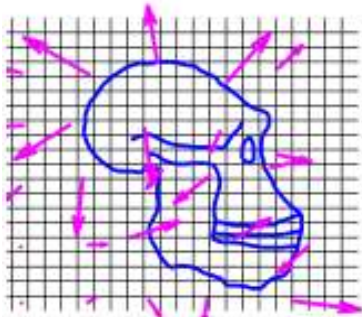
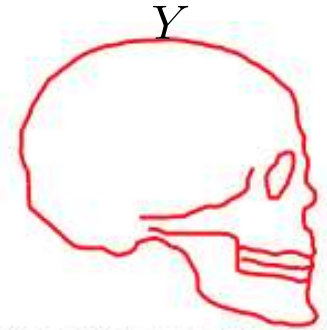
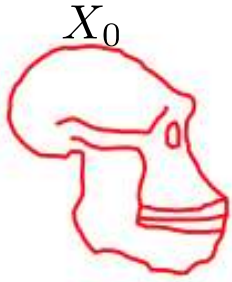
$$E(S_0) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + \underbrace{\|v(0, \cdot)\|_K^2}_{L(S_0)}$$

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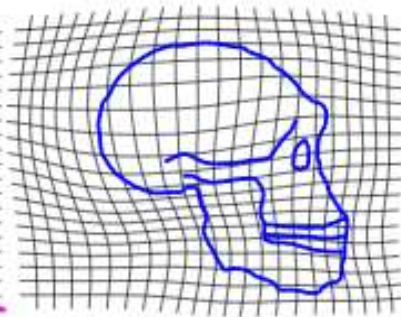
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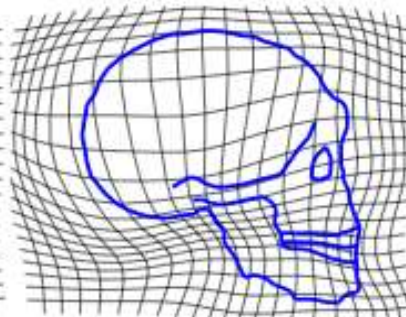
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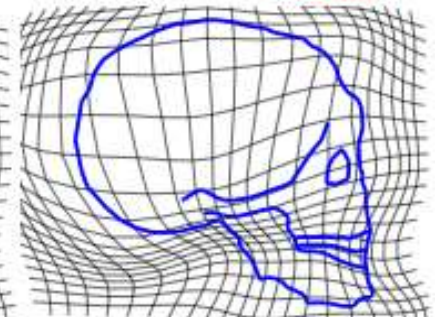
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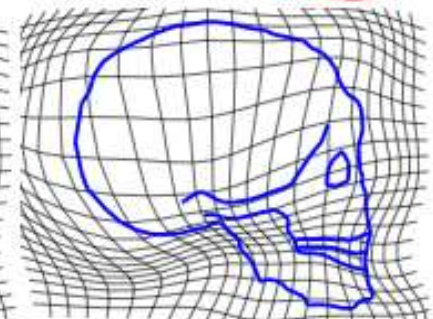
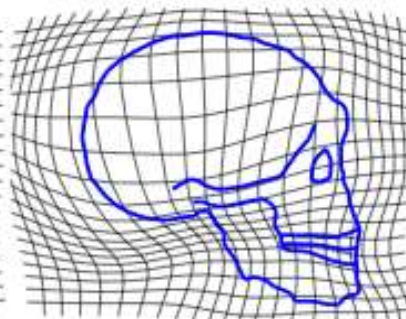
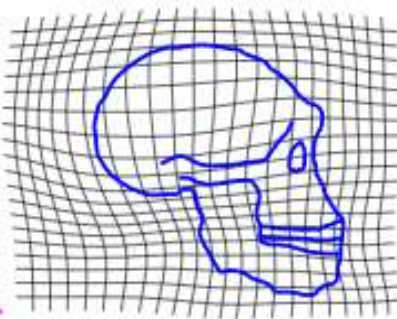
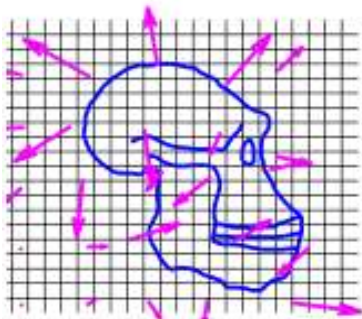
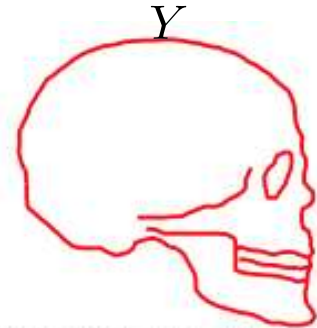
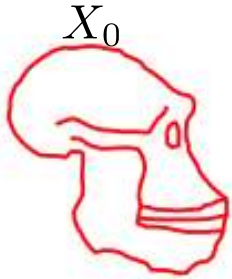


$X(0.66)$



$X(1)$

Forms and deformation



$$E(S(t)) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + \int_0^1 L(S(t)) dt$$

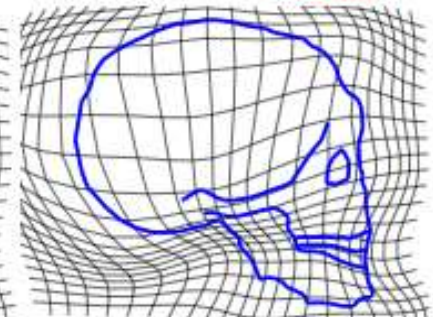
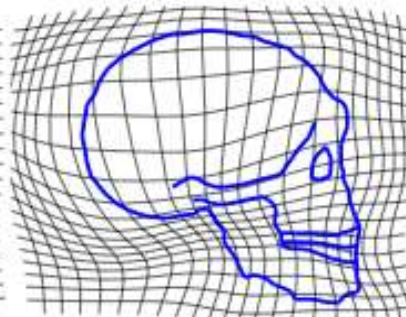
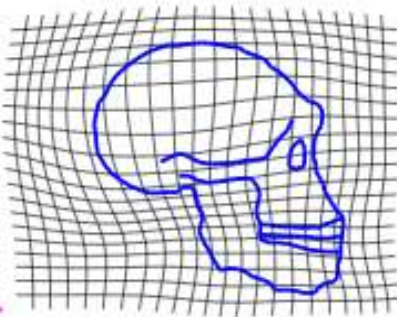
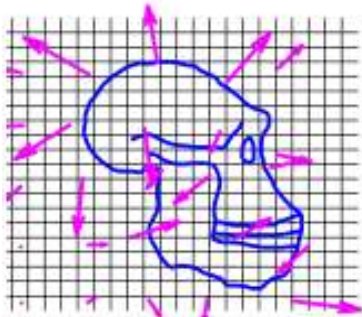
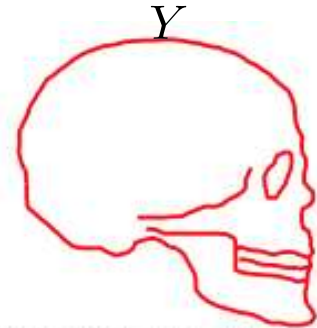
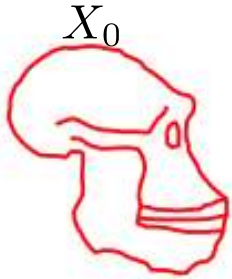
$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

$$E(S_0) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

Same theoretical solutions, different algorithms

Forms and deformation



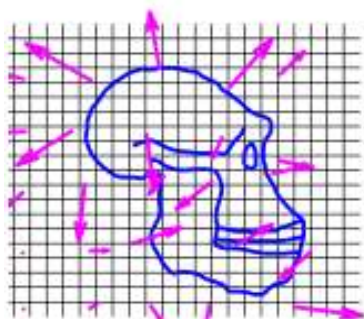
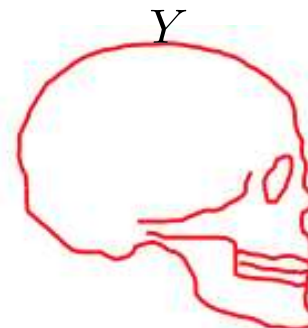
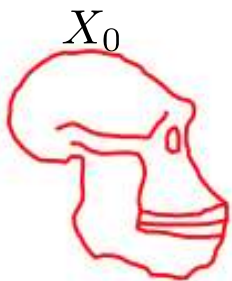
$$E(S(t)) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

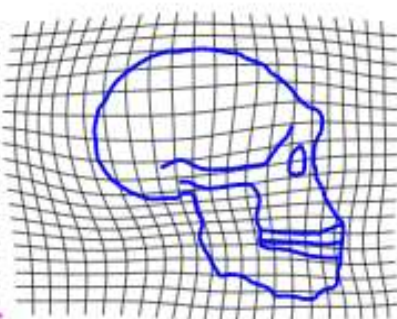
$$E(S_0) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

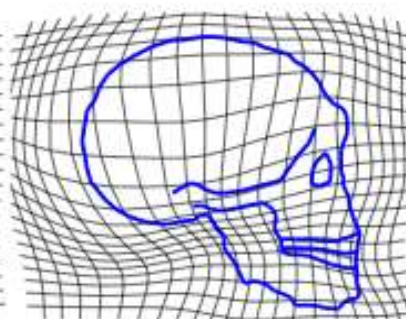
Forms and deformation



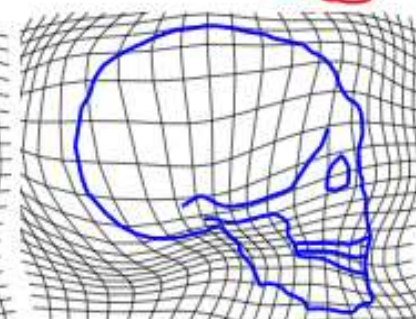
$X(0)$



$X(0.33)$



$X(0.66)$



$X(1)$

$$E(\mathbf{S}(t)) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + \int_0^1 L(\mathbf{S}(t)) dt$$

$$\text{s.t. } \dot{X}(t) = G(\mathbf{S}(t), X(t)) \quad X(0) = X_0$$

$$\nabla_{\mathbf{S}(t)} E = \partial_2 G^T \theta(t) + \nabla_{\mathbf{S}(t)} L$$

$$\text{with } \dot{\theta}(t) = -\partial_1 G(t)^T \theta(t) \quad \theta(1) = \nabla_{X(1)} D$$

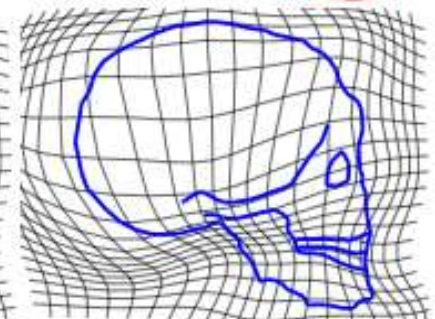
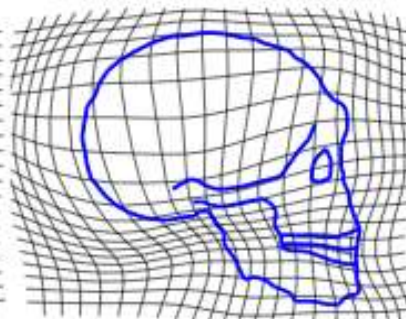
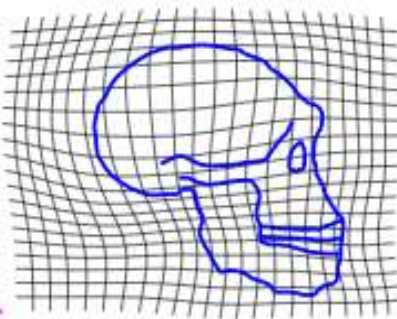
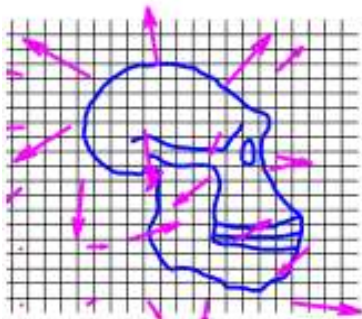
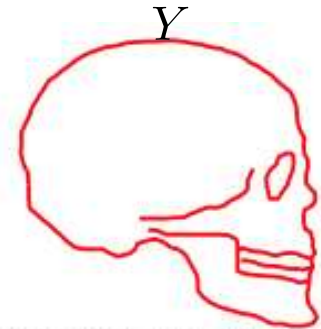
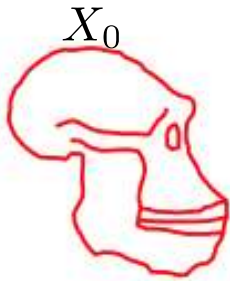
$$E(\mathbf{S}_0) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + L(\mathbf{S}_0)$$

$$\text{s.t. } \begin{cases} \dot{X}(t) = G(\mathbf{S}(t), X(t)) & X(0) = X_0 \\ \dot{\mathbf{S}}(t) = F(\mathbf{S}(t)) & \mathbf{S}(0) = \mathbf{S}_0 \end{cases}$$

$$\nabla_{\mathbf{S}_0} E = \xi(0) + \nabla_{\mathbf{S}_0} L$$

$$\text{with } \begin{cases} \dot{\theta}(t) = -\partial_1 G(t)^T \theta(t) & \theta(1) = \nabla_{X(1)} D \\ \dot{\xi}(t) = -\partial_2 G^T \theta(t) - d_{\mathbf{S}(t)} F^T \xi(t) & \xi(1) = 0 \end{cases}$$

Forms and deformation



$$E(S(t)) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

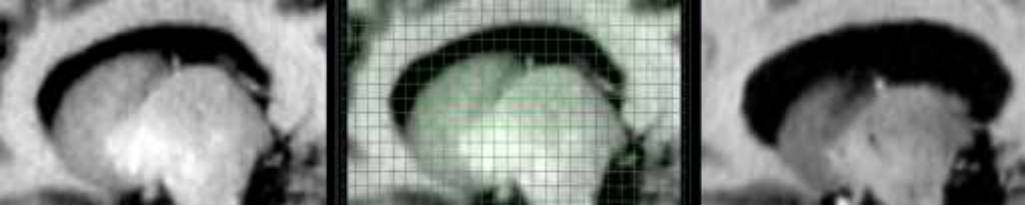
$$E(S_0) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

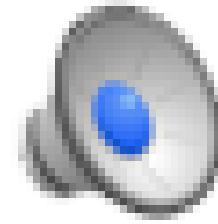
$D(X(1), Y) = \|X(1) - Y\|$, or norm of currents, norm of varifolds, norm between images, etc..

[Glaunès'05, Durrleman'08, Charon'13, ...]

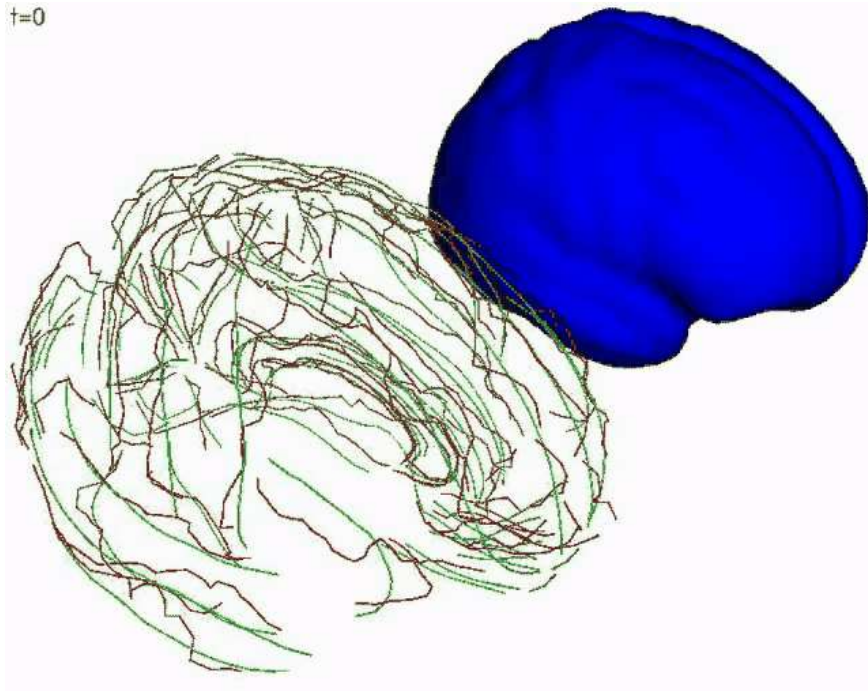
Forms and deformations



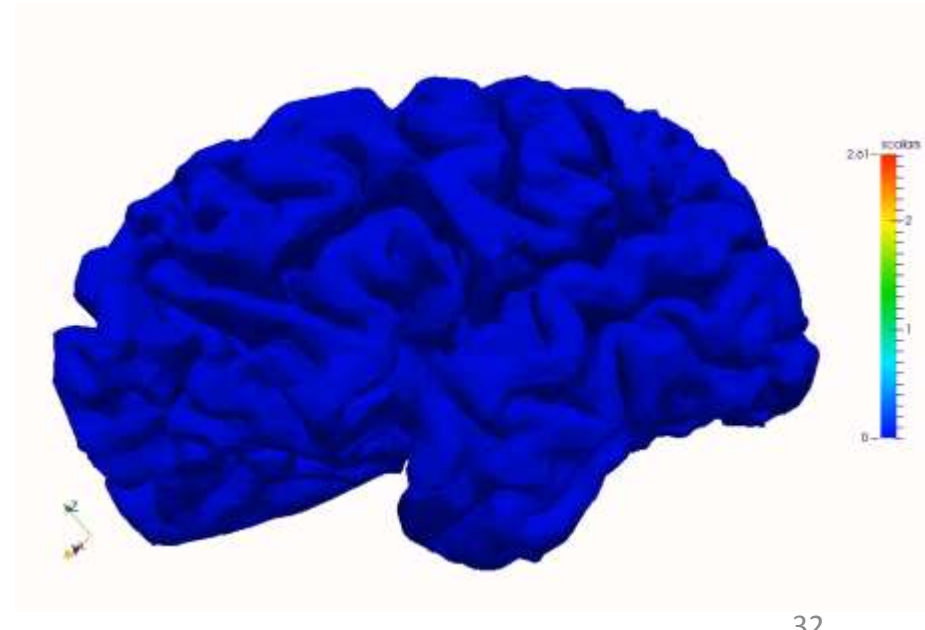
Atlas-to-patient registration of basal ganglia [Fouquier et al. DBSMC'14]



Deformation of white matter tracts [Gori et al. MICCAI'13]



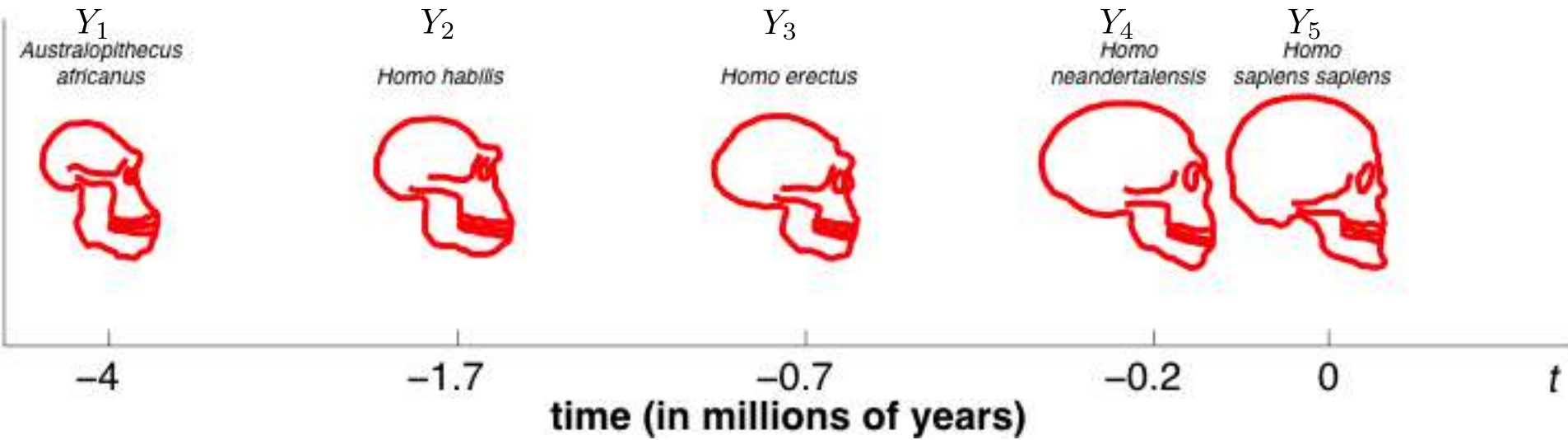
Registration of sulcal curves [Durrleman et al. Media'08]



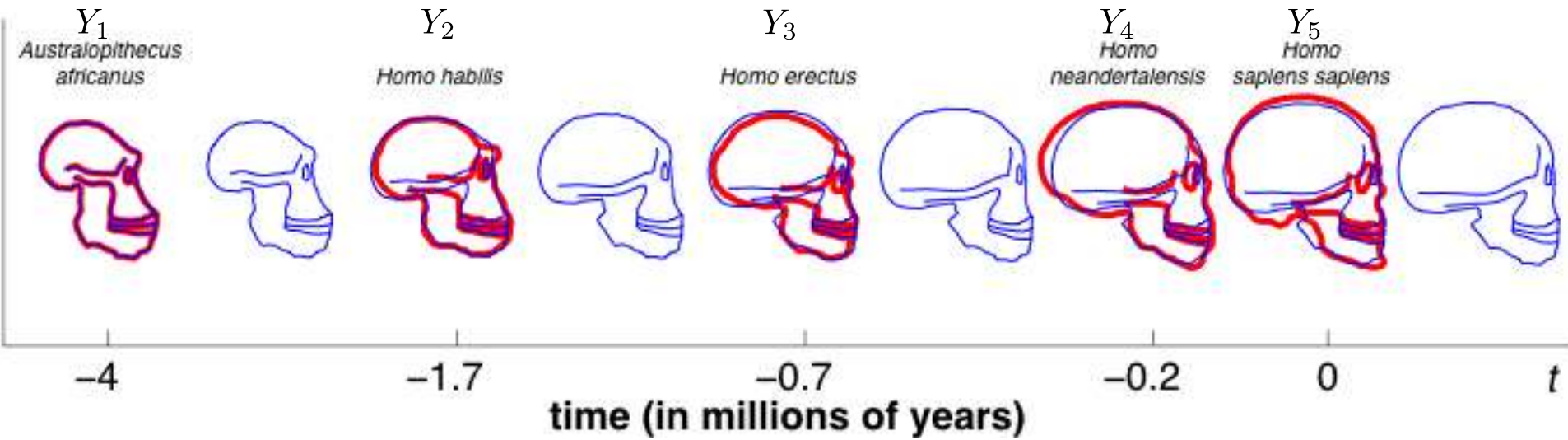
Registration cortical surface between baseline and follow-up

Regression of time-series shape data

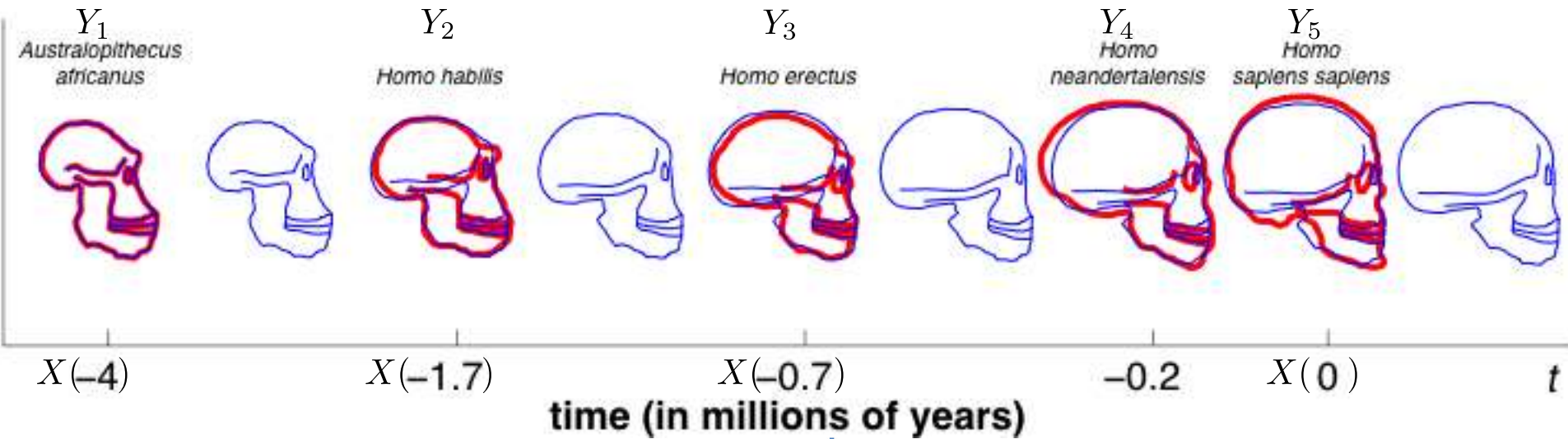
Regression of time-series shape data



Regression of time-series shape data



Regression of time-series shape data



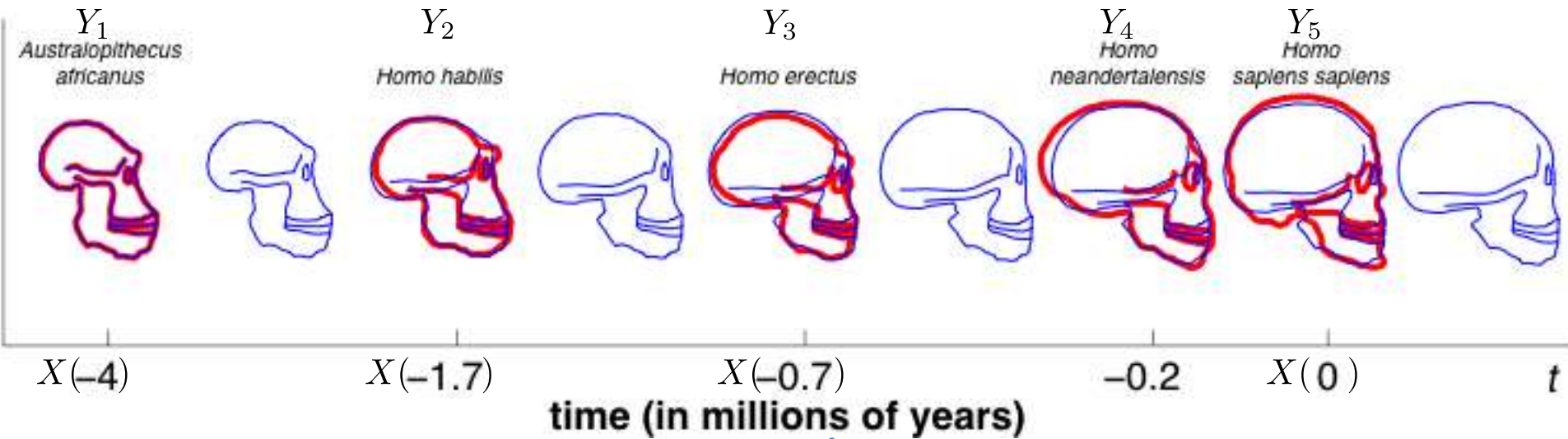
$$E(S(t)) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

$$E(S_0) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

Regression of time-series shape data



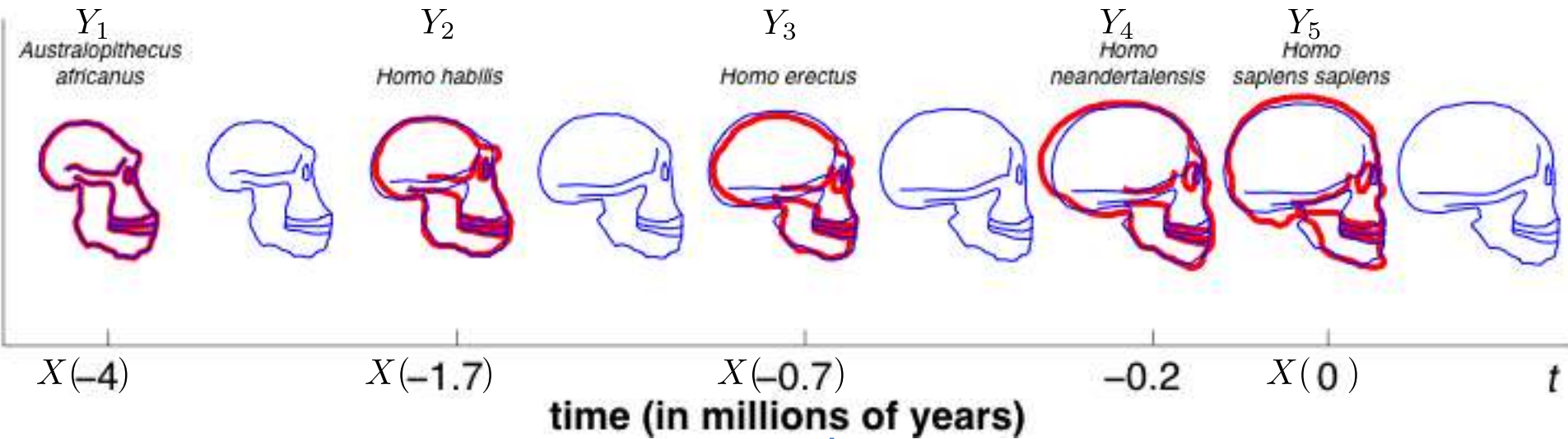
$$E(S(t)) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

$$E(S_0) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

Regression of time-series shape data



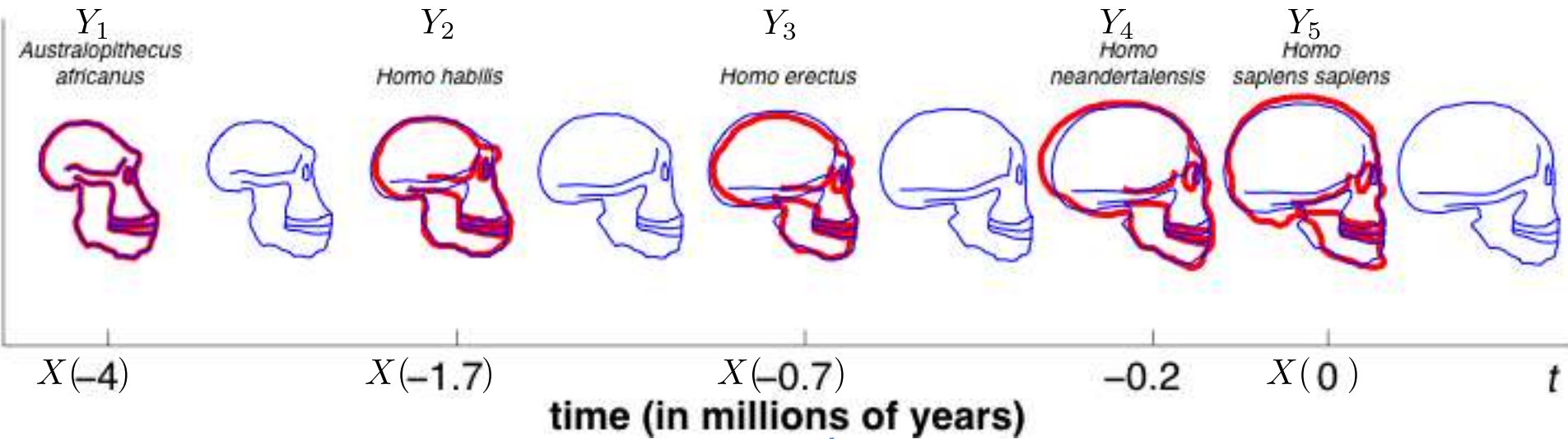
$$E(S(t)) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

$$E(S_0) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

Regression of time-series shape data



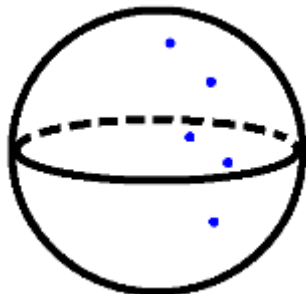
$$E(S(t)) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

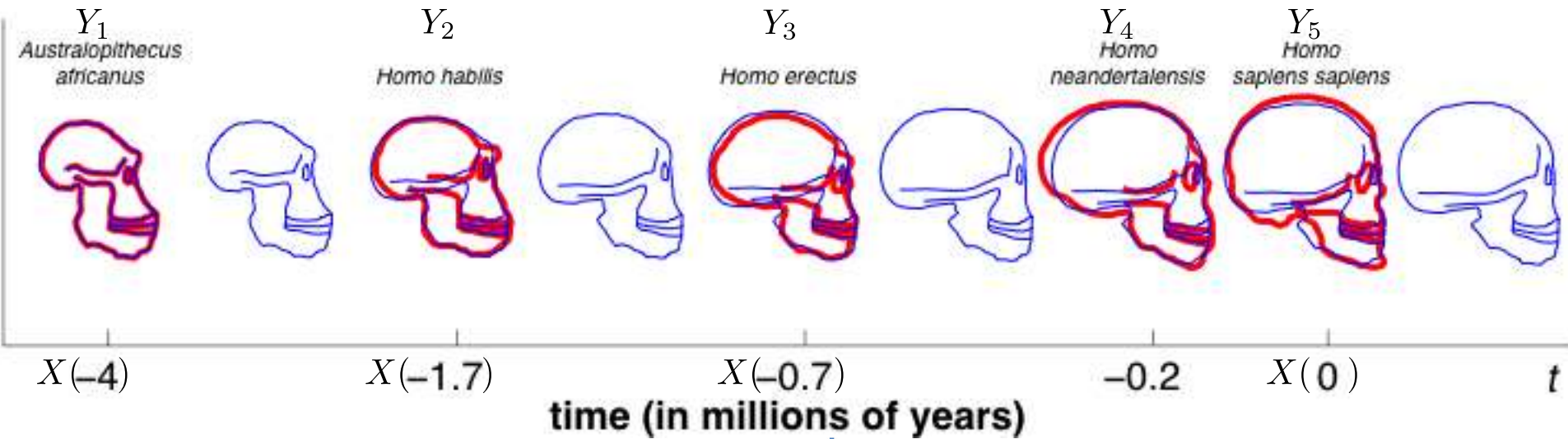
$$E(S_0) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

Piecewise geodesic solution



Regression of time-series shape data



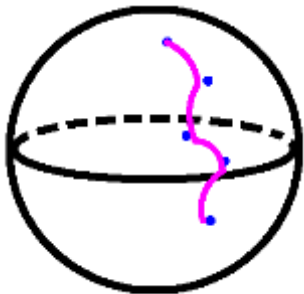
$$E(S(t)) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

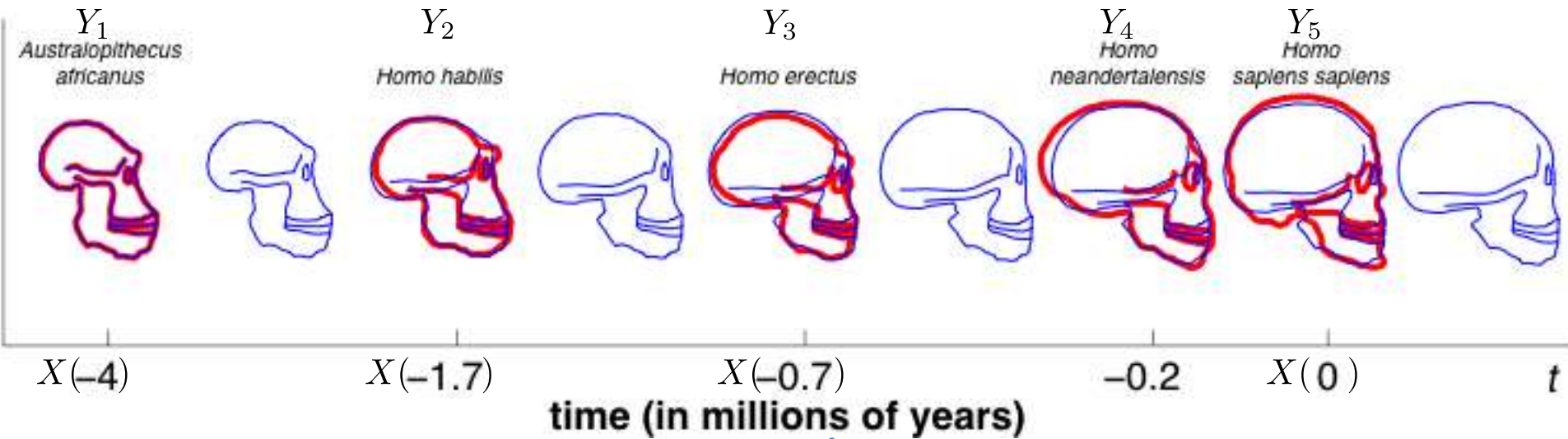
$$E(S_0) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

Piecewise geodesic solution



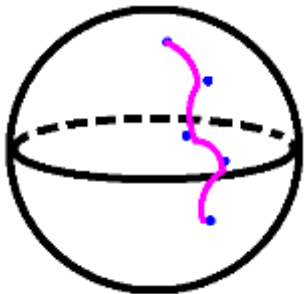
Regression of time-series shape data



$$E(S(t)) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

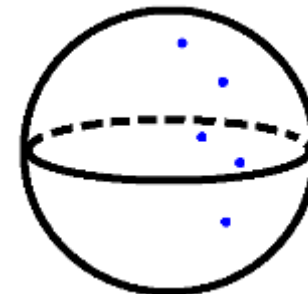
Piecewise geodesic solution



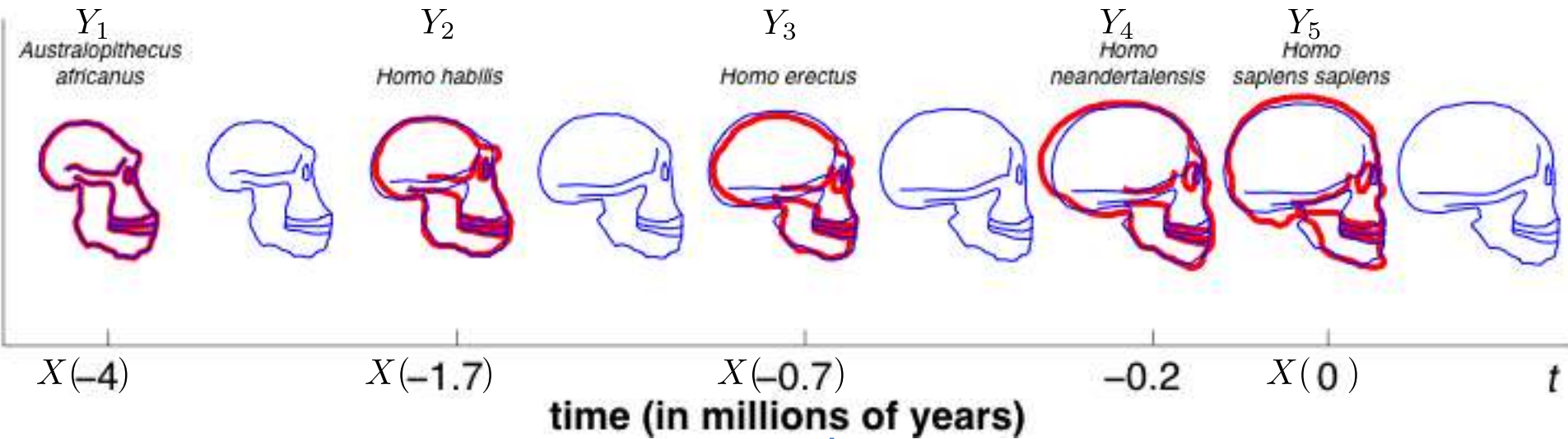
$$E(S_0) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

geodesic solution



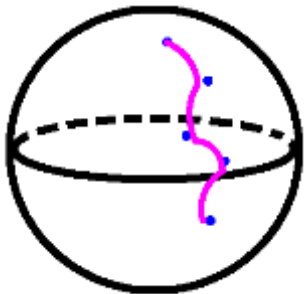
Regression of time-series shape data



$$E(S(t)) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

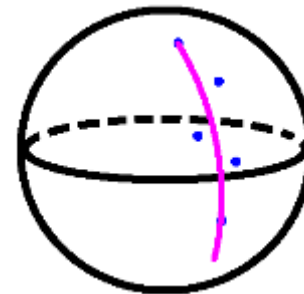
Piecewise geodesic solution



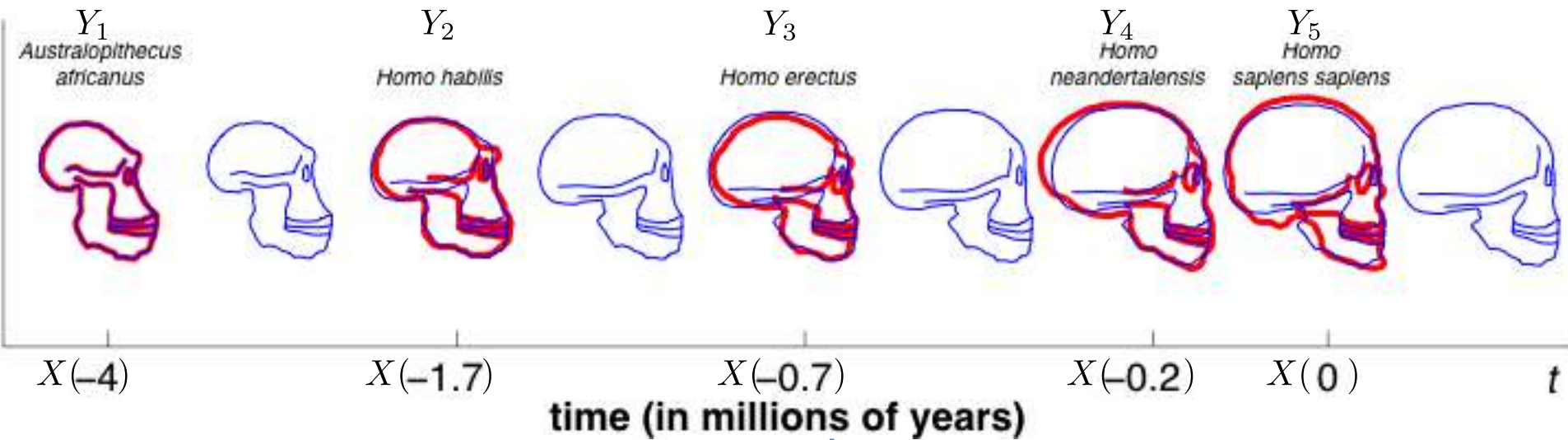
$$E(S_0) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

geodesic solution



Regression of time-series shape data



$$E(S(t)) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

$$E(S_0) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + L(S_0)$$

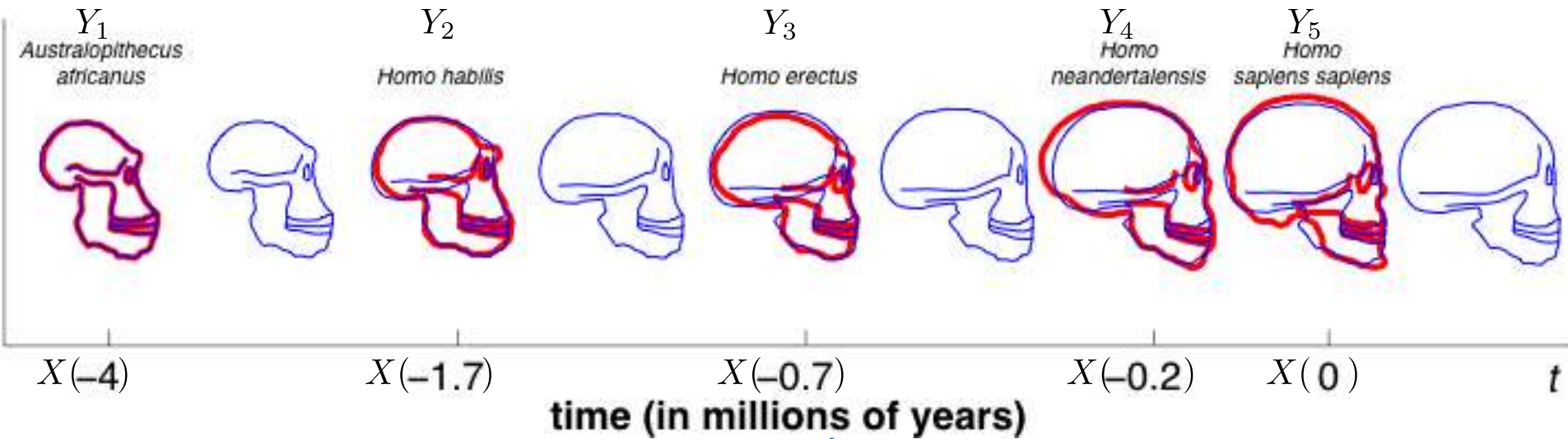
$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

Piecewise geode

lution



Regression of time-series shape data



$$E(S(t)) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + \int_0^1 L(S(t)) dt$$

$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

Piecewise geodesic solution

$$\nabla_{S(t)} E = \partial_2 G^T \theta(t) + \nabla_{S(t)} L$$

$$with \quad \dot{\theta}(t) = -\partial_1 G(t)^T \theta(t) - \sum_i \delta(t - t_i) \nabla_{X(t_i)} D \quad and \quad \theta(T) = 0$$

$$E(S_0) = \frac{1}{2\sigma^2} \sum_i D(X(t_i), Y_i)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

geodesic solution

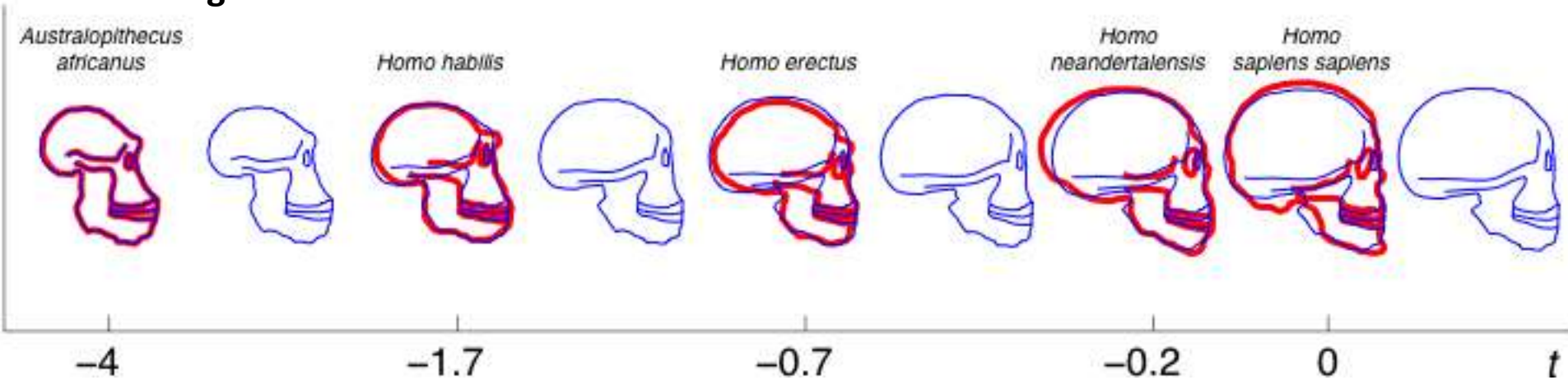
$$\nabla_{S_0} E = \xi(0) + \nabla_{S_0} L$$

$$\dot{\xi}(t) = -\partial_2 G^T \theta(t) - d_{S(t)} F^T \xi(t) \quad \xi(T) = 0$$

$$\nabla_{X_0} E = \theta(0)$$

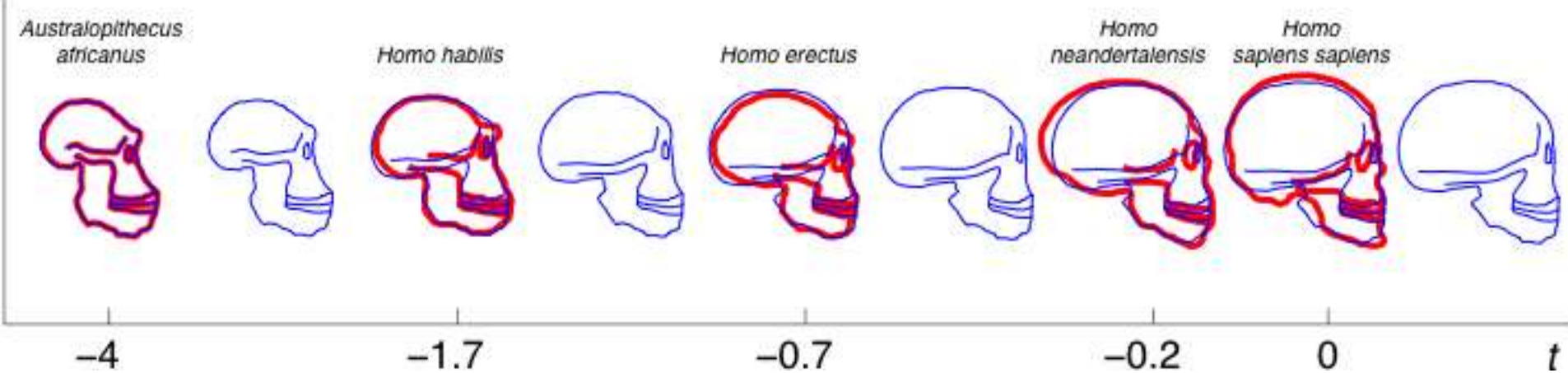
Regression of time-series shape data

Geodesic regression: fixed baseline



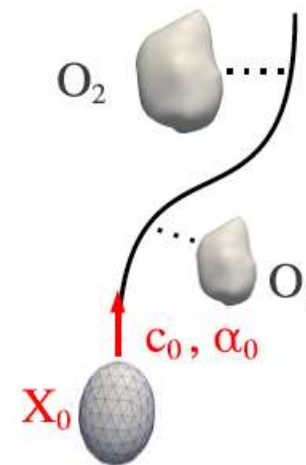
Regression of time-series shape data

Geodesic regression: fixed baseline



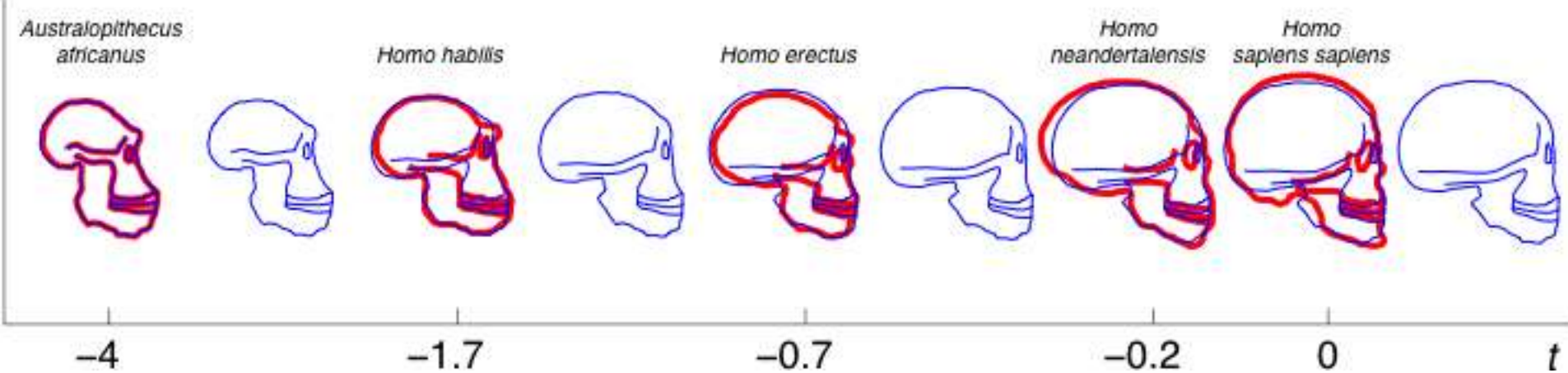
Geodesic regression: estimated baseline

- Joint optimization:
 - Estimation of a baseline (intercept)
 - Estimation of initial momenta and control points (slope)



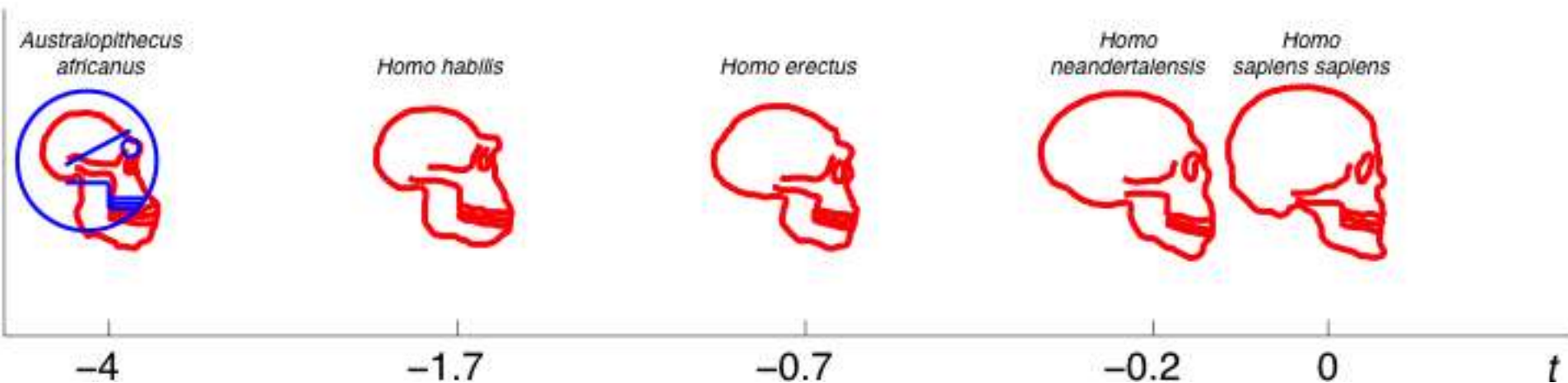
Regression of time-series shape data

Geodesic regression: fixed baseline



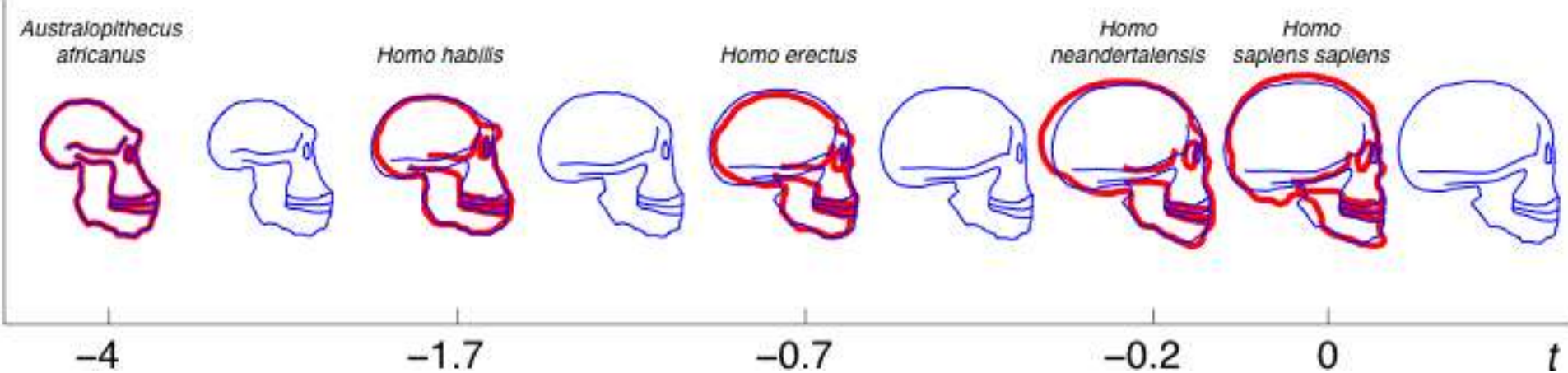
Geodesic regression: estimated baseline

Input:



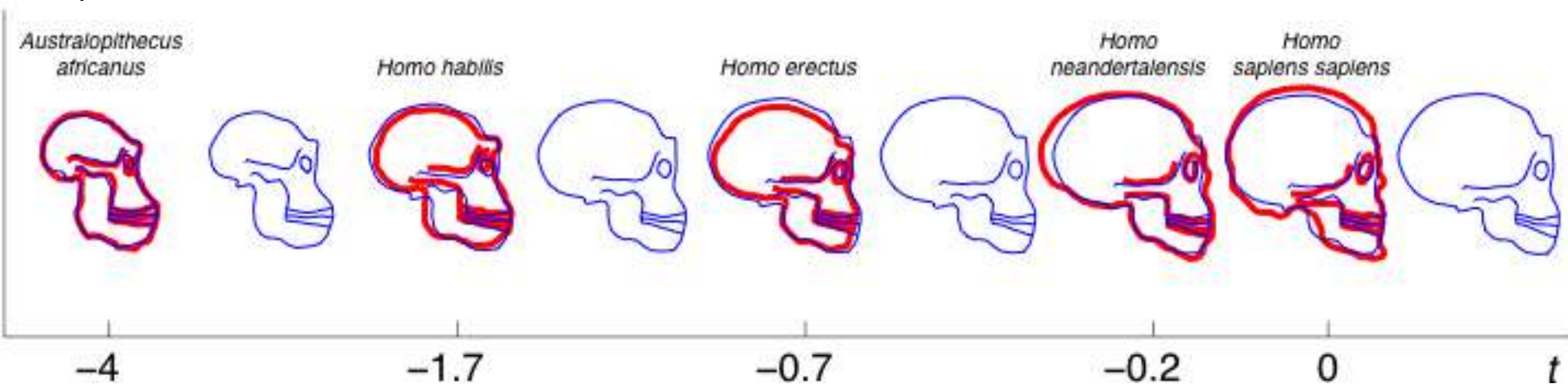
Regression of time-series shape data

Geodesic regression: fixed baseline

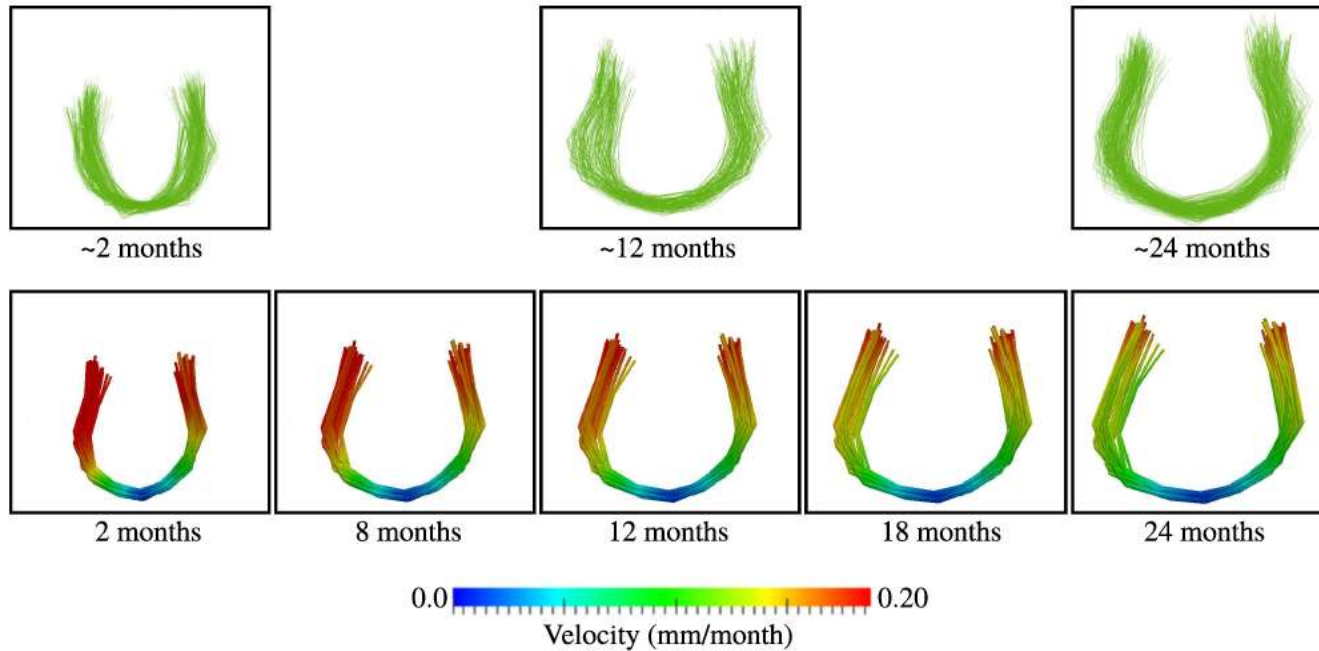


Geodesic regression: estimated baseline

Output:

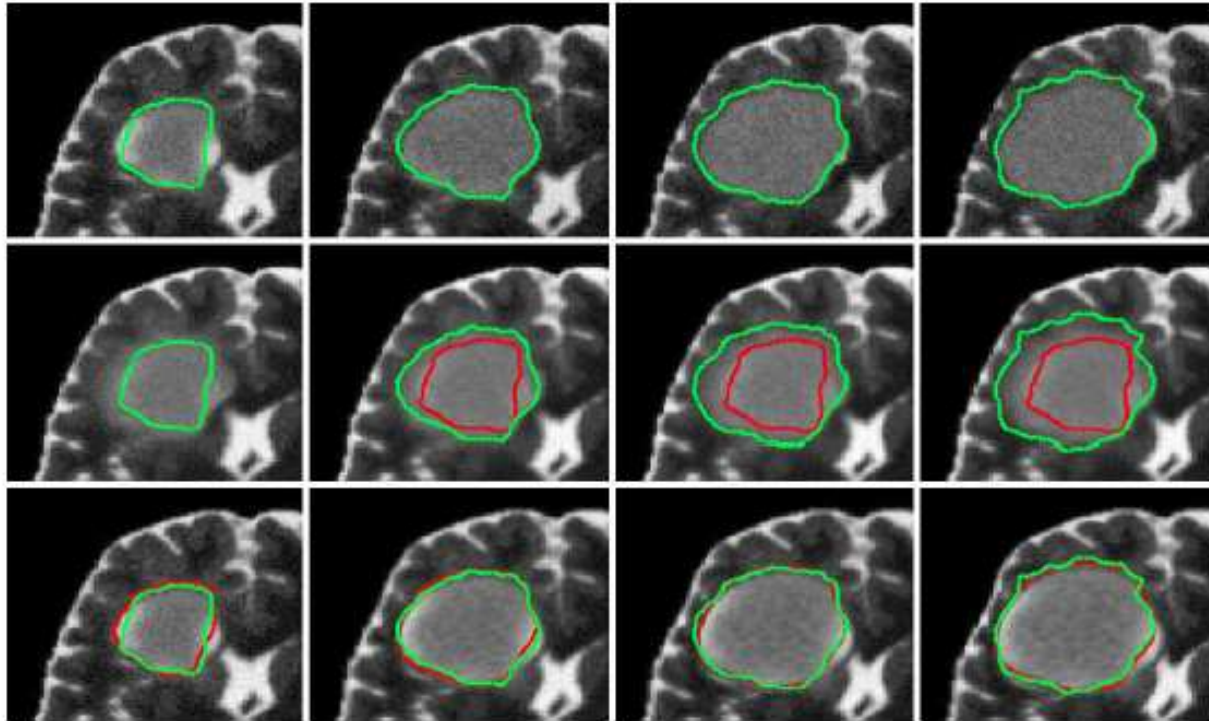


Regression of time-series shape data



Growth of the genu fiber tract [Fishbaugh et al. IPMI'13]

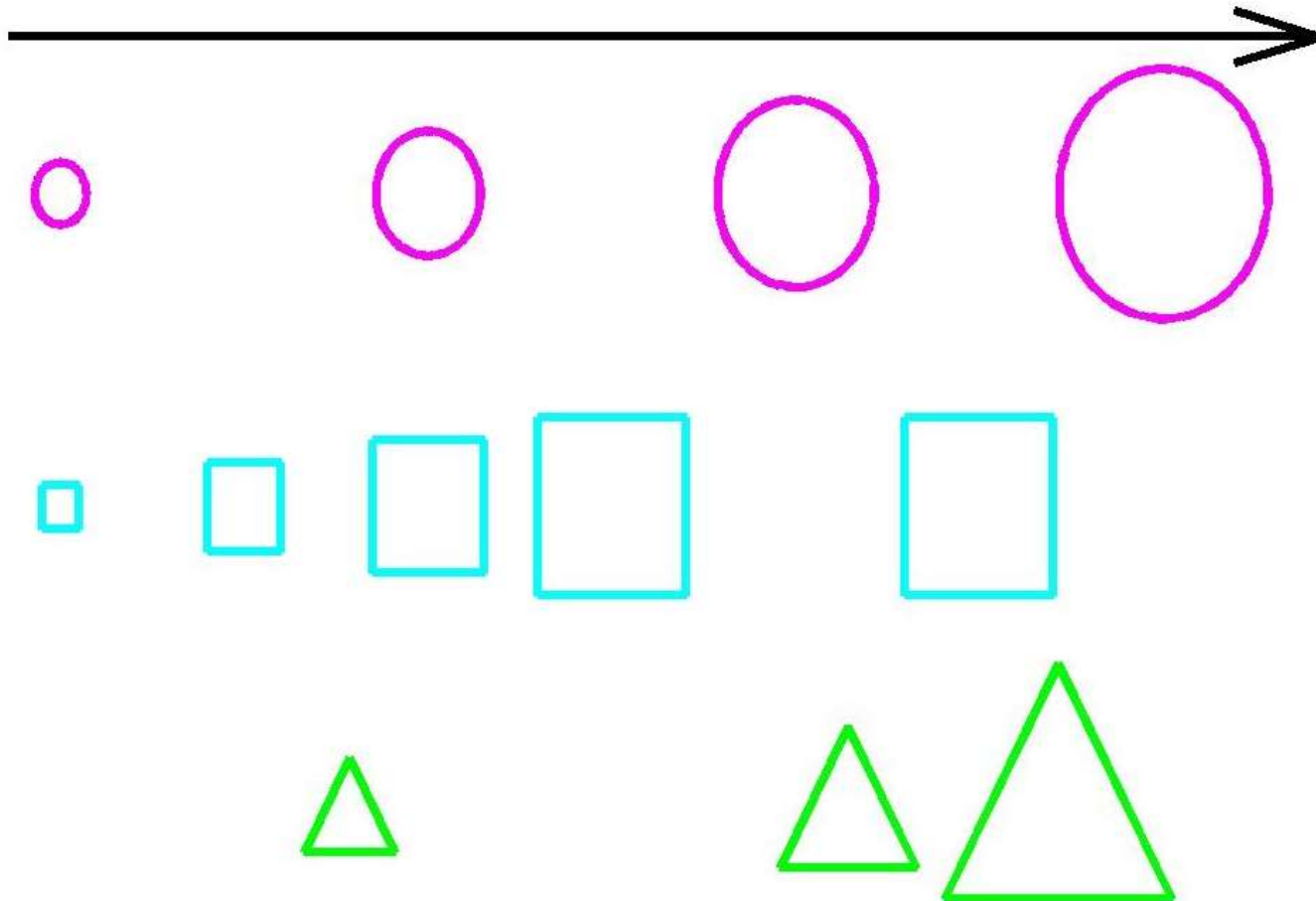
Regression of time-series shape data



Geodesic regression of joint image and surface data [Fishbaugh et al. MICCAI'13]

Longitudinal Data Analysis

Longitudinal Data Analysis



Repeated measurements of a series of subjects. Subjects differ in:

- Shape
- Pace of development

Longitudinal Data Analysis

- Comparison between two lineages (toy example)



-1600

-1400

-1200

-1000

-800

-600

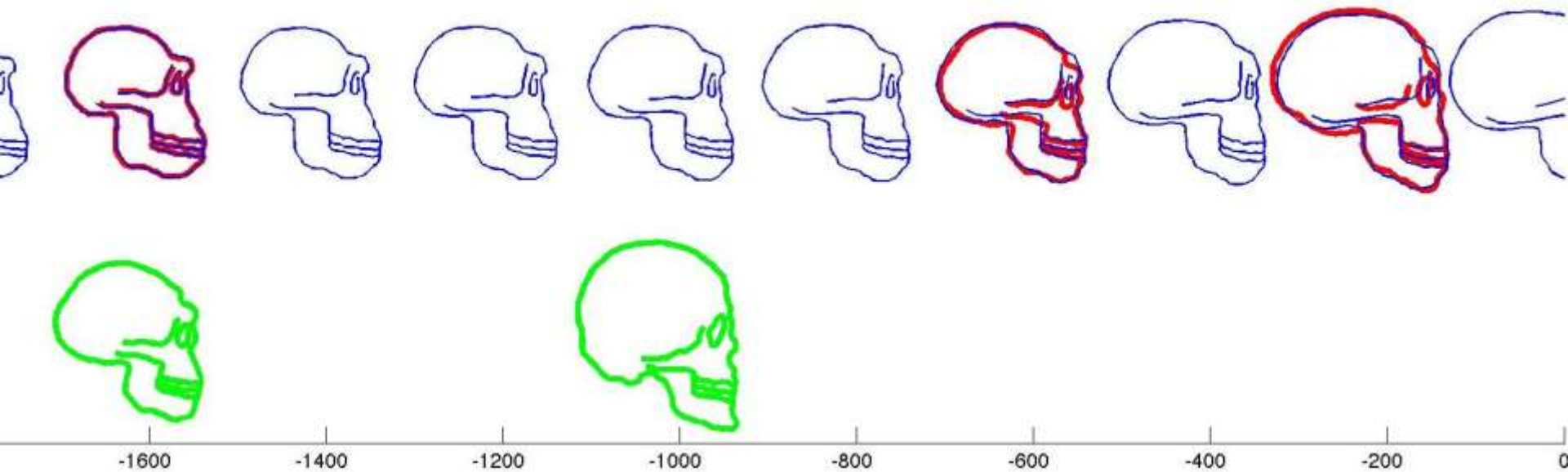
-400

-200

0

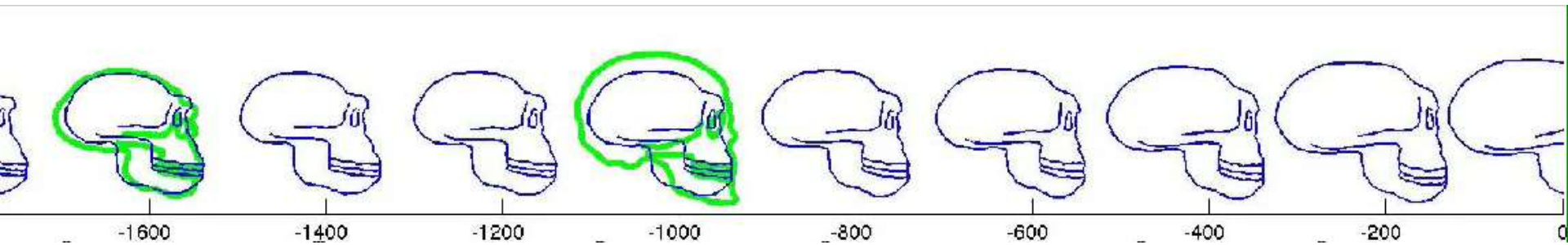
Longitudinal Data Analysis

- Comparison between two lineages (toy example)



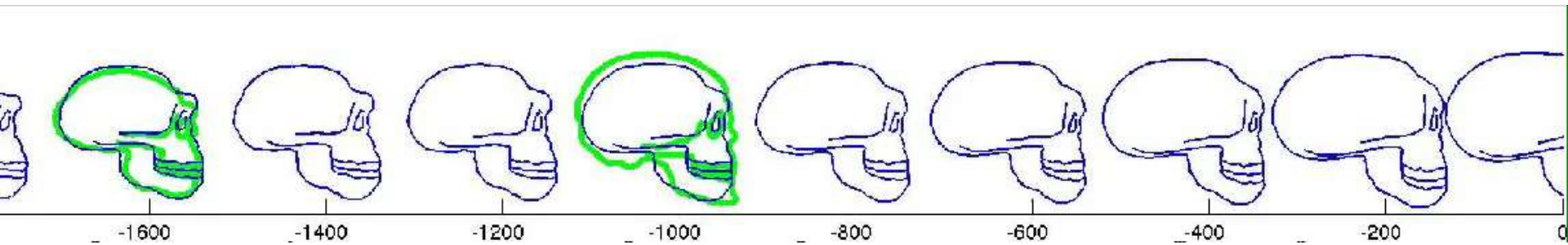
Longitudinal Data Analysis

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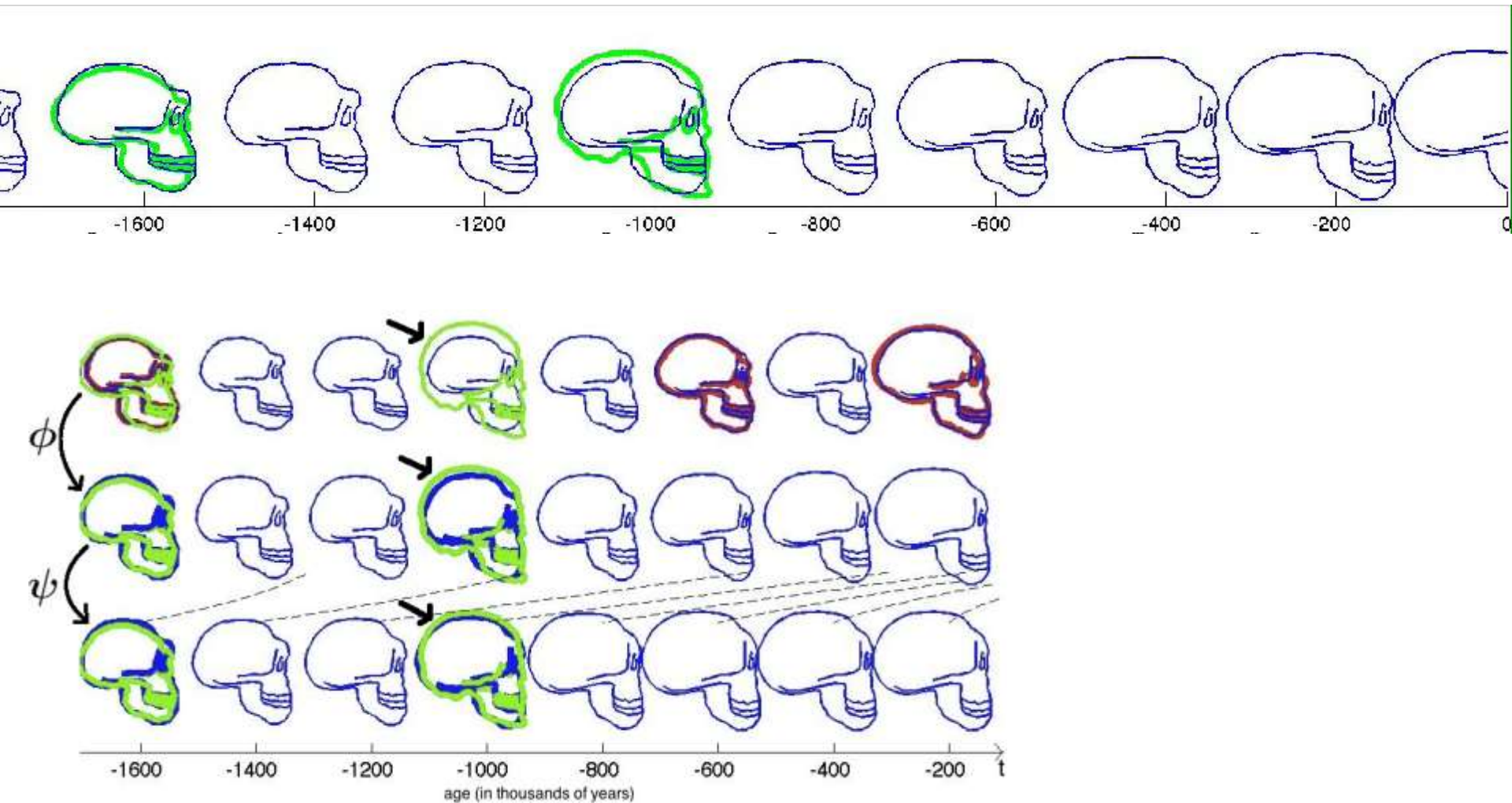
Longitudinal Data Analysis

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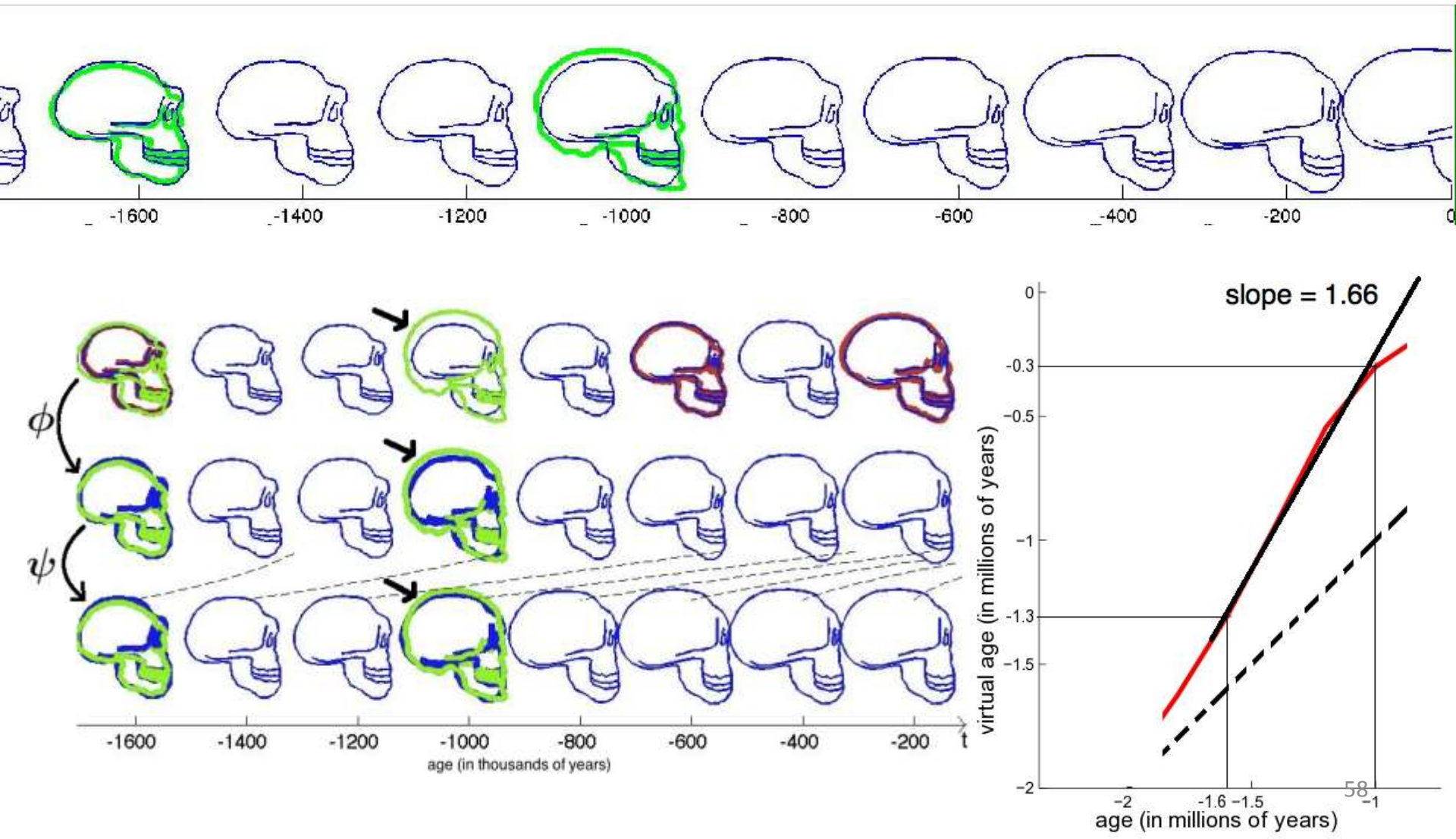
Longitudinal Data Analysis

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Longitudinal Data Analysis

- Comparison between two lineages (toy example)

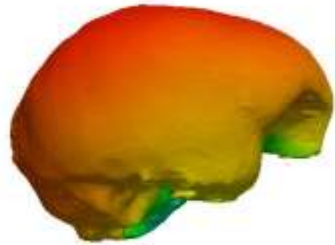


Longitudinal Data Analysis

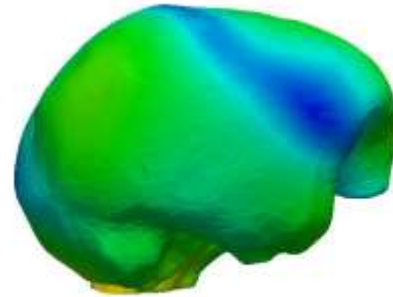
- Compare regression between subjects [Durrleman et al. JHE'11, IJCV'13]

Longitudinal Data Analysis

- Compare regression between subjects [Durrleman et al. JHE'11, IJCV'13]



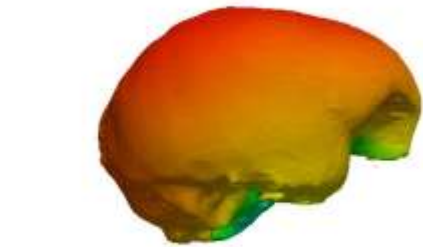
Bonobos



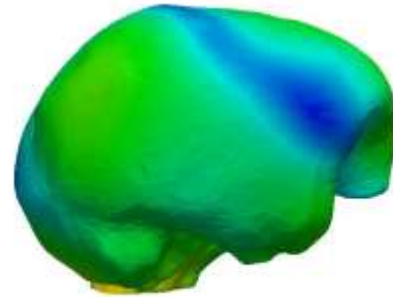
Chimpanzees

Longitudinal Data Analysis

- Compare regression between subjects [Durrleman et al. JHE'11, IJCV'13]



Bonobos



Chimpanzees

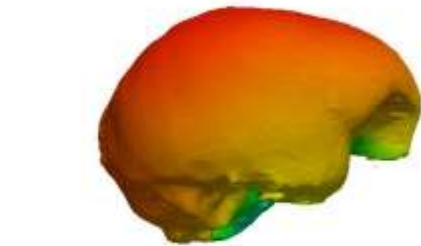
Morphological changes



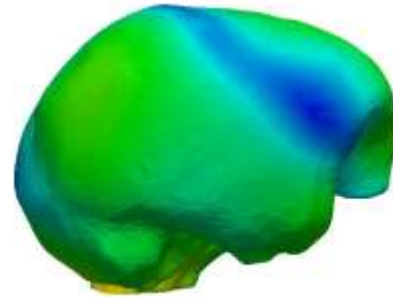
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Longitudinal Data Analysis

- Compare regression between subjects [Durrleman et al. JHE'11, IJCV'13]



Bonobos

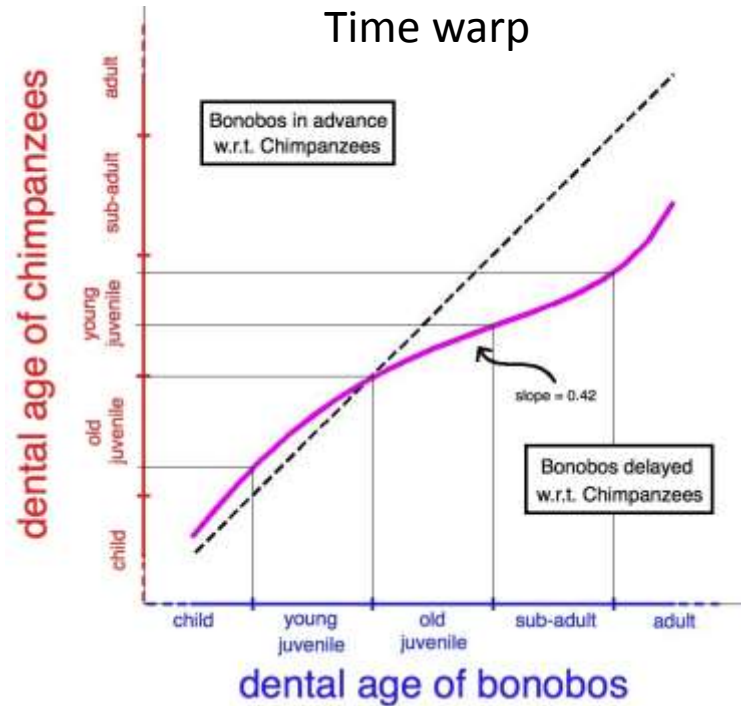


Chimpanzees

Morphological changes



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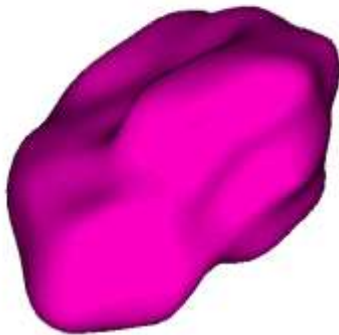


Longitudinal Data Analysis

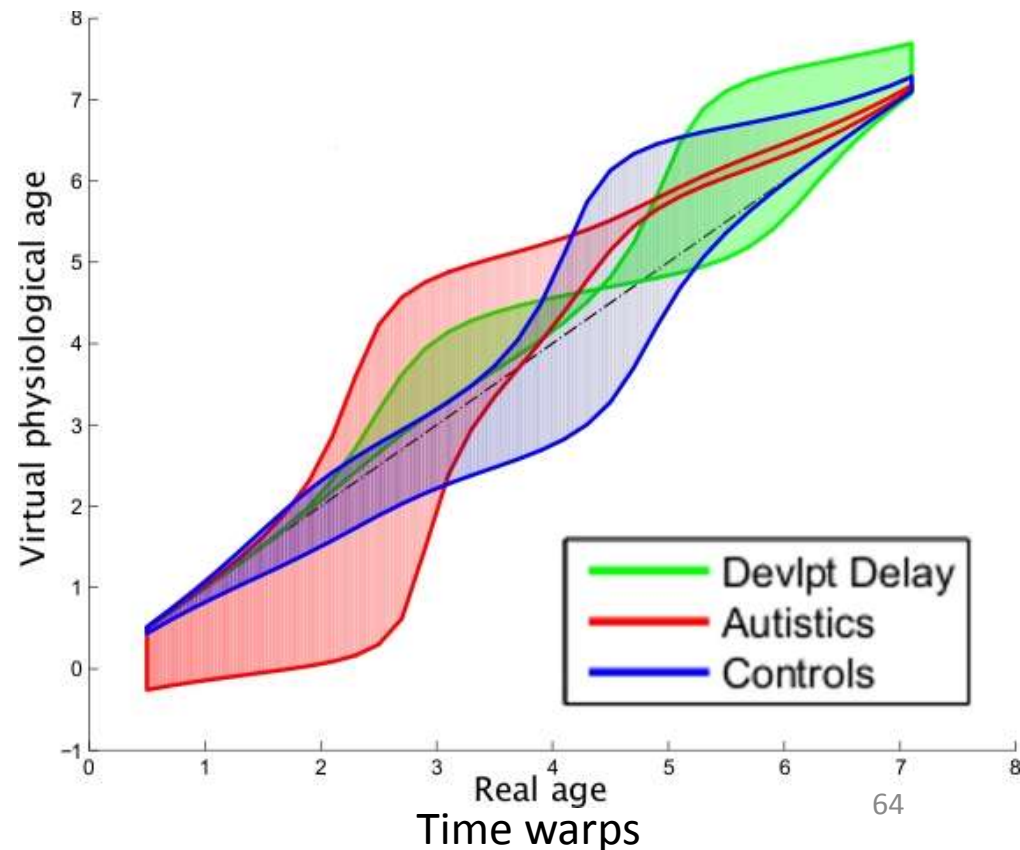
- Multiple subjects comparison: [Durrleman et al. IJCV'13]
 - Construction of an average growth scenario
 - Spatiotemporal deformation of the average scenario to each subject

Longitudinal Data Analysis

- Multiple subjects comparison: [Durrleman et al. IJCV'13]
 - Construction of an average growth scenario
 - Spatiotemporal deformation of the average scenario to each subject
- Developmental delays in autistic children:
 - 2 scans (initial age 2-3 years, follow-up 4-5 years)
 - 12 subjects (4 autistics, 4 developmental delays, 4 controls)

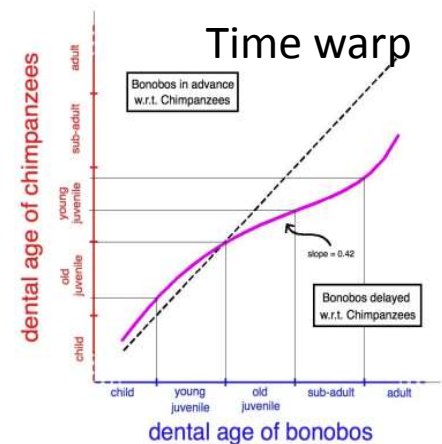
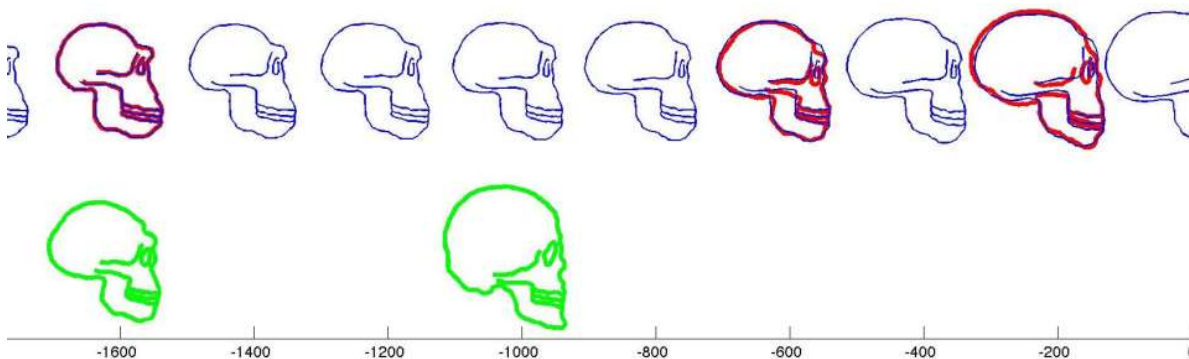


Average growth scenario



Conclusion

- **An approach to biological shape analysis based on deformations**
- **Regression of time series shape data:**
 - Piecewise geodesic regression
 - Geodesic regression
 - Other alternatives include:
 - Acceleration-controlled (continuously differentiable trajectories) [Fishbaugh'11]
 - Riemannian splines (perturbation of Hamiltonian equations) [Vialard'10]
- **Statistics of longitudinal data sets:**
 - Morphological deformation
 - Time-warp
- **Joint work with:** J. Fishbaugh, G. Gerig, X. Pennec, M. Prastawa, A. Trouvé



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Deformetrica

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Tutorials

Check out our tutorials! Find more help in the [Get Help](#) section with the user's manual and source code documentation.

Gallery

Visit our gallery to find out what Deformetrica can do. Just few examples... and we are waiting for your contributions!

Deformetrica is a software for the statistical analysis of 2D and 3D shape data. It essentially computes deformations of the ambient 2D or 3D ambient space, which, in turn, warp any object embedded in this space, whether this object is a curve, a surface, a structured or unstructured set of points, or any combination of them.

Deformetrica comes with two applications:

- registration, which computes the best possible deformation between two sets of objects,
- atlas construction, which computes an average object configuration from a collection of object sets, and the deformations from this average to each sample in the collection.

Deformetrica has very little requirements about the data it can deal with. In particular, it does *not* require point correspondence between objects!

Have more insights into what Deformetrica can do by checking out our [tutorials](#) and [gallery](#). Enjoy!

The Deformetrica team

LATEST NEWS

[Deformetrica is released!](#) March 13, 2014

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