Statistics of Longitudinal Shape Data

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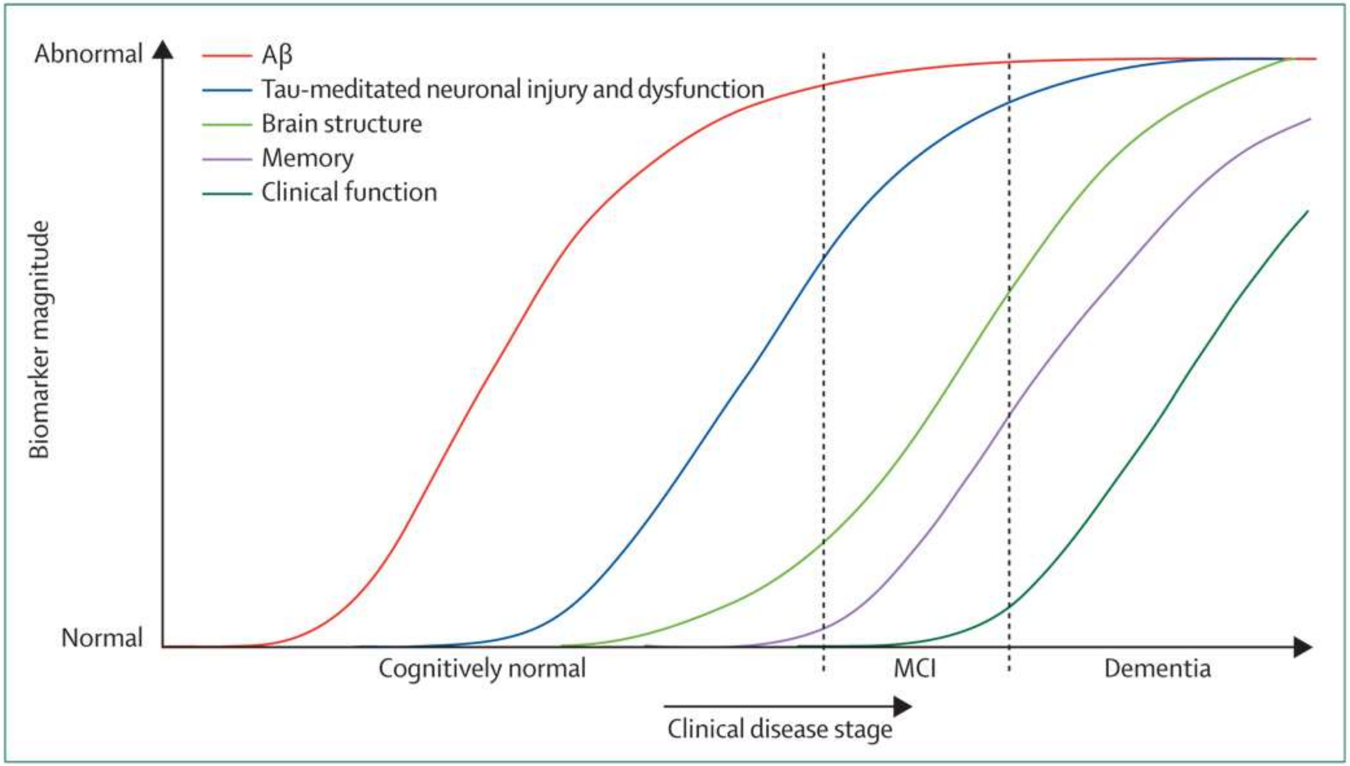
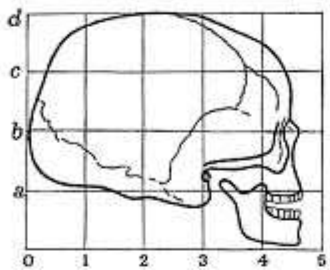


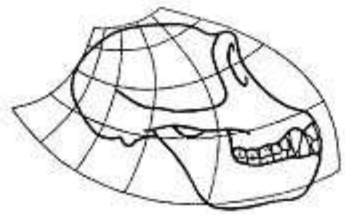
Jack et al. Lancet Neurol'10

- Need to study the dynamics of anatomical alterations for:
  - To monitor disease progression
  - Detect subjects at risk
  - Classify subjects according to patterns of anatomical alterations
- Challenges:
  - Shape data: image, 3D surface meshes, point sets, etc.
  - Infer dynamics from few time points
  - Average inter-subject differences → normative scenario

Forms and Deformations

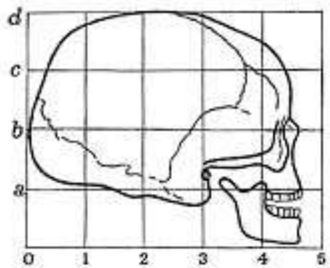
Forms and deformation

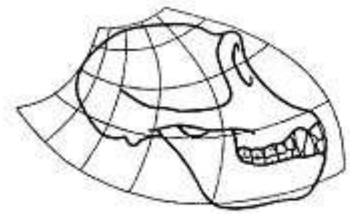




Complex *differences in shape can be described by* simple *space deformation*

Forms and deformation





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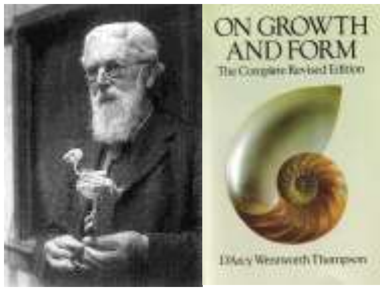

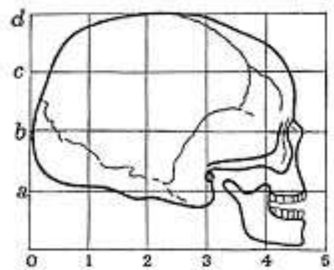

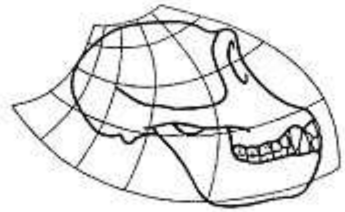

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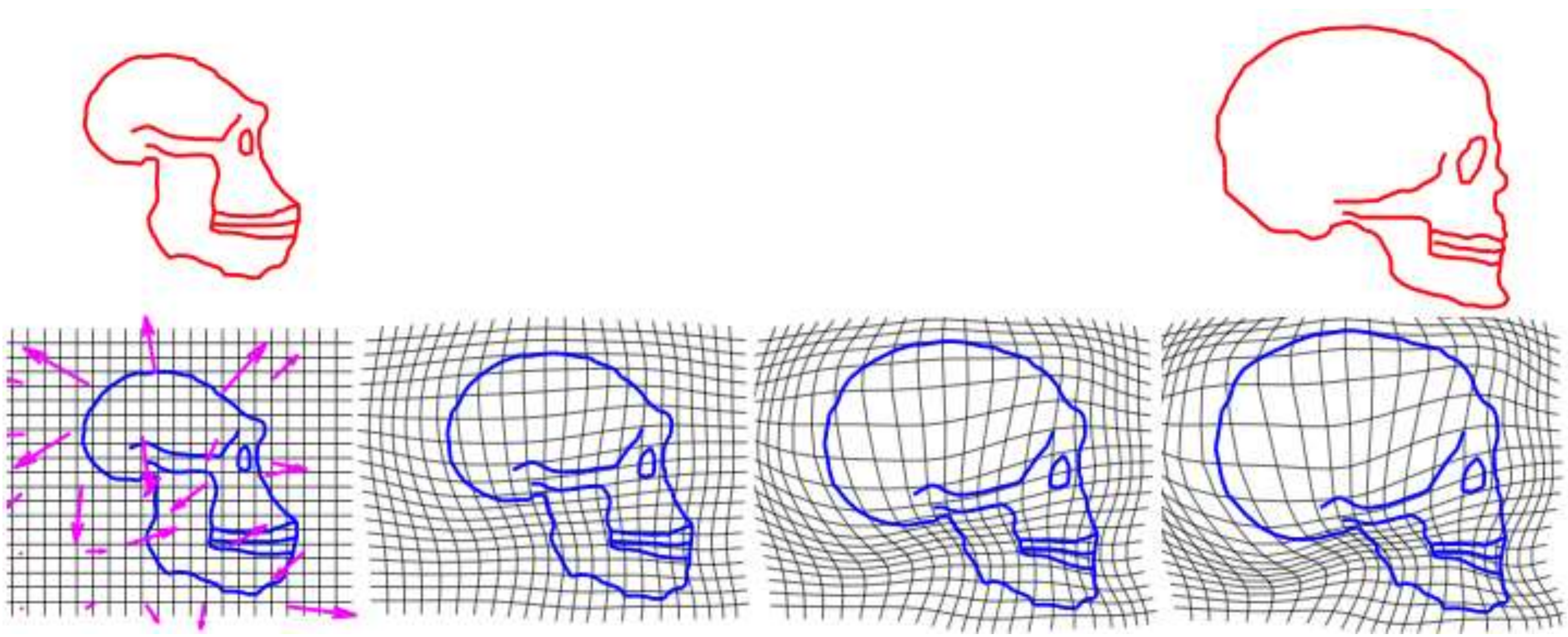

Durrleman et al. MedIA'09

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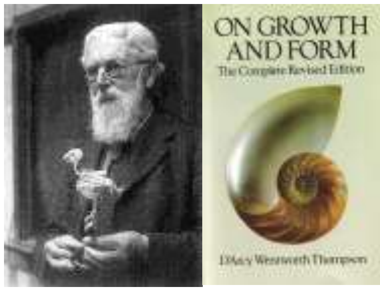

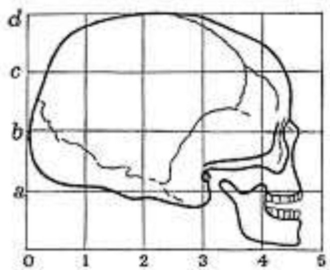

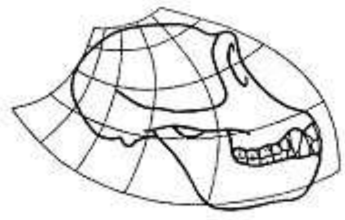

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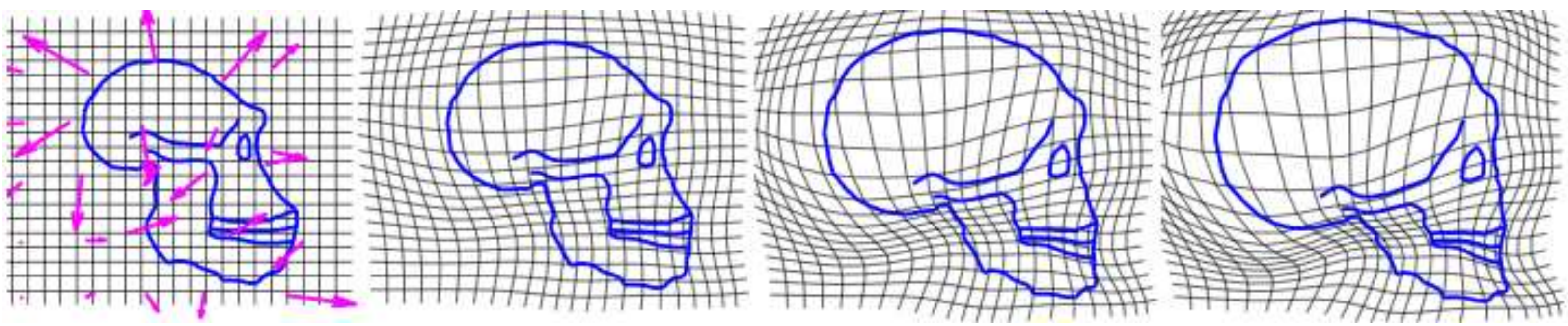

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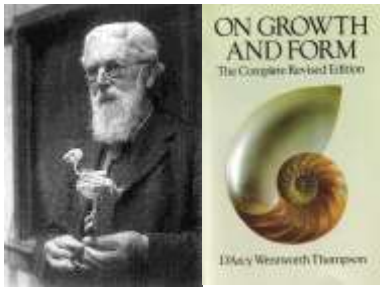

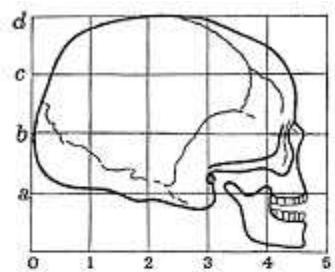

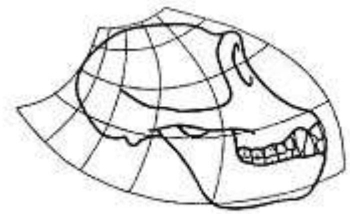

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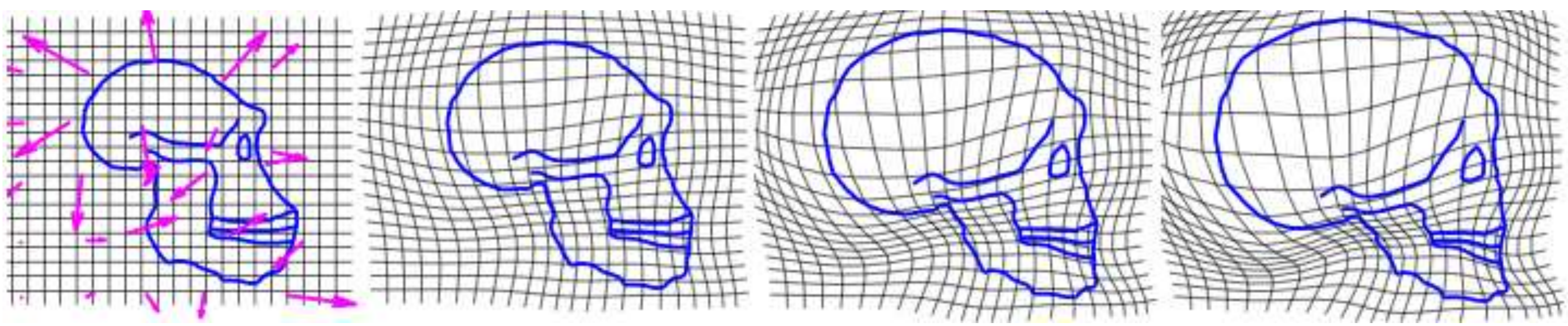

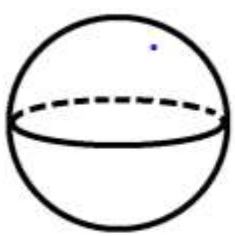

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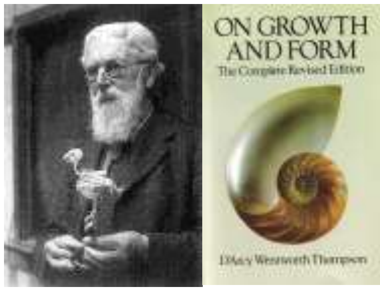

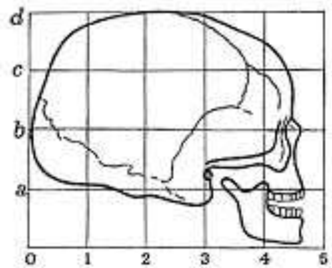

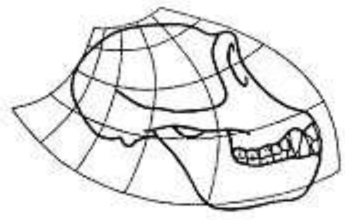

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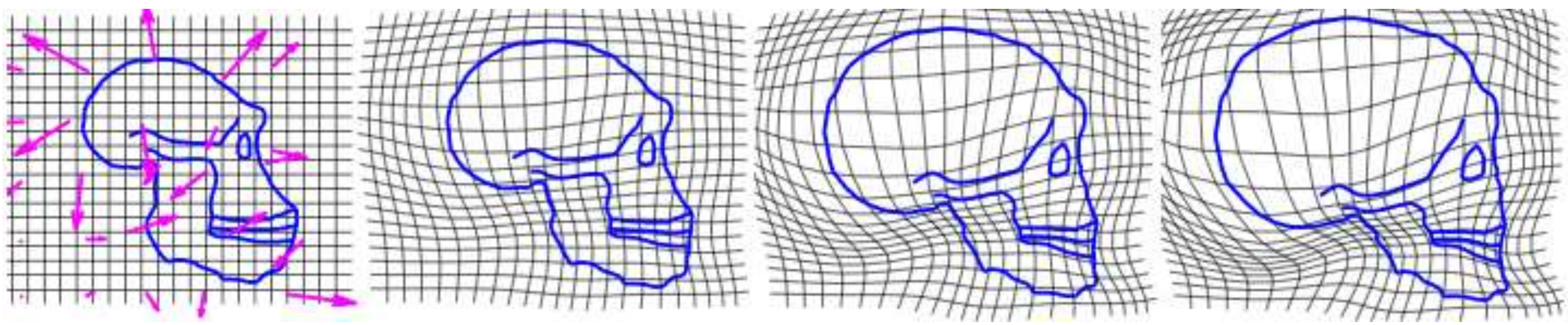

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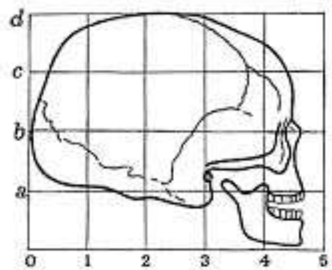

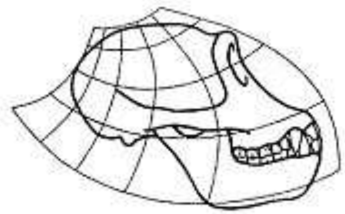

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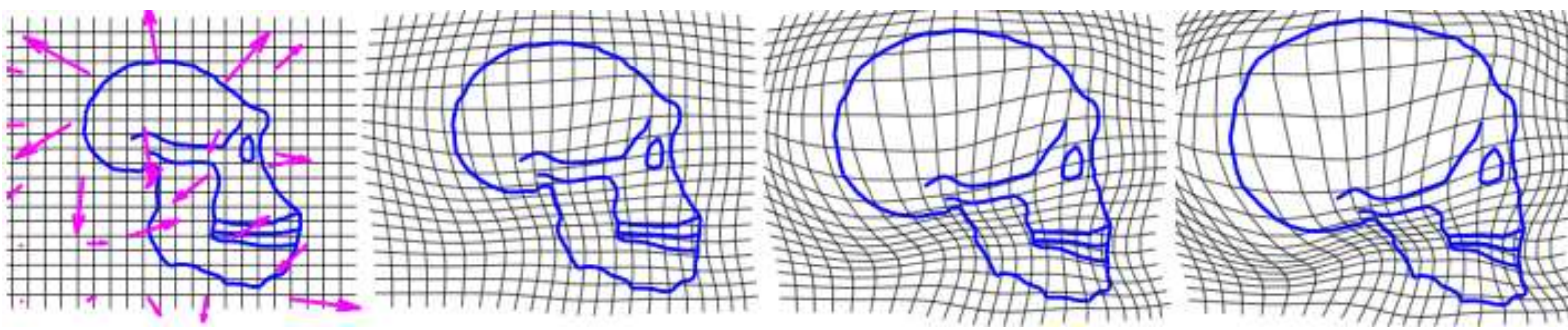

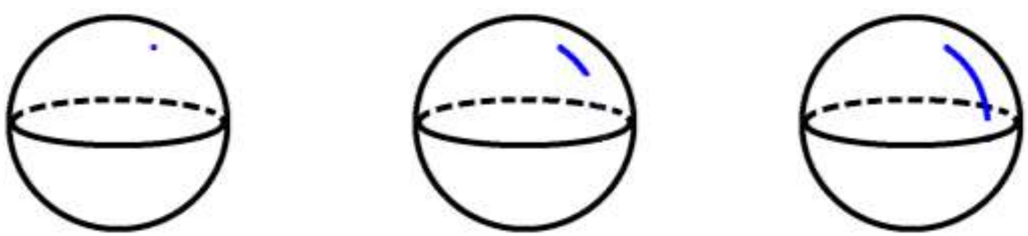

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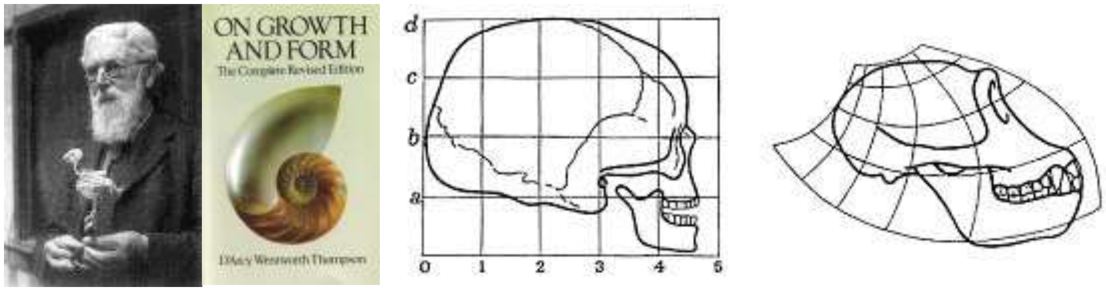

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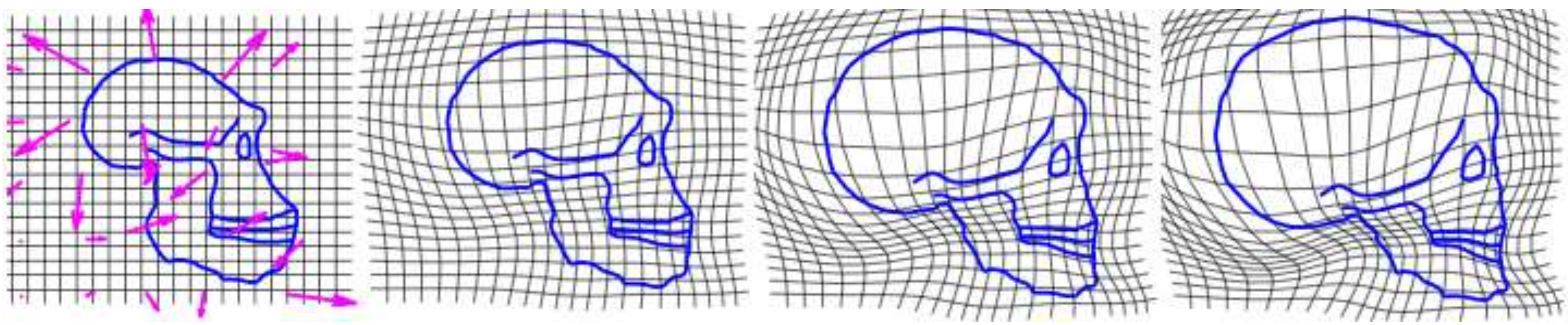

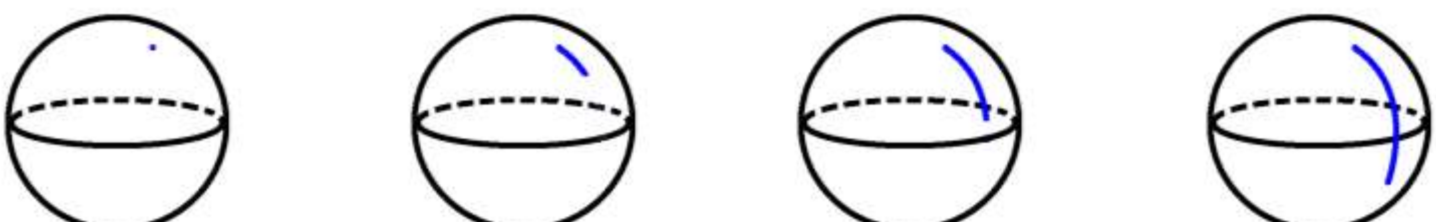

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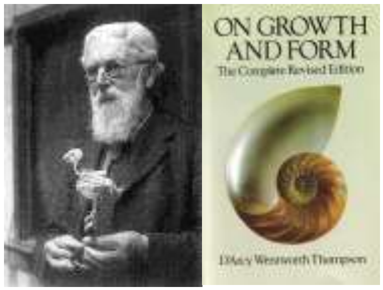

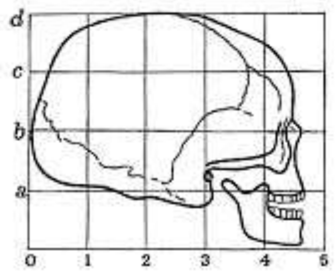

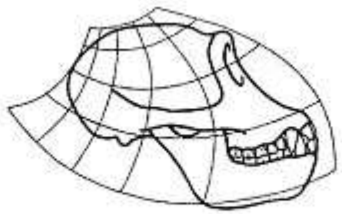

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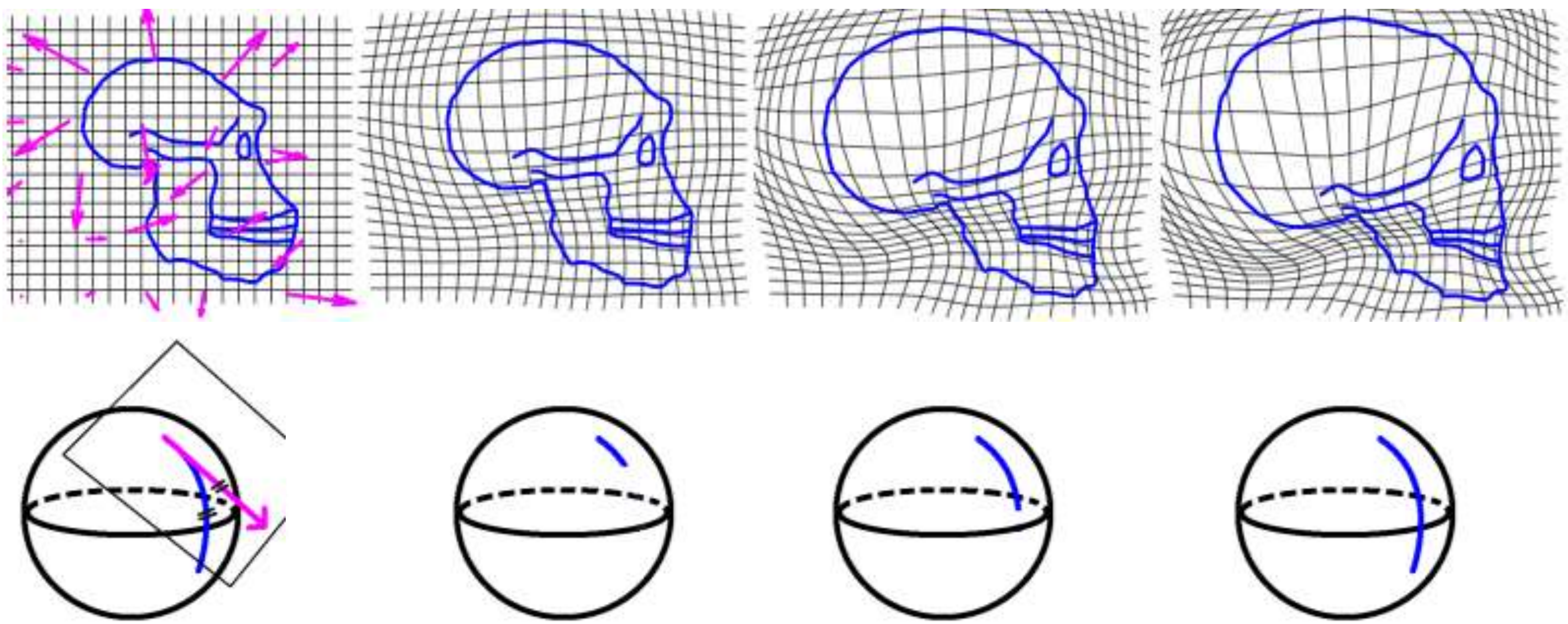

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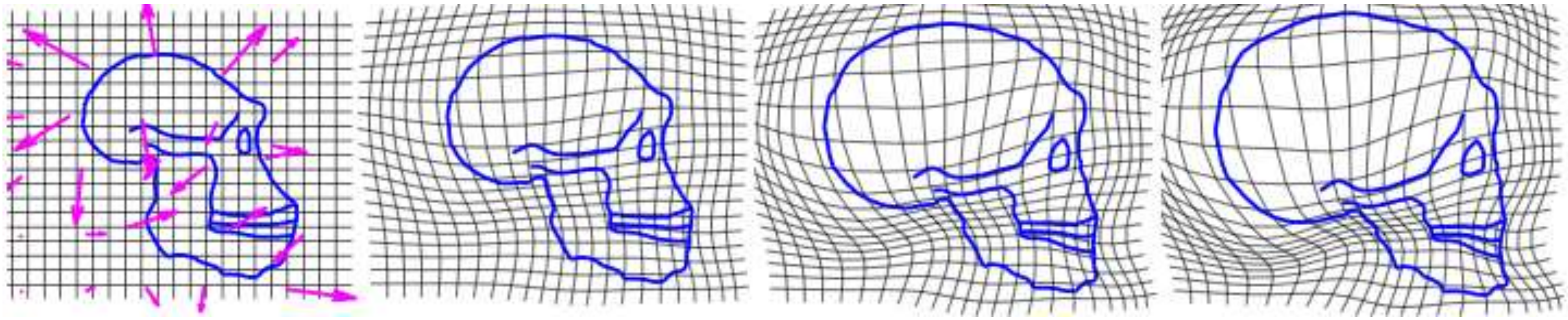

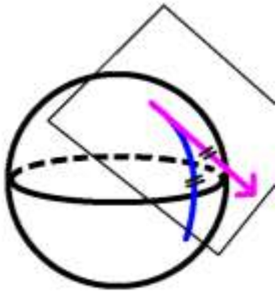

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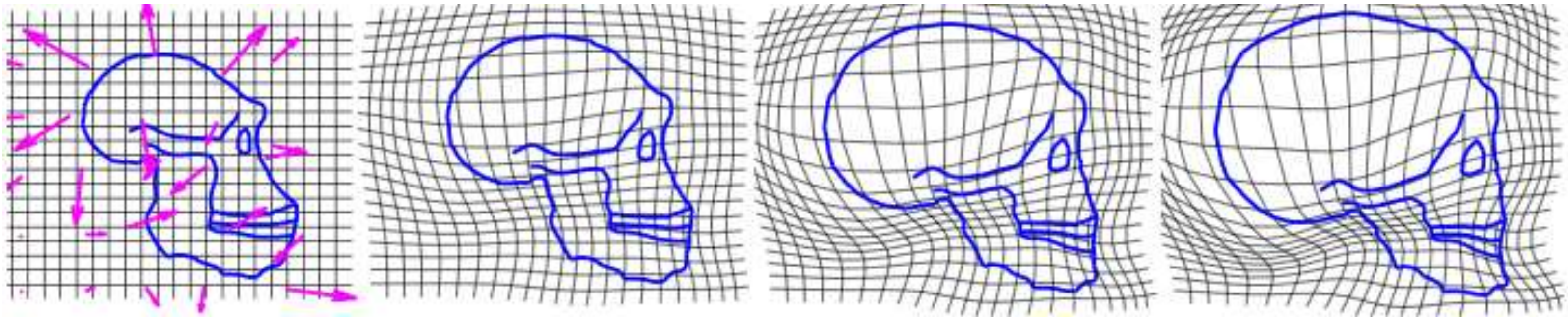

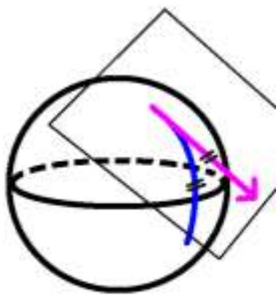

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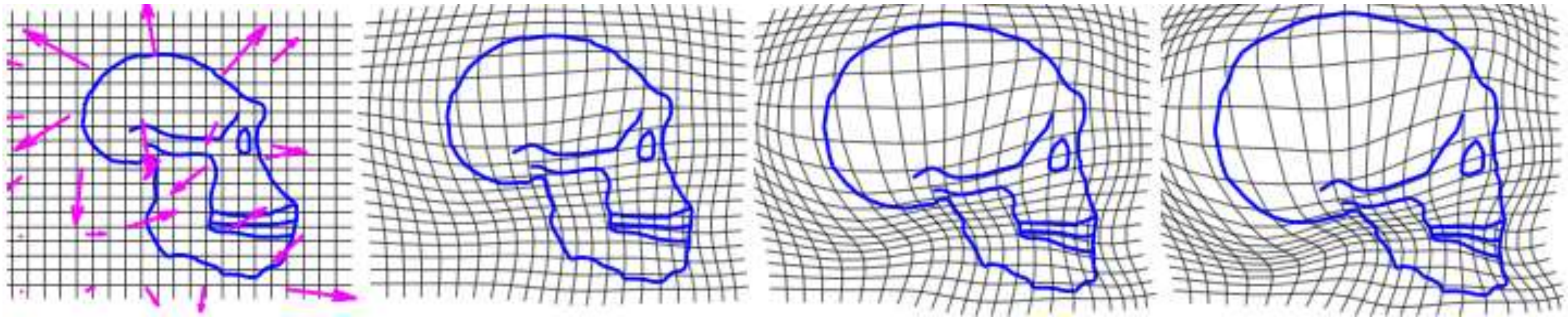
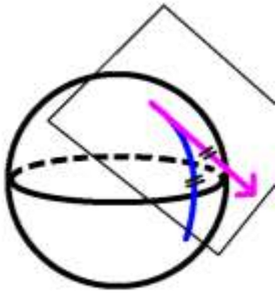
Forms and deformation





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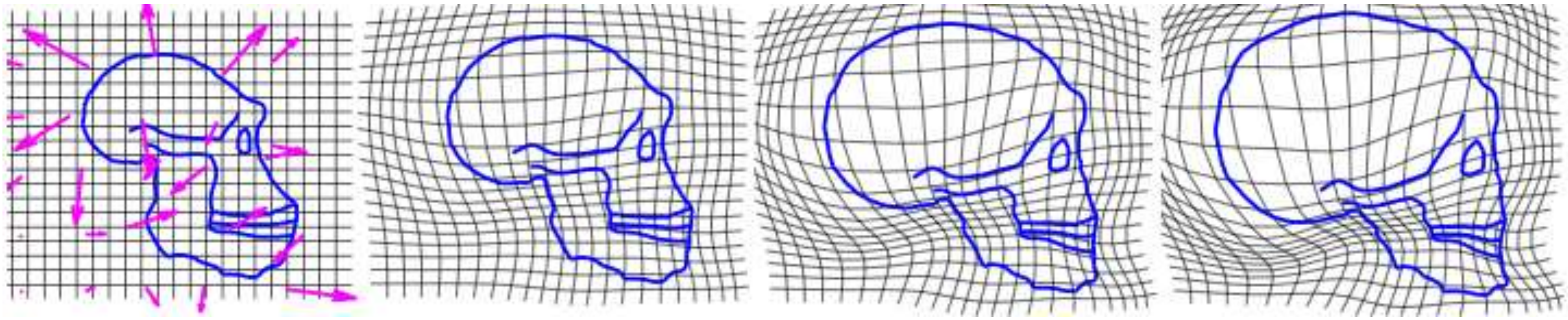

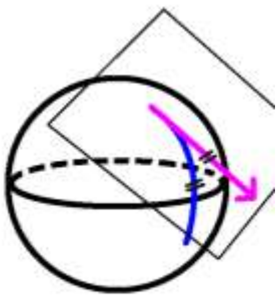

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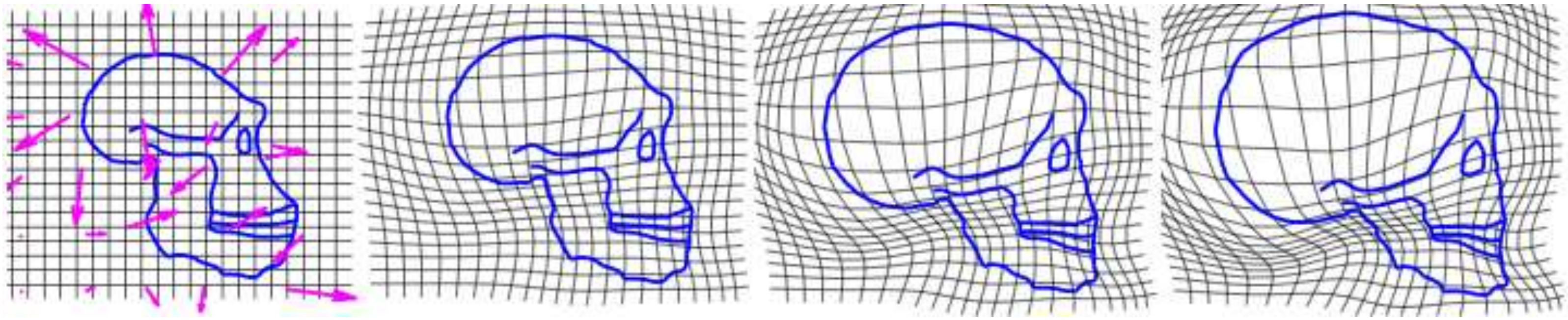

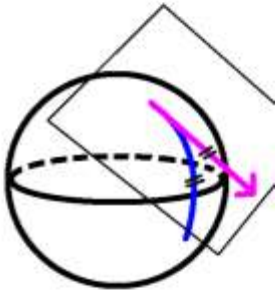

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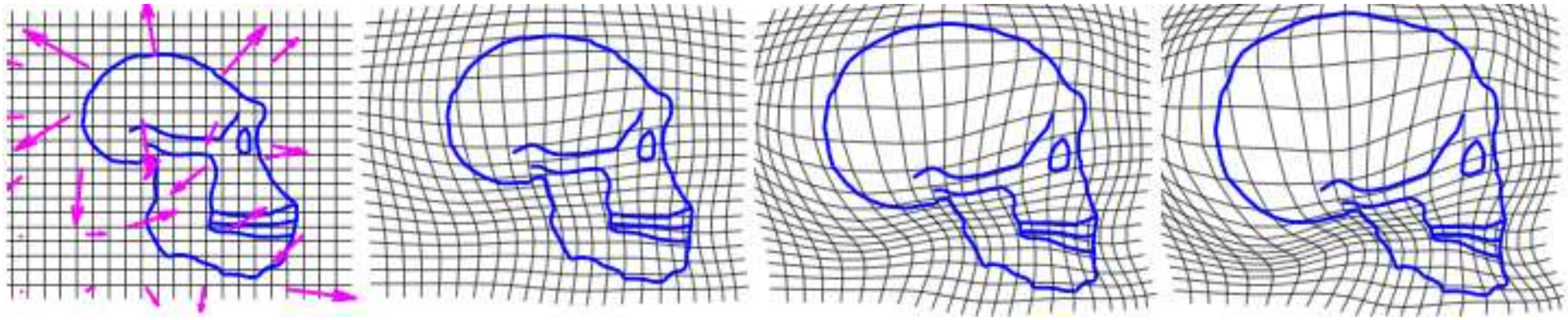

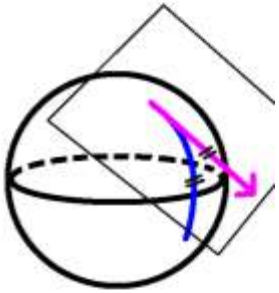

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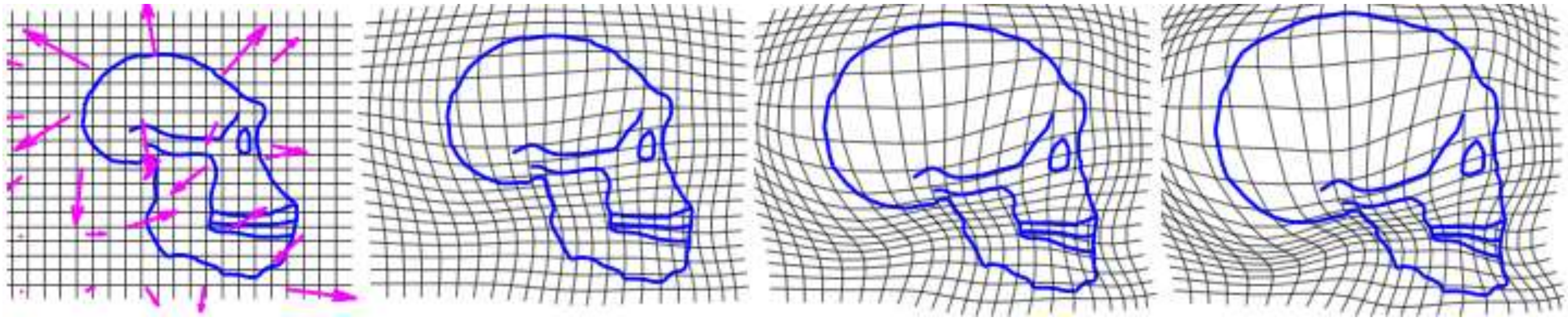

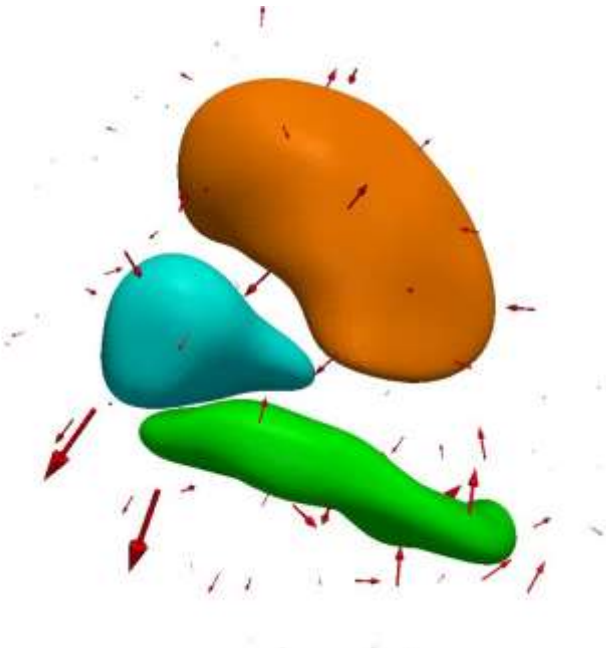

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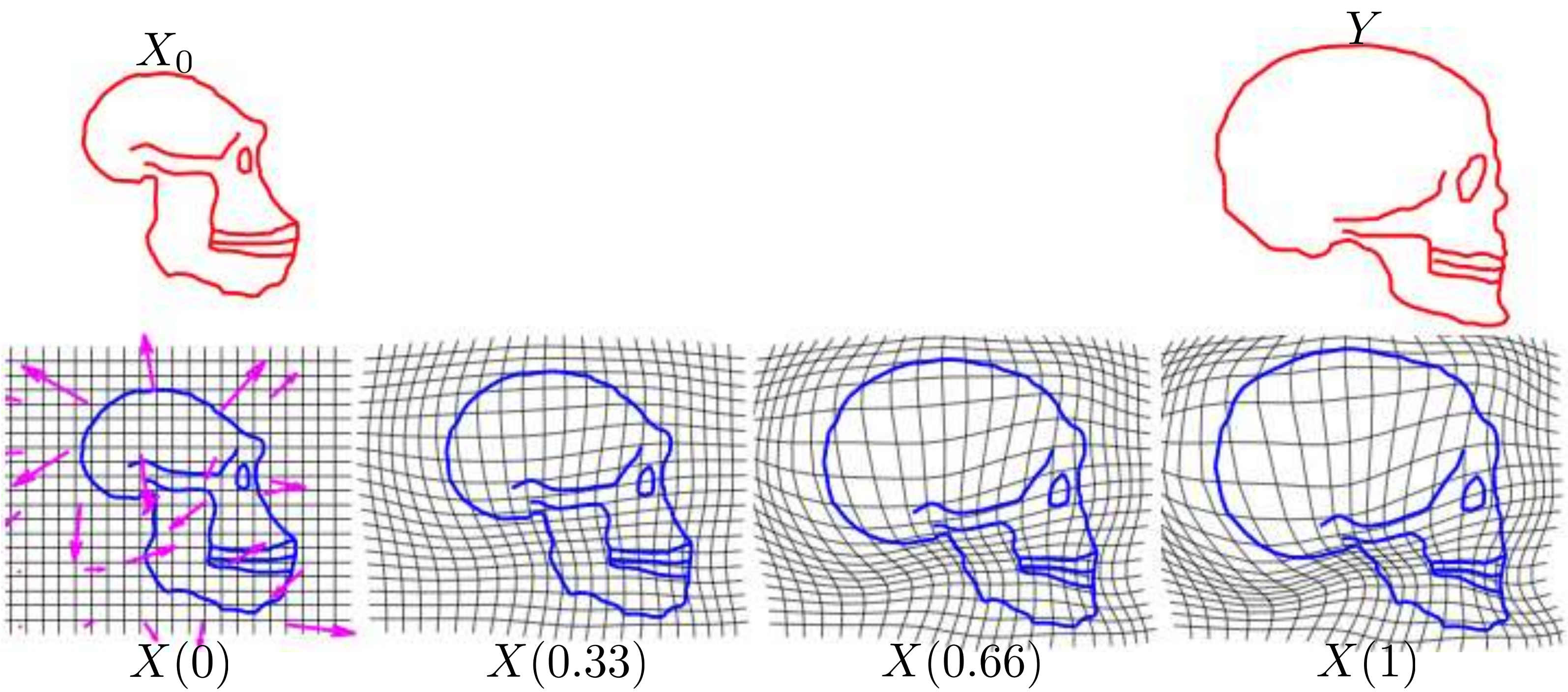

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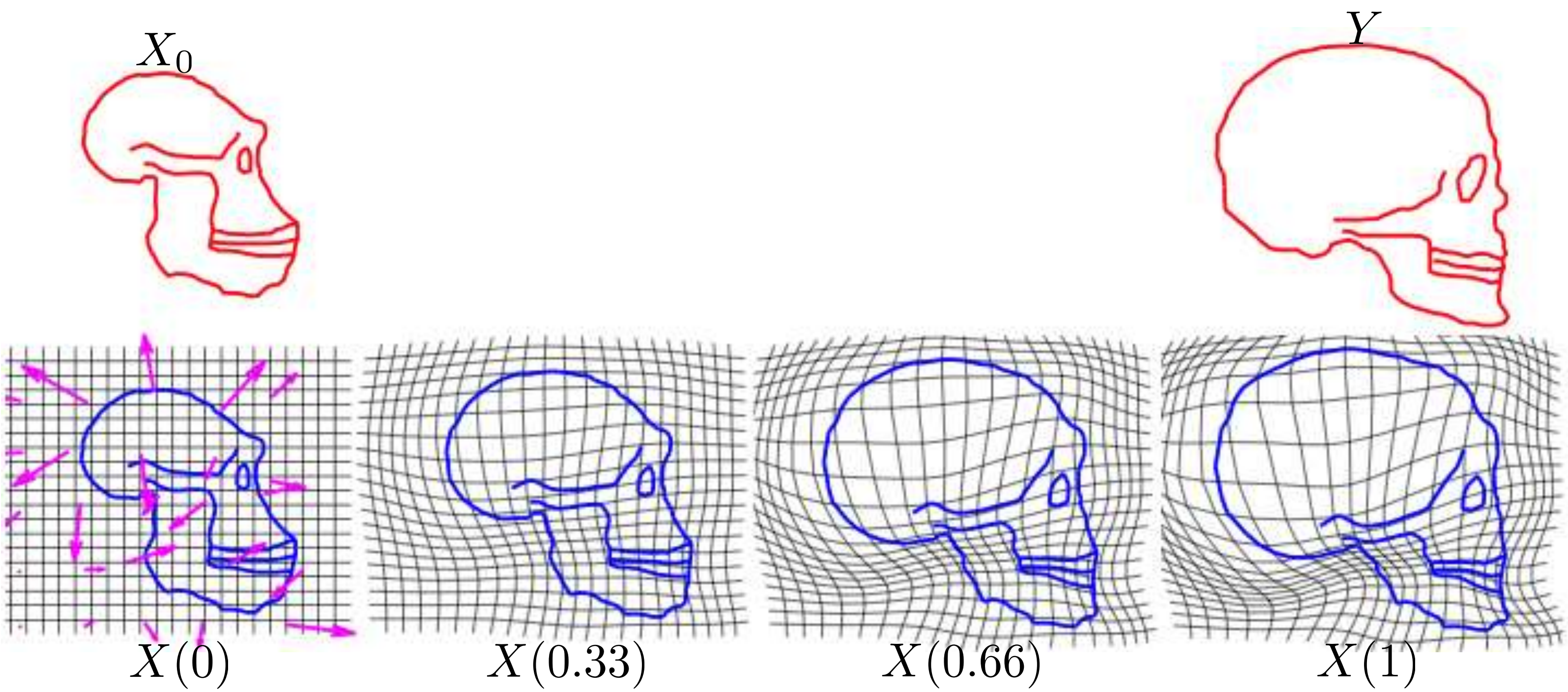
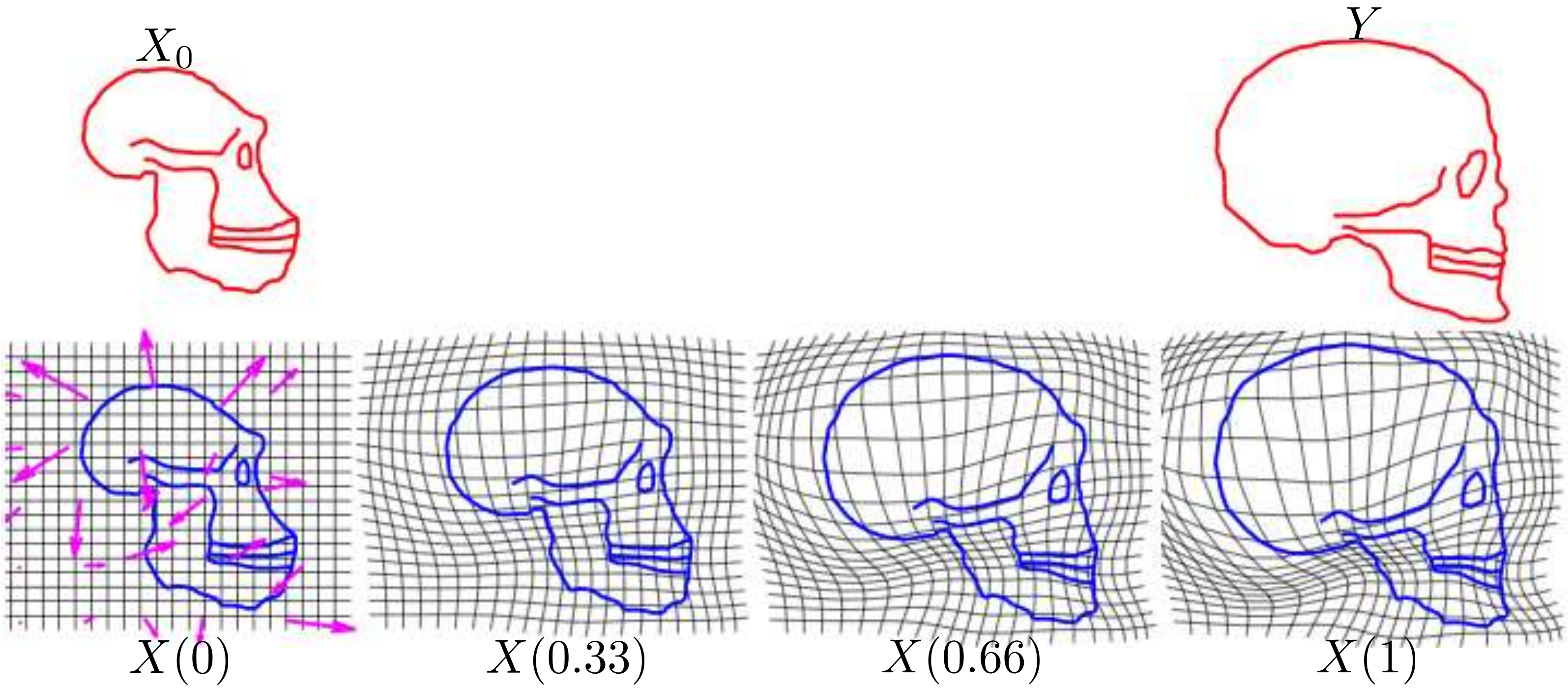


$$E\Big(S(t)\Big) = \frac{1}{2\sigma^2} D(X(1), Y)^2 + \int_0^1 \|v(t,.)\|_K^2$$

Forms and deformation



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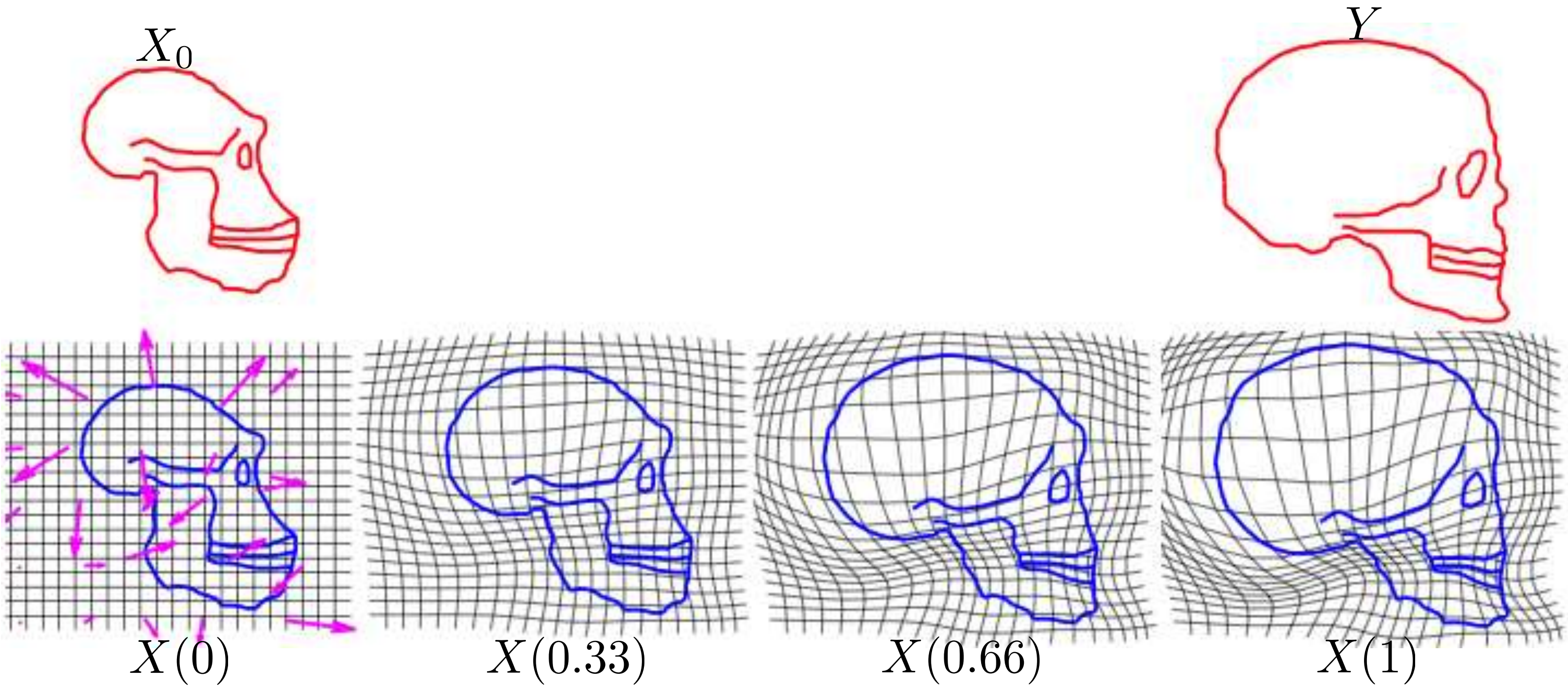

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The solution is a geodesic path!

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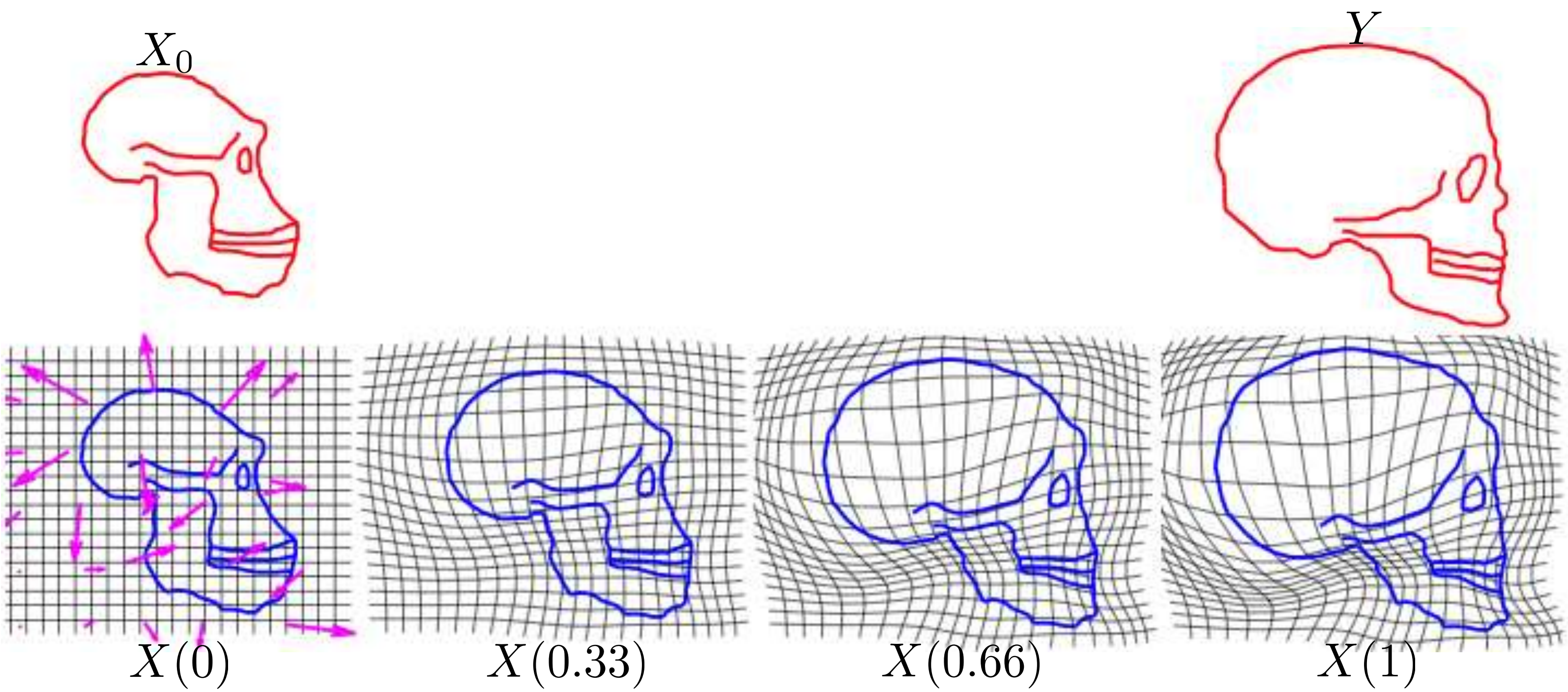

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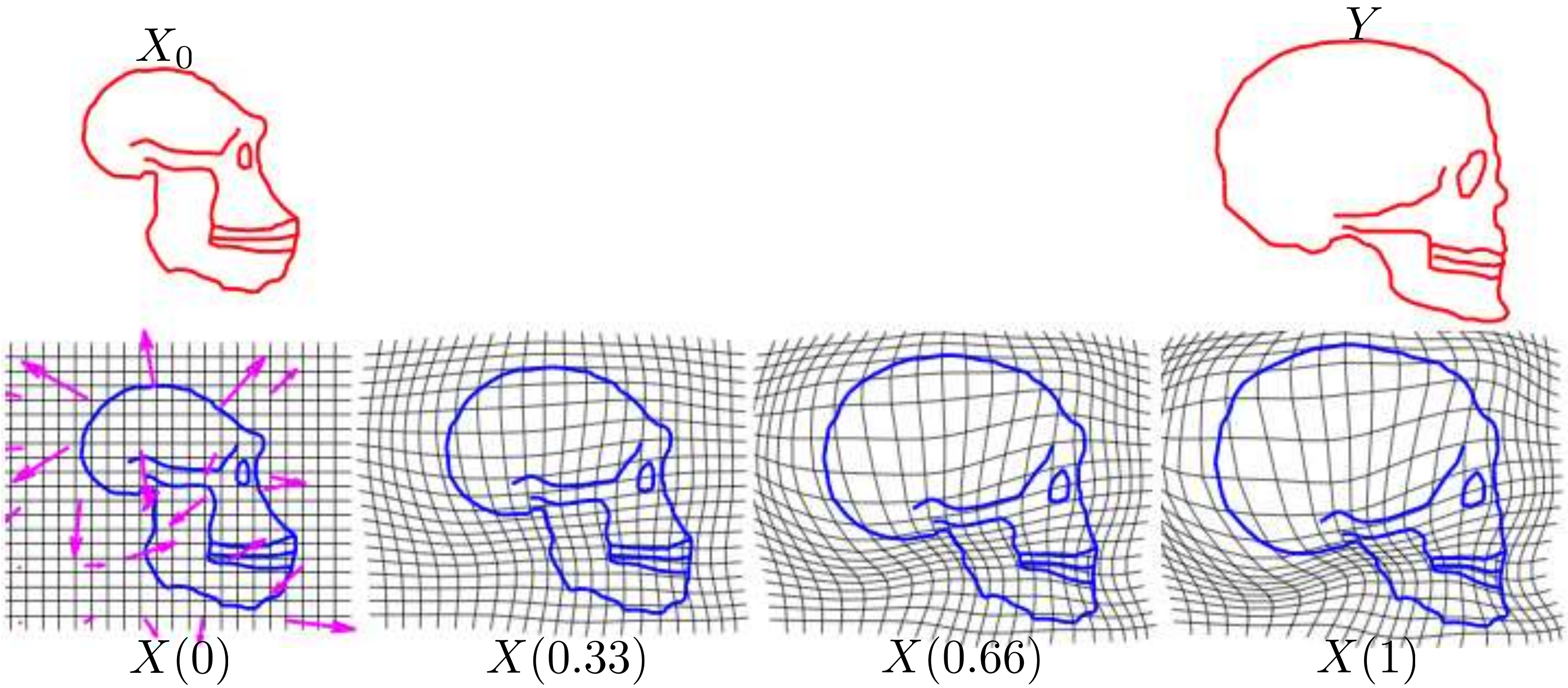

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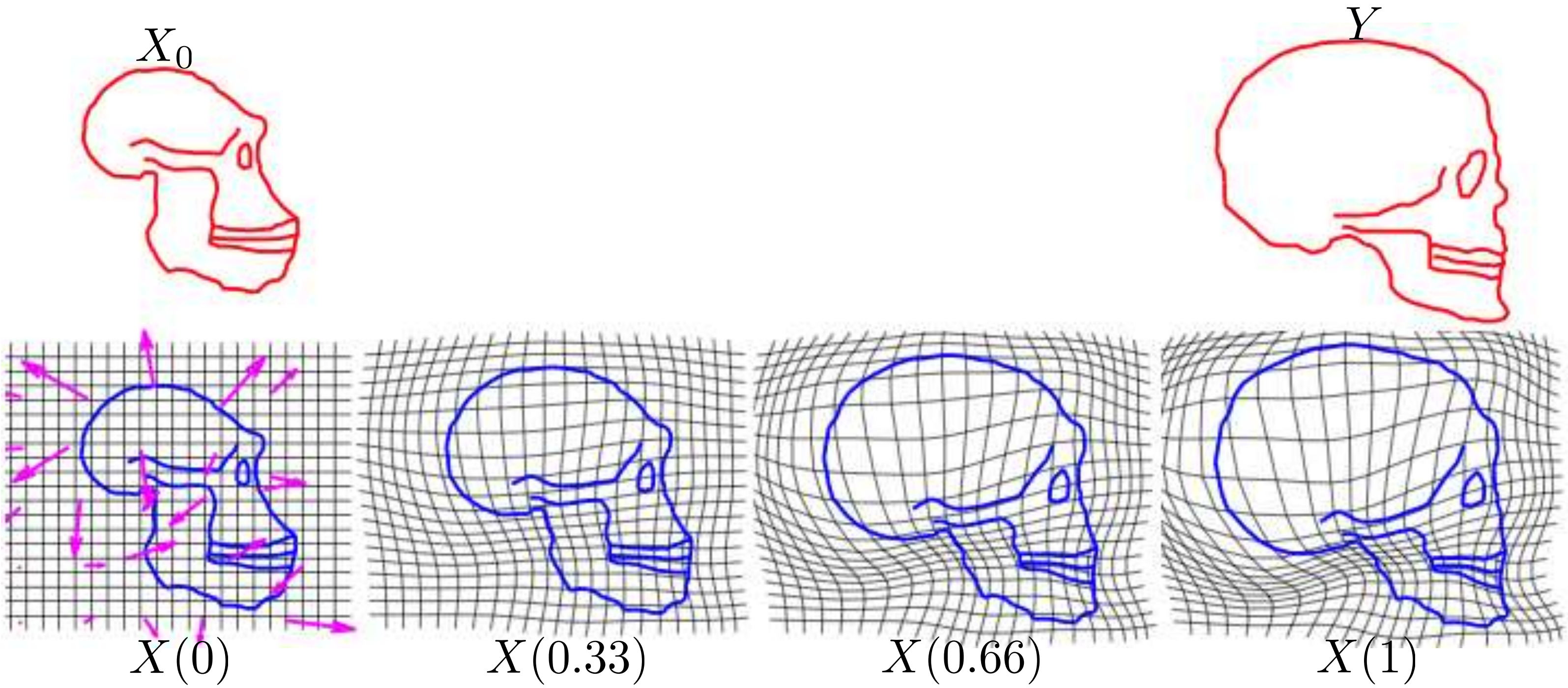

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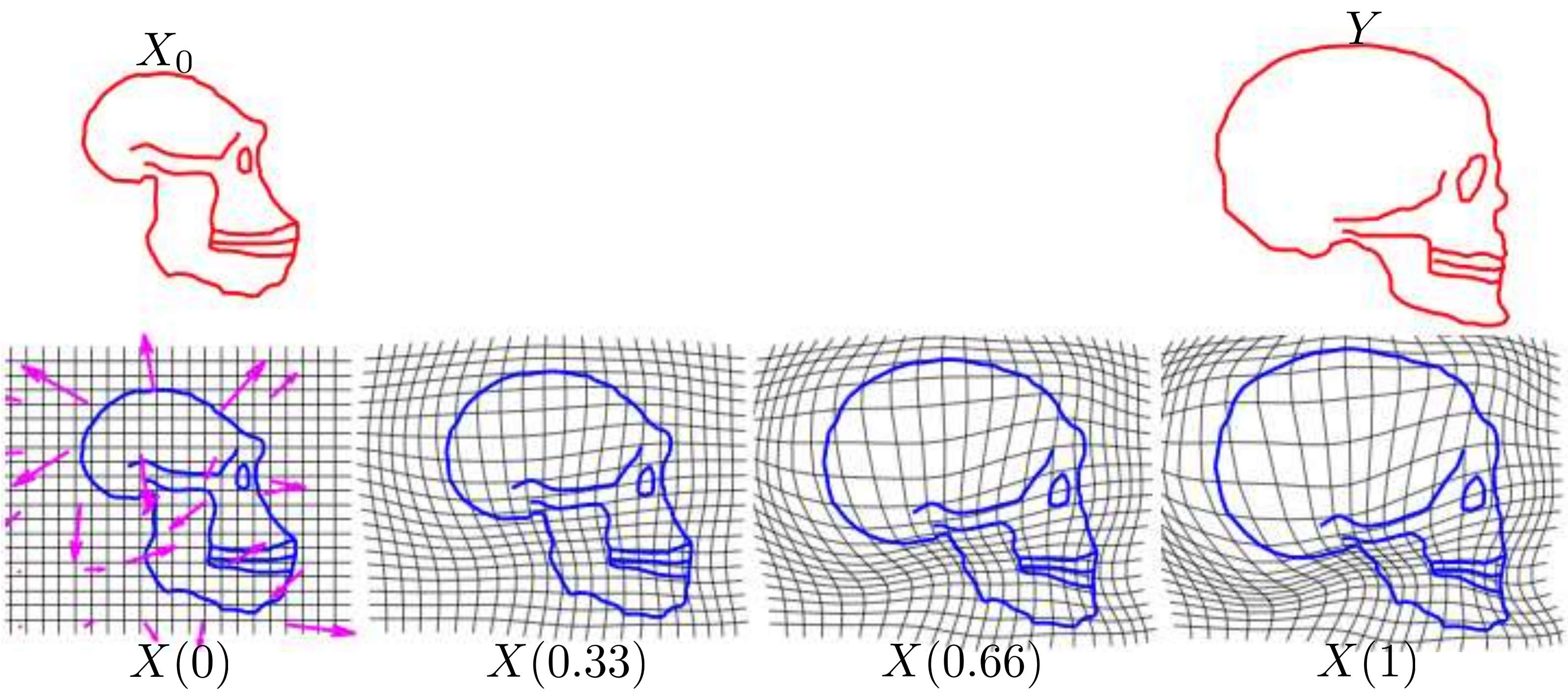
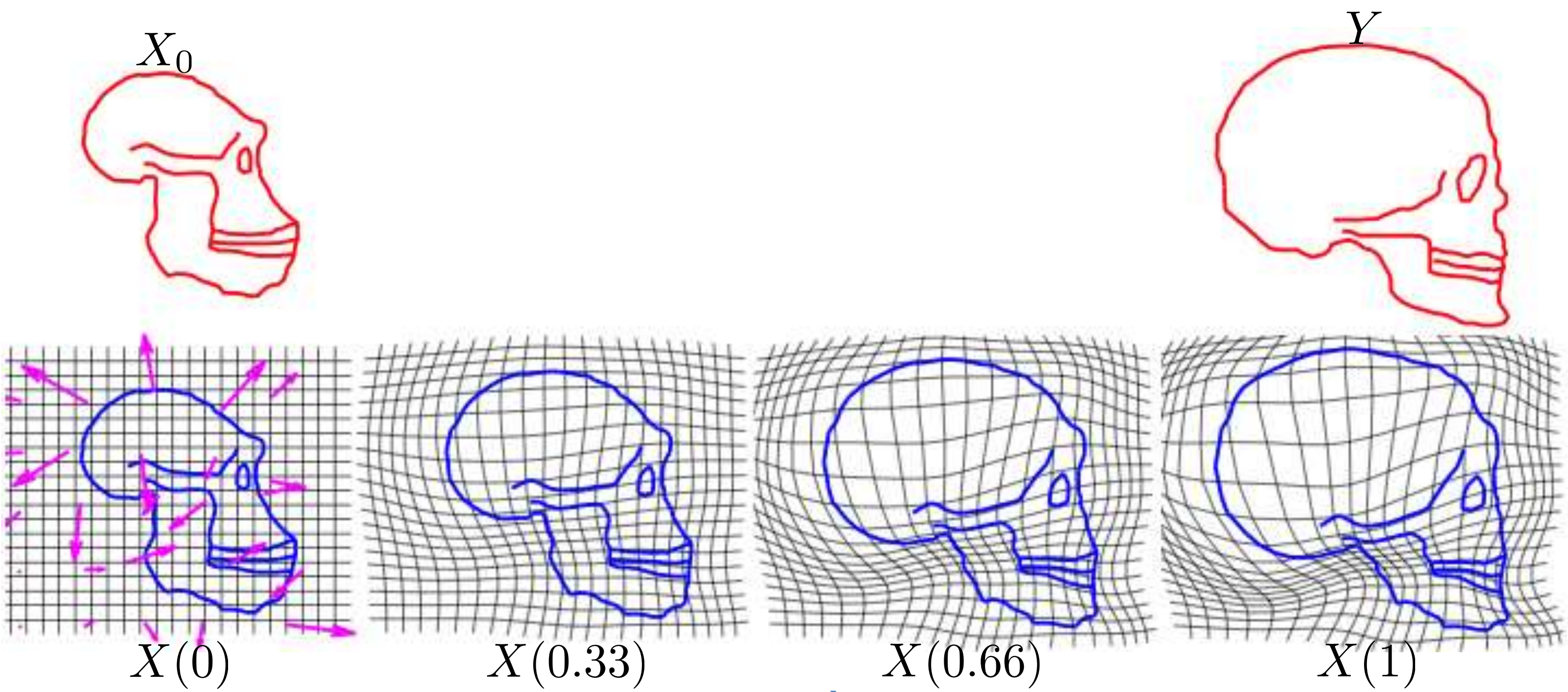
Forms and deformation



Forms and deformation



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$$s.t. \quad \dot{X}(t) = G(S(t), X(t)) \quad X(0) = X_0$$

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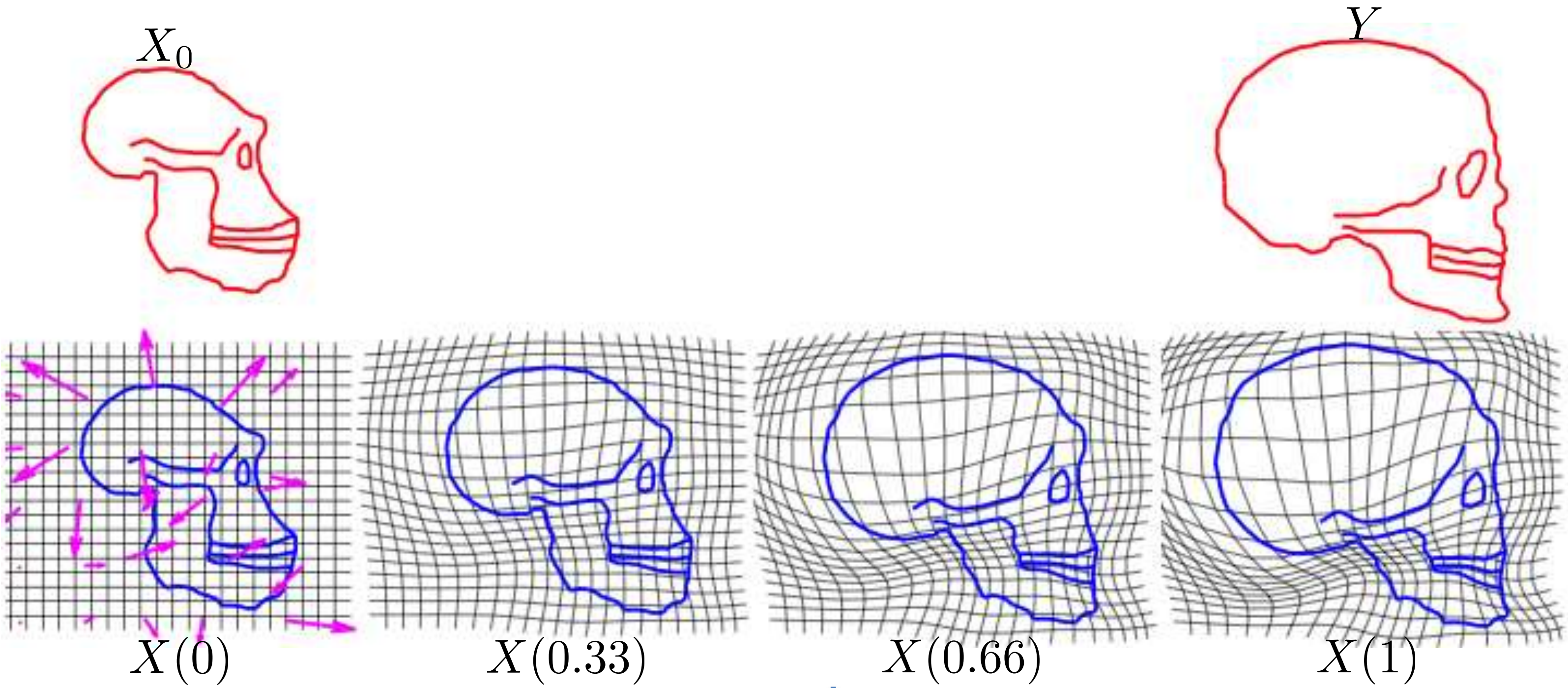
Same theoretical solutions, different algorithms

Forms and deformation



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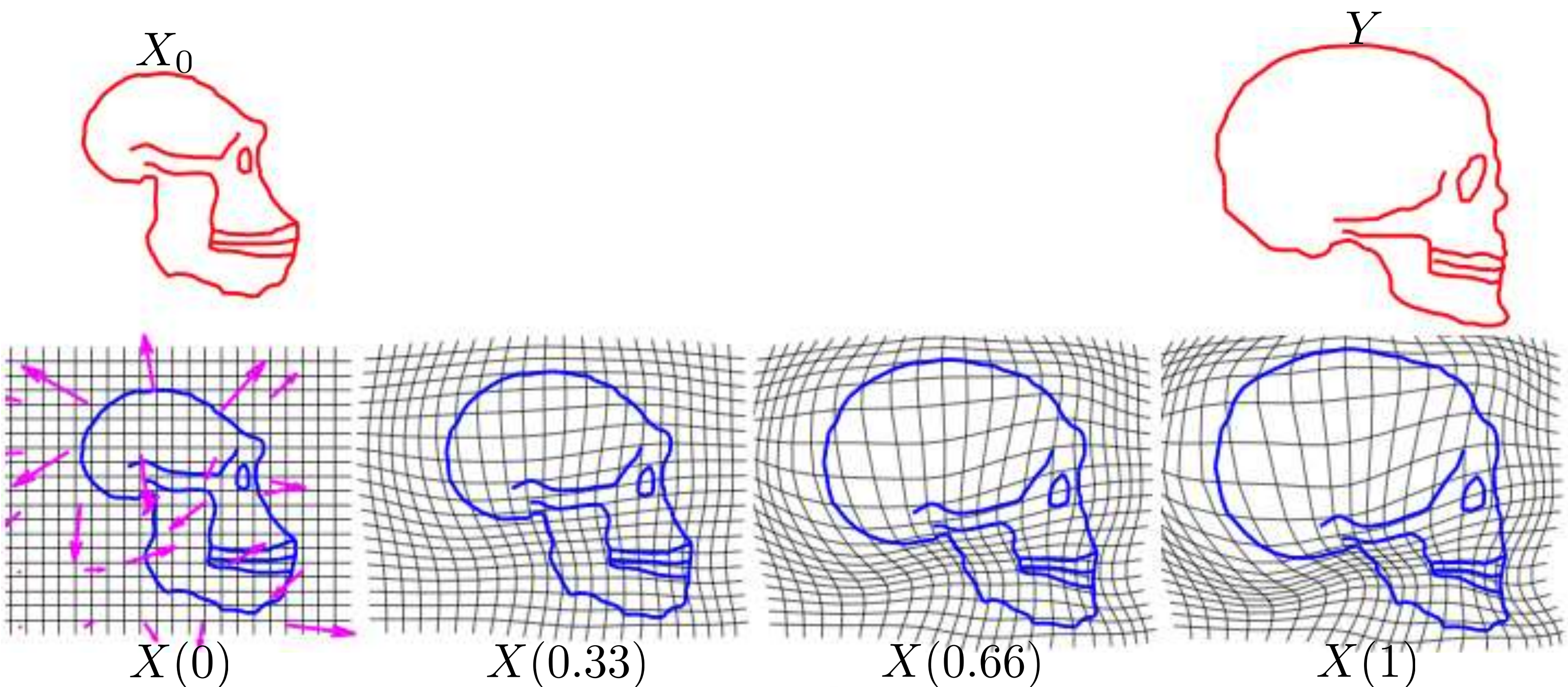
Forms and deformation



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$$\nabla_{S(t)} E = \partial_2 G^T \theta(t) + \nabla_{S(t)} L$$

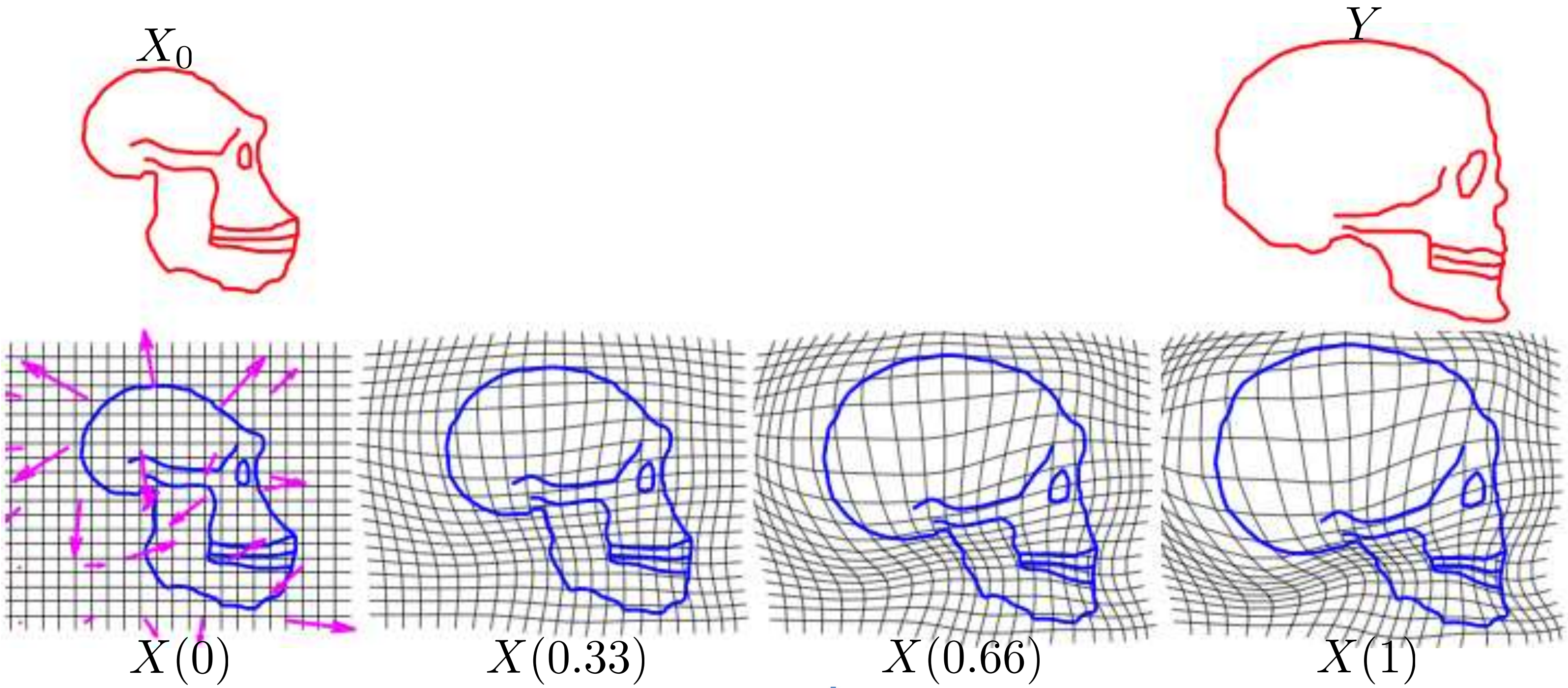
$$with \quad \dot{\theta}(t) = -\partial_1 G(t)^T \theta(t) \quad \theta(1) = \nabla_{X(1)} D$$

$$\nabla_{S_0} E = \xi(0) + \nabla_{S_0} L$$

$$with \quad \dot{\theta}(t) = -\partial_1 G(t)^T \theta(t) \quad \theta(1) = \nabla_{X(1)} D$$

$$\dot{\xi}(t) = -\partial_2 G^T \theta(t) - d_{S(t)} F^T \xi(t) \quad \xi(1) = 0$$

Forms and deformation



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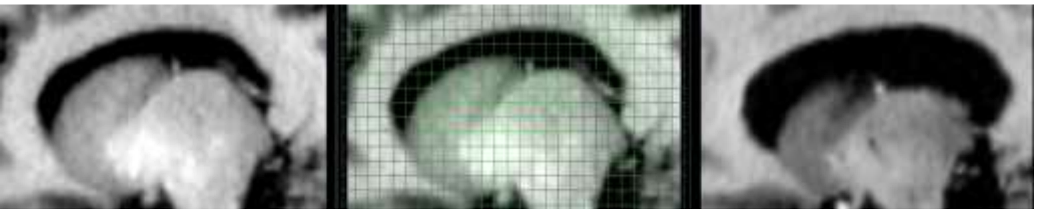
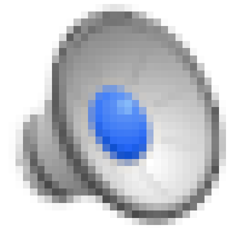
$D(X(1), Y) = \|X(1) - Y\|$, or norm of currents, norm of varifolds, norm between images, etc..

[Glaunès'05, Durrleman'08, Charon'13, …]

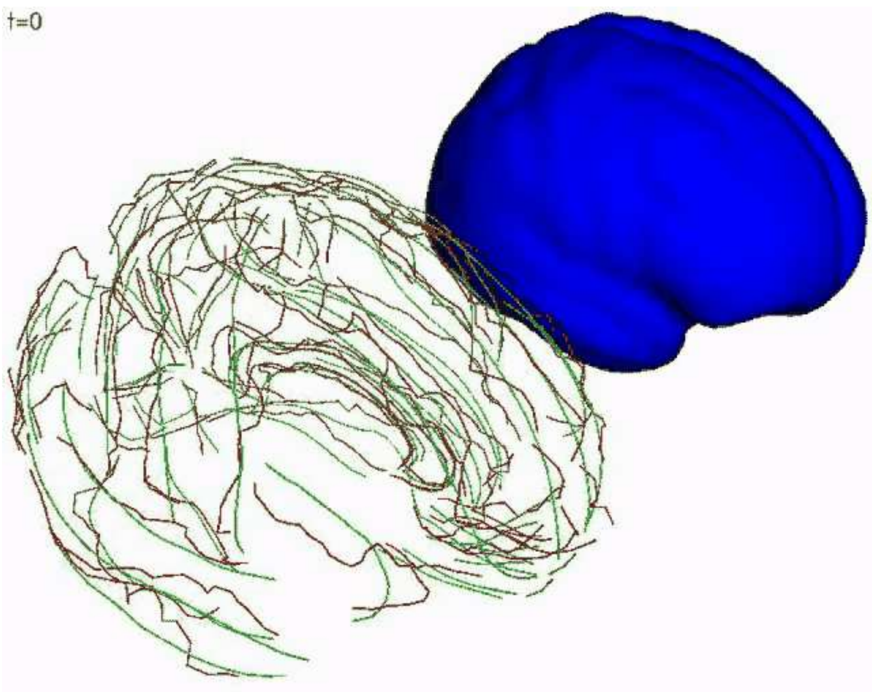
Forms and deformations



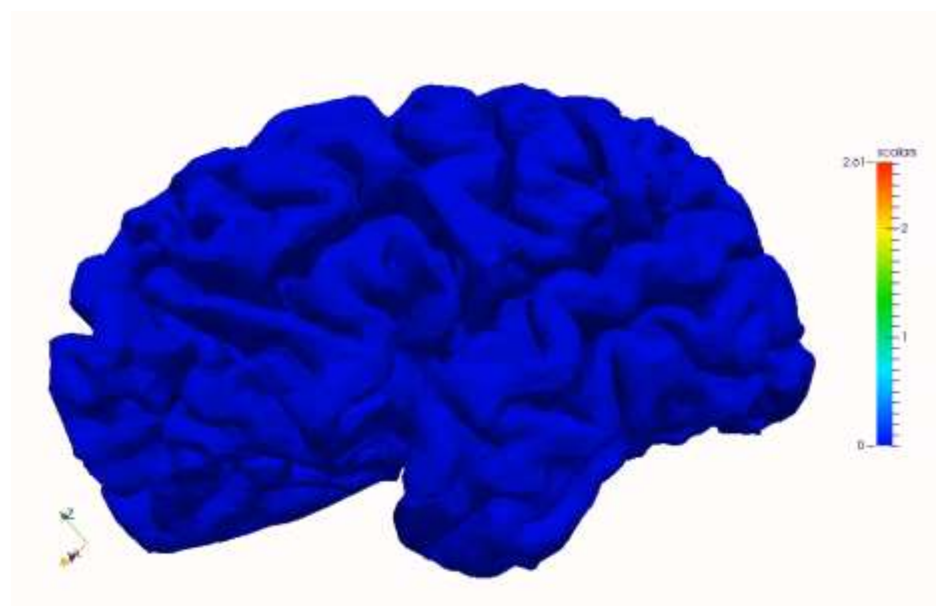
Atlas-to-patient registration of basal ganglia [Fouquier et al. DBSMC'14]



Deformation of white matter tracts [Gori et al. MICCAI'13]



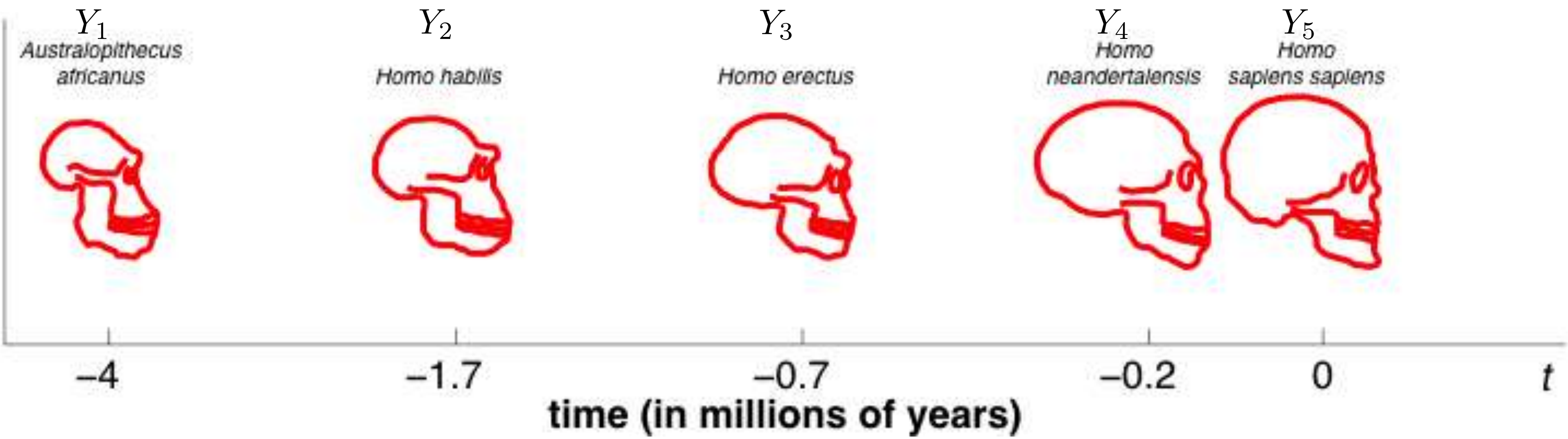
Registration of sulcal curves [Durrleman et al. Media'08]



Registration cortical surface between baseline and follow-up

Regression of time-series shape data

Regression of time-series shape data



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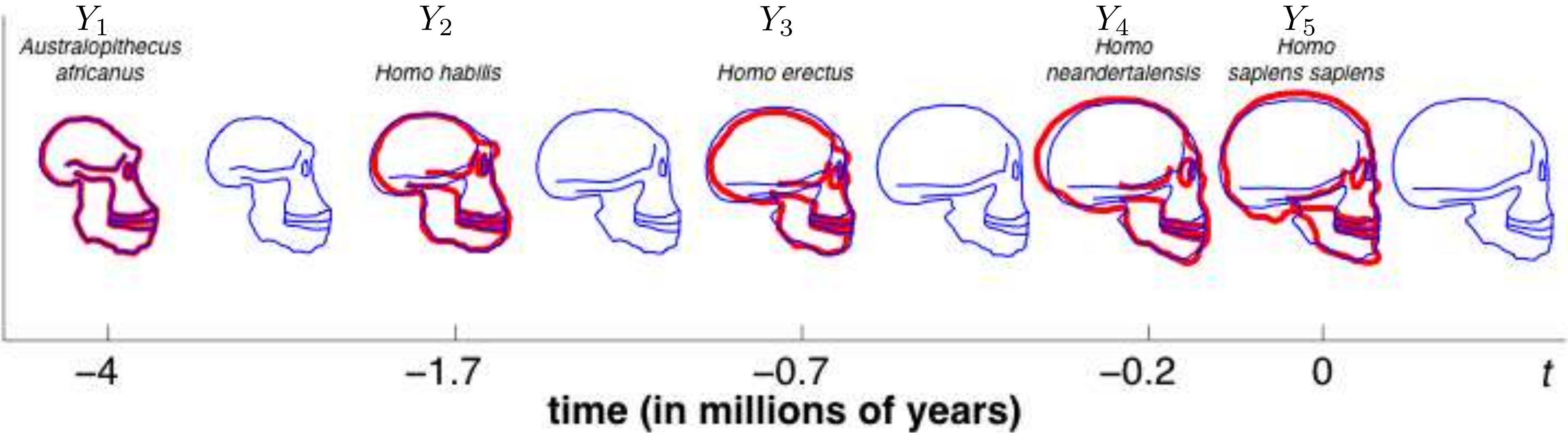

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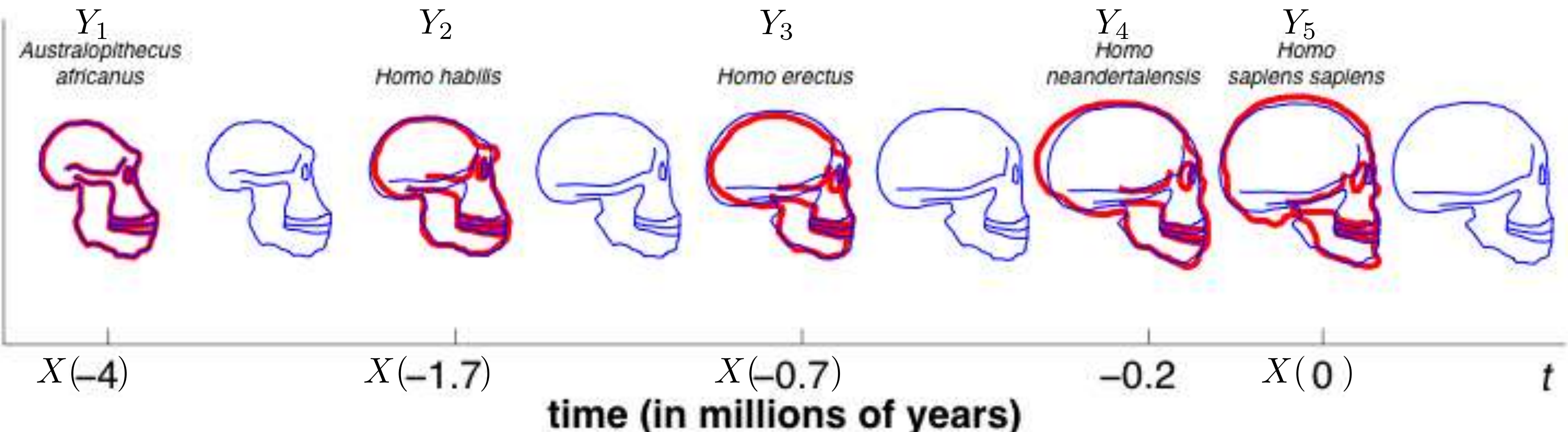

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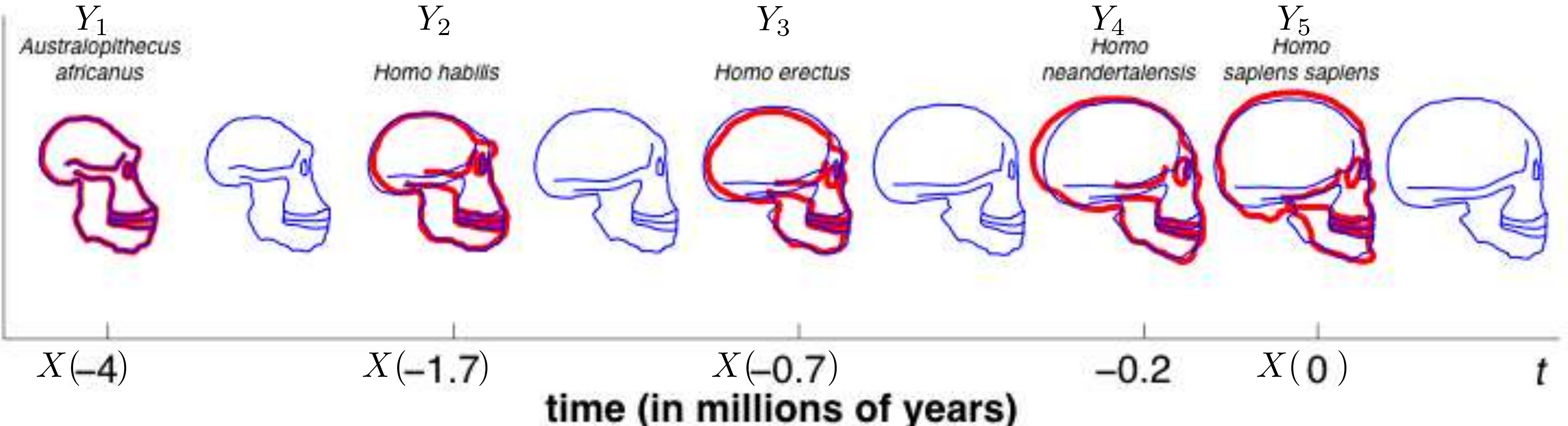

$$E\Big(S(t)\Big) = \frac{1}{2\sigma^2}\sum_i D(X(t_i), Y_i)^2 + \int_0^1 L(S(t))dt$$

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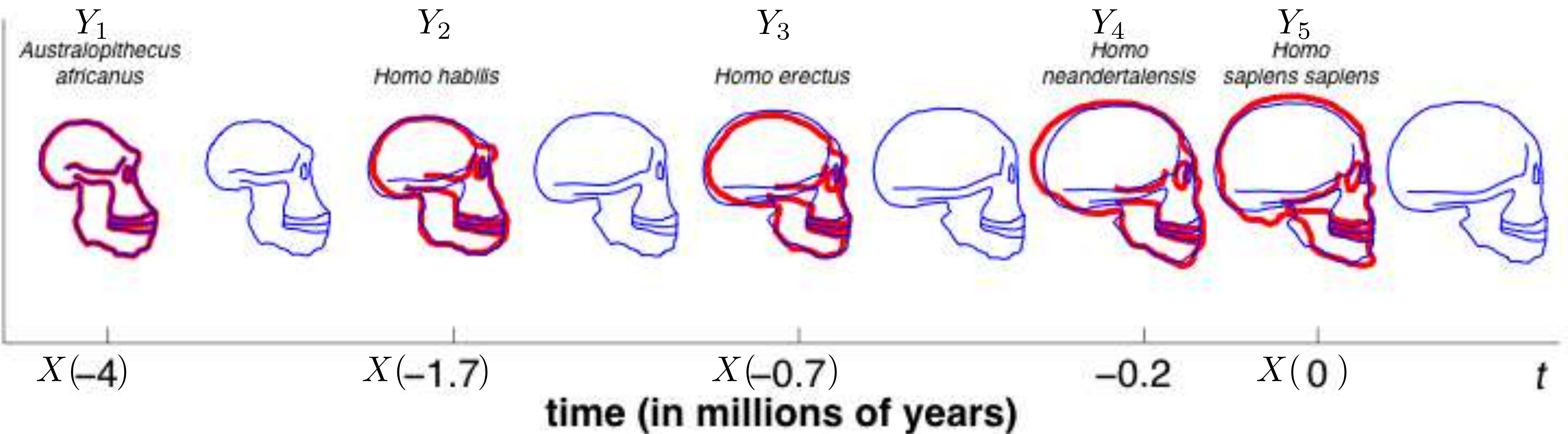

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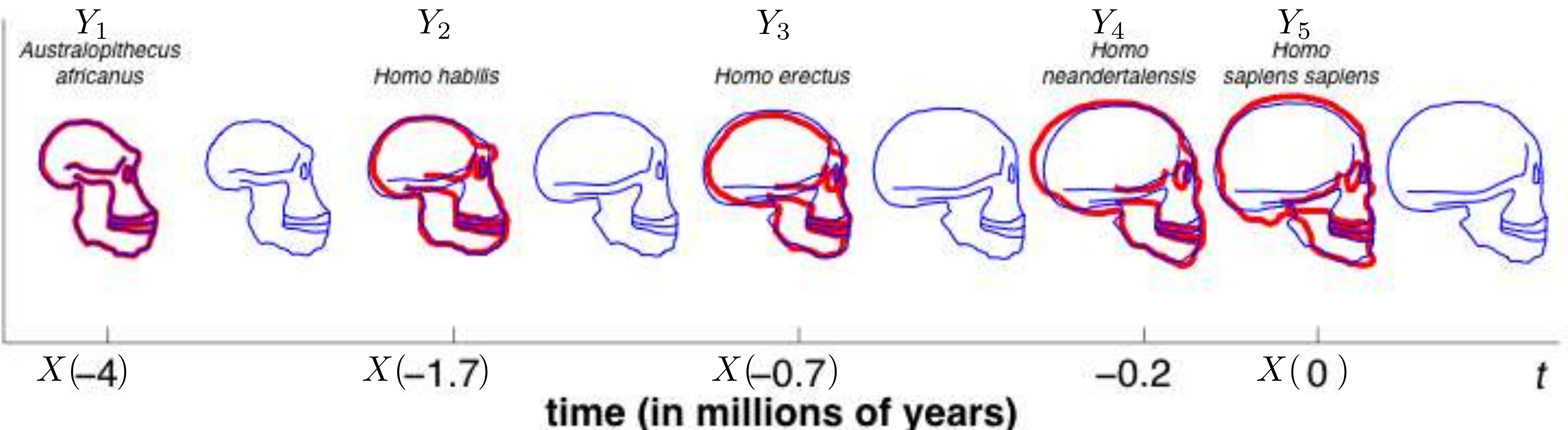

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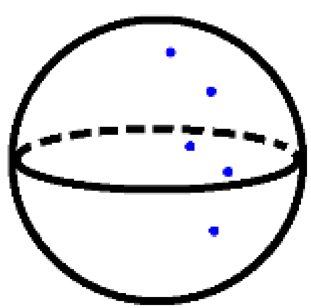
Piecewise geodesic solution



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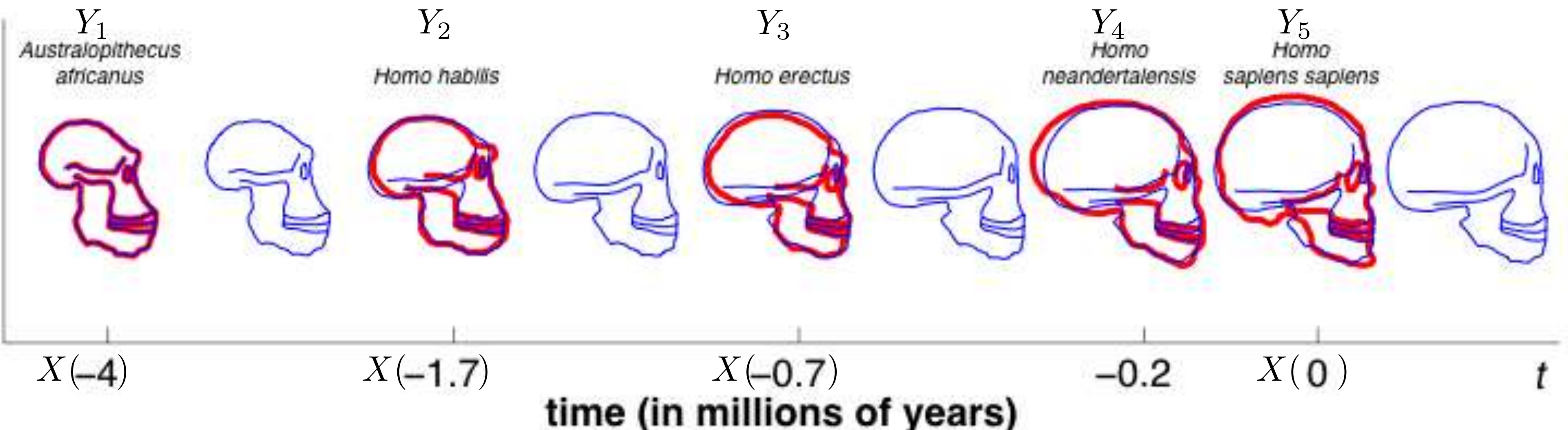

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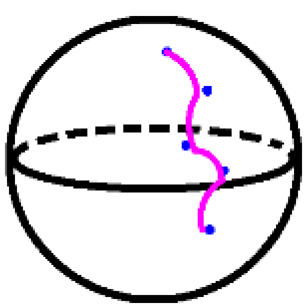

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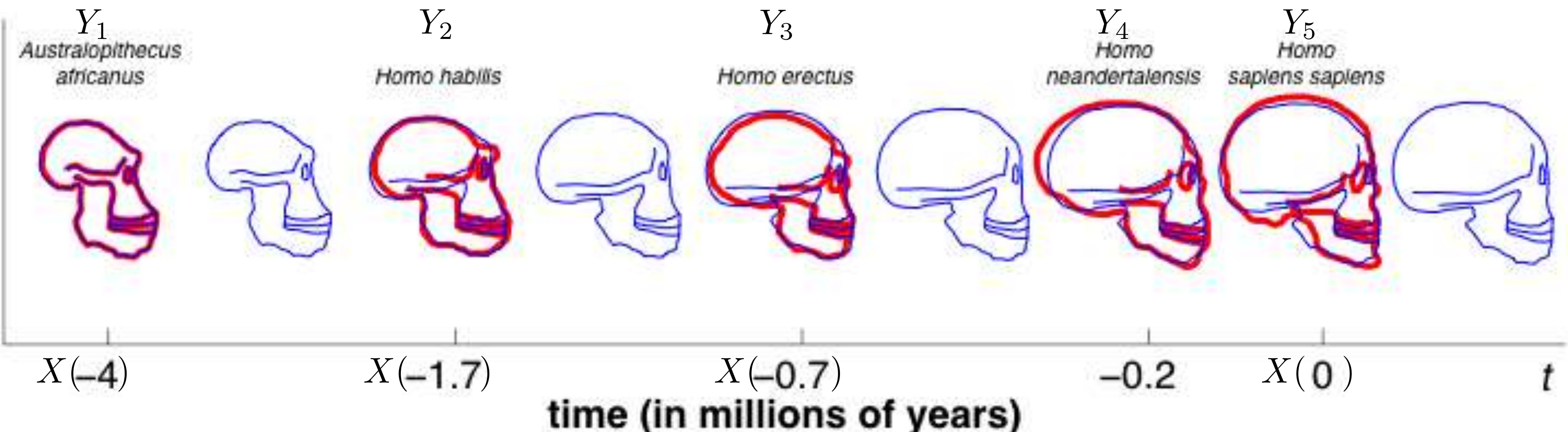

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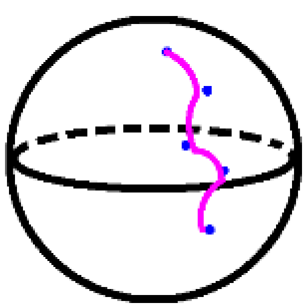

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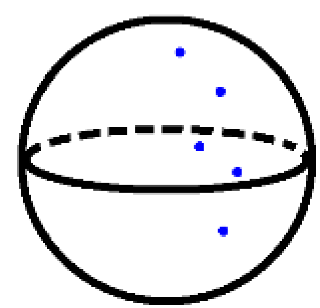
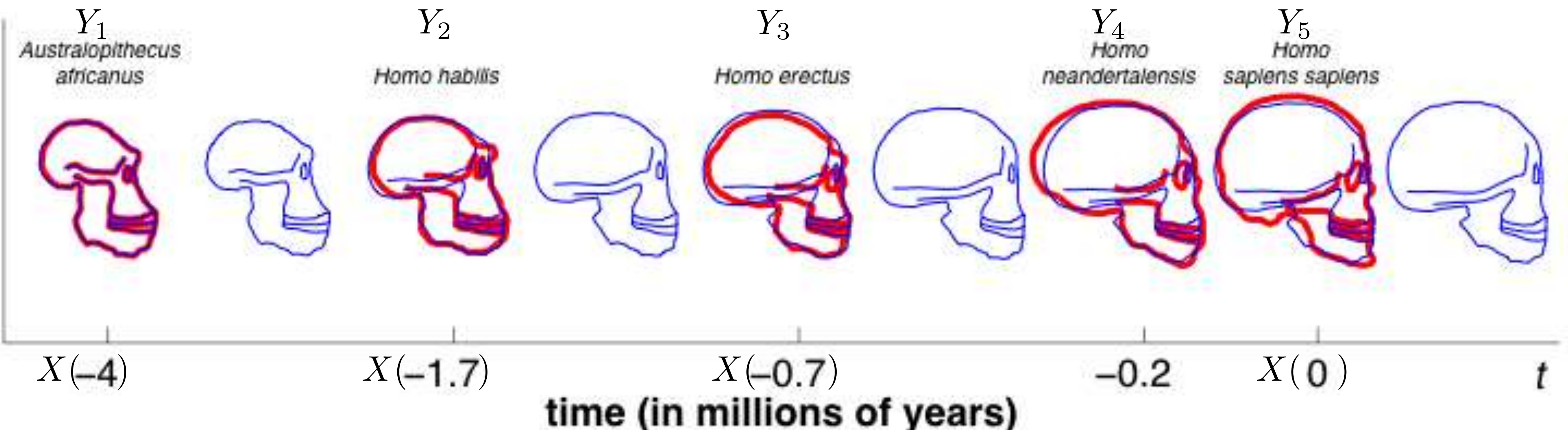
geodesic solution



Regression of time-series shape data



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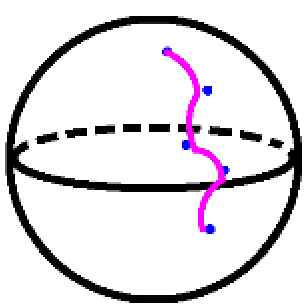

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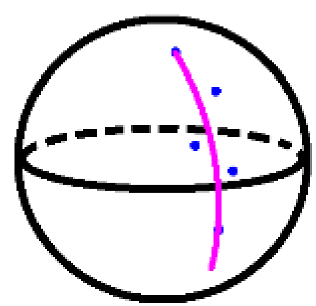

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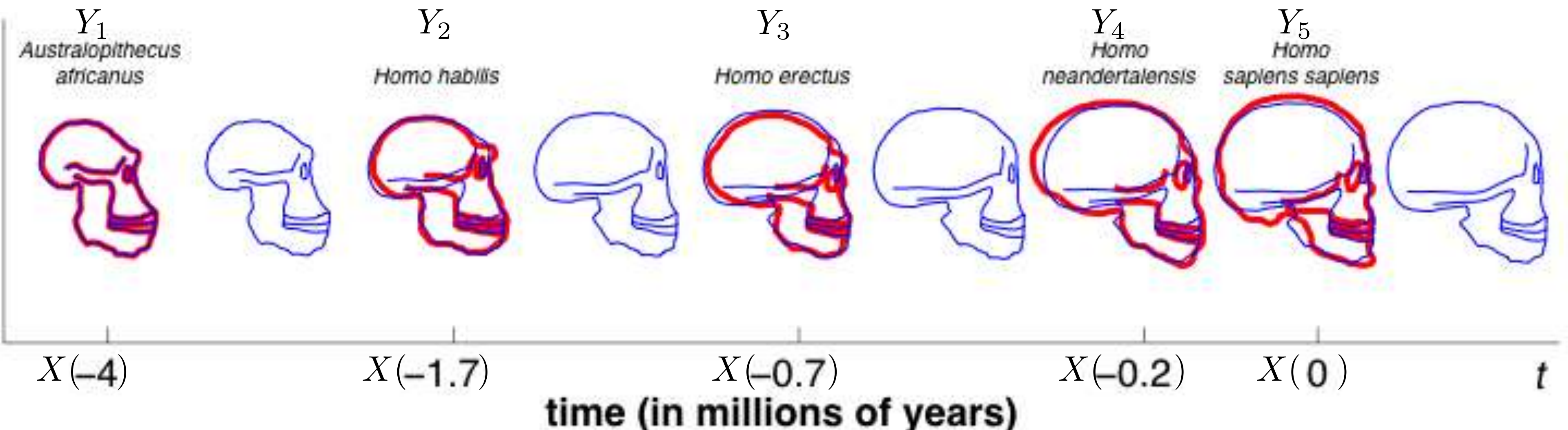

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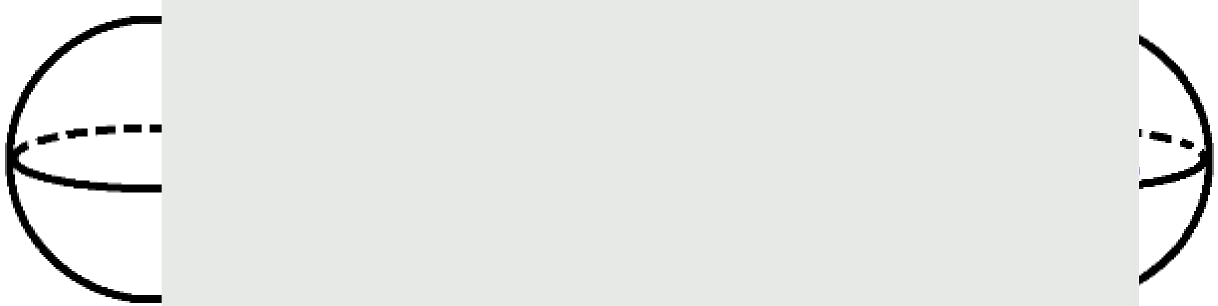
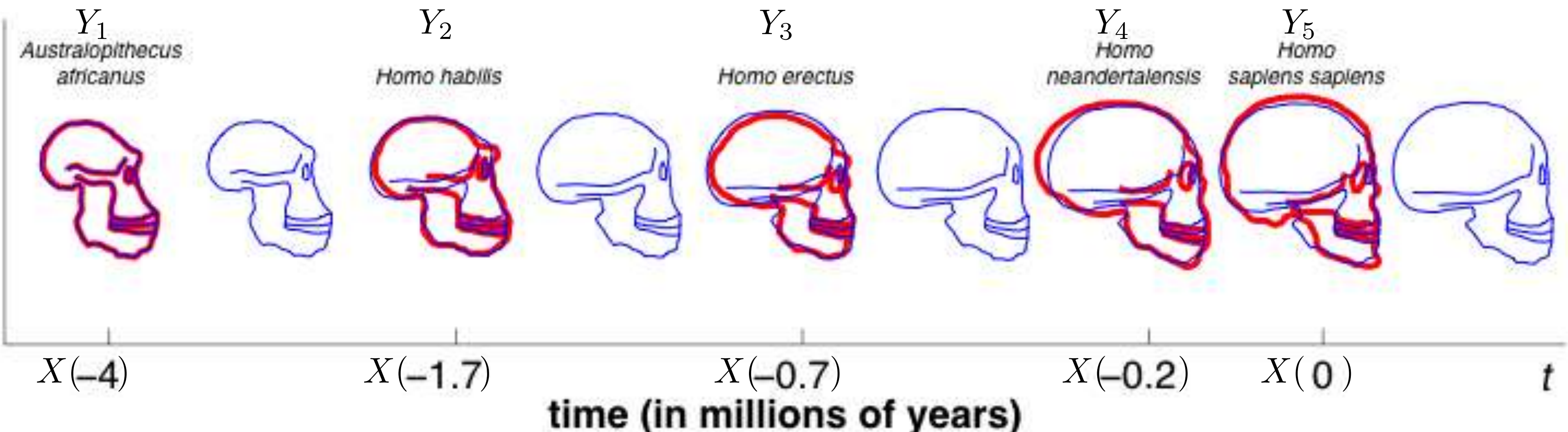
Piecewise geode … lution



Regression of time-series shape data



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Piecewise geodesic solution

$$\nabla_{S(t)} E = \partial_2 G^T \theta(t) + \nabla_{S(t)} L$$

$$E\Big(S_0\Big) = \frac{1}{2\sigma^2}\sum_i D(X(t_i), Y_i)^2 + L(S_0)$$

$$s.t. \quad \begin{cases} \dot{X}(t) = G(S(t), X(t)) & X(0) = X_0 \\ \dot{S}(t) = F(S(t)) & S(0) = S_0 \end{cases}$$

geodesic solution

$$\nabla_{S_0} E = \xi(0) + \nabla_{S_0} L$$

$$with \quad \dot{\theta}(t) = -\partial_1 G(t)^T \theta(t) - \sum_i \delta(t - t_i)\nabla_{X(t_i)} D \quad and \quad \theta(T) = 0$$

$$\dot{\xi}(t) = -\partial_2 G^T \theta(t) - d_{S(t)} F^T \xi(t) \qquad \xi(T) = 0$$

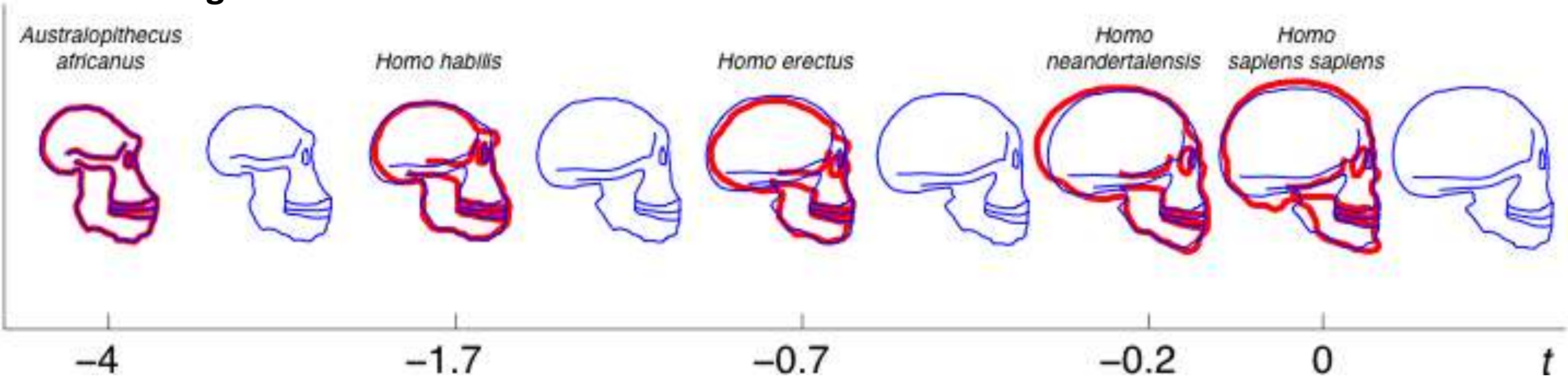
$$\nabla_{X_0} E = \theta(0)$$

Durrleman et al. MICCAI'09, IJCV'13,

Fishbaugh et al. IPMI'13, GSI'13, ISBI'14

Regression of time-series shape data

Geodesic regression: fixed baseline



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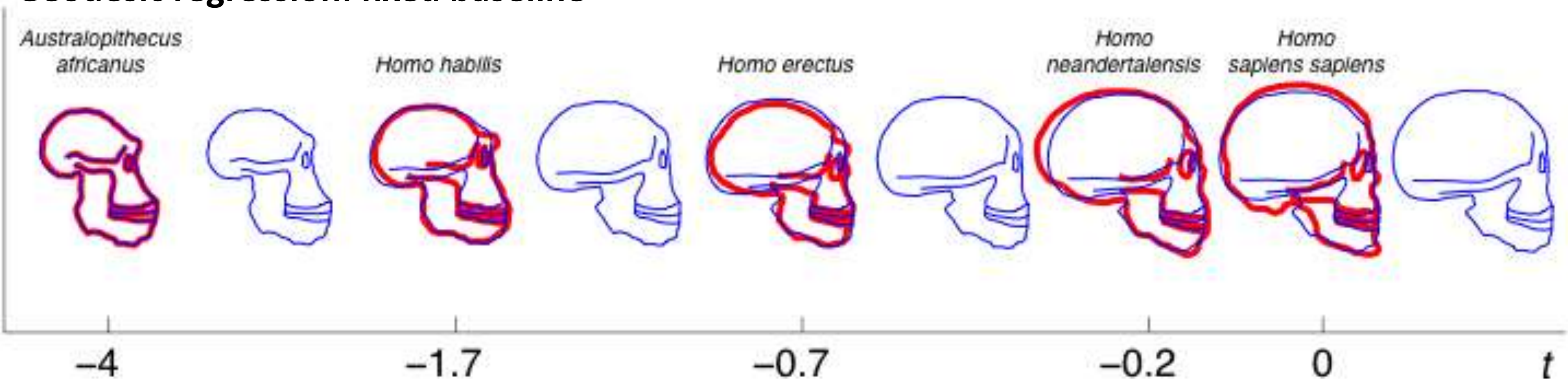
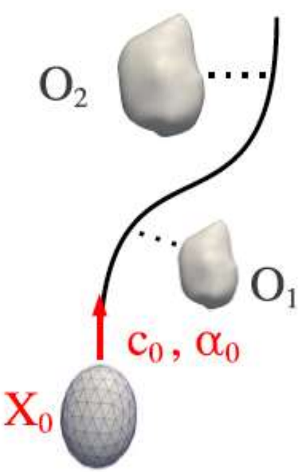
Regression of time-series shape data

Geodesic regression: fixed baseline



Geodesic regression: estimated baseline

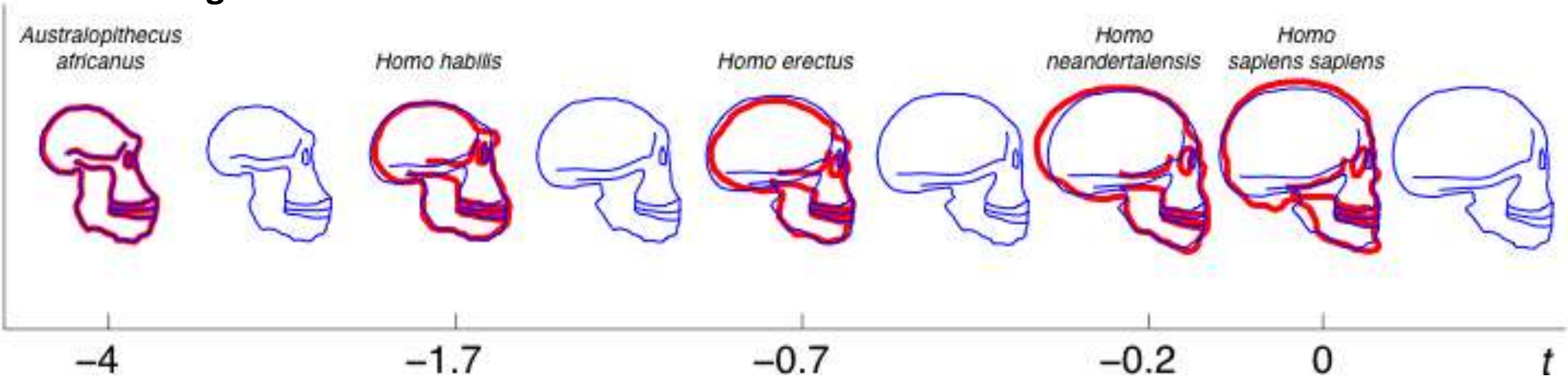
- Joint optimization:
  - Estimation of a baseline (intercept)
  - Estimation of initial momenta and control points (slope)



Durrleman et al. MICCAI'09, IJCV'13, Fishbaugh et al. IPMI'13, GSI'13, ISBI'14

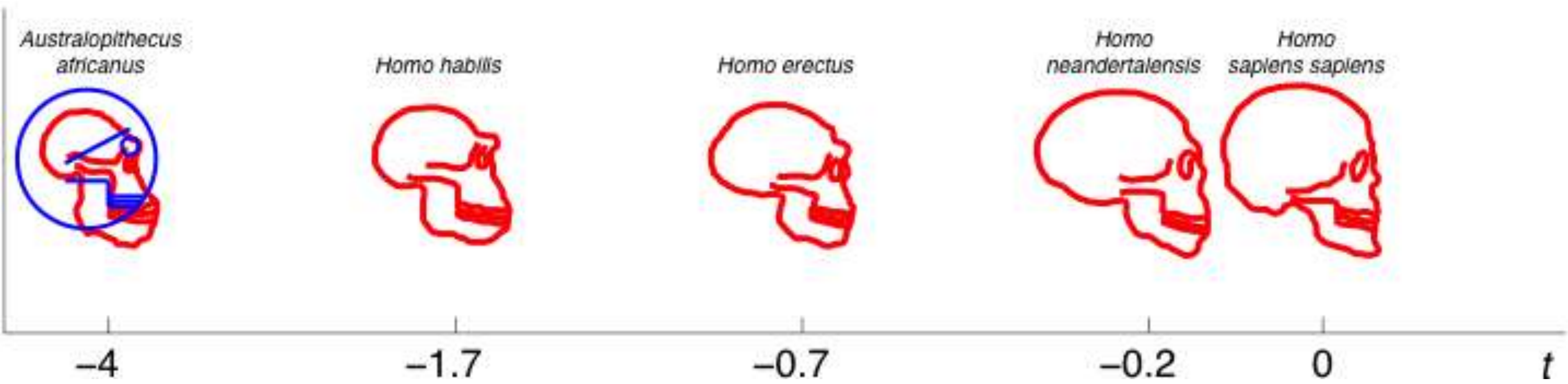
Regression of time-series shape data

Geodesic regression: fixed baseline



Geodesic regression: estimated baseline

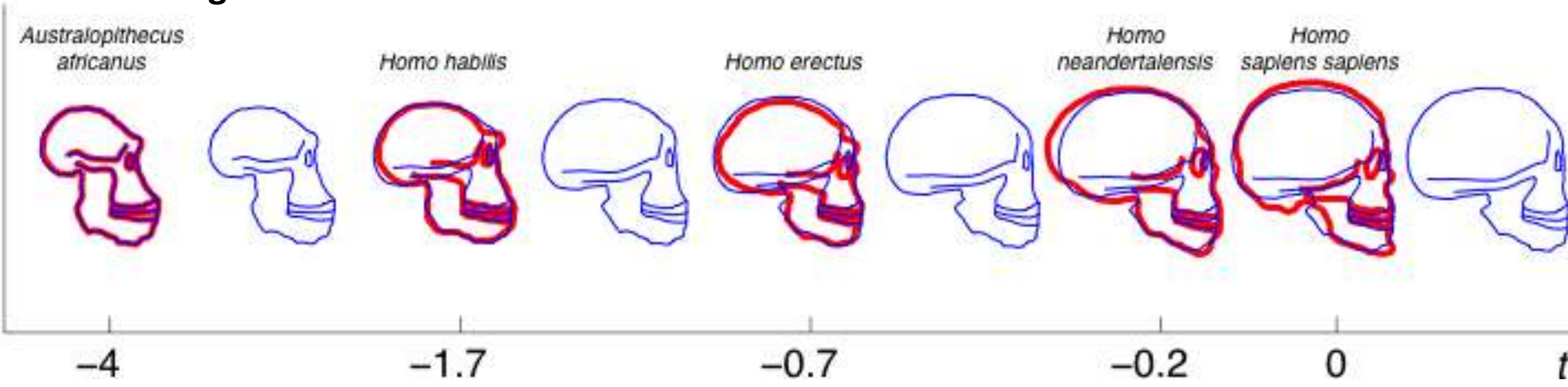
Input:



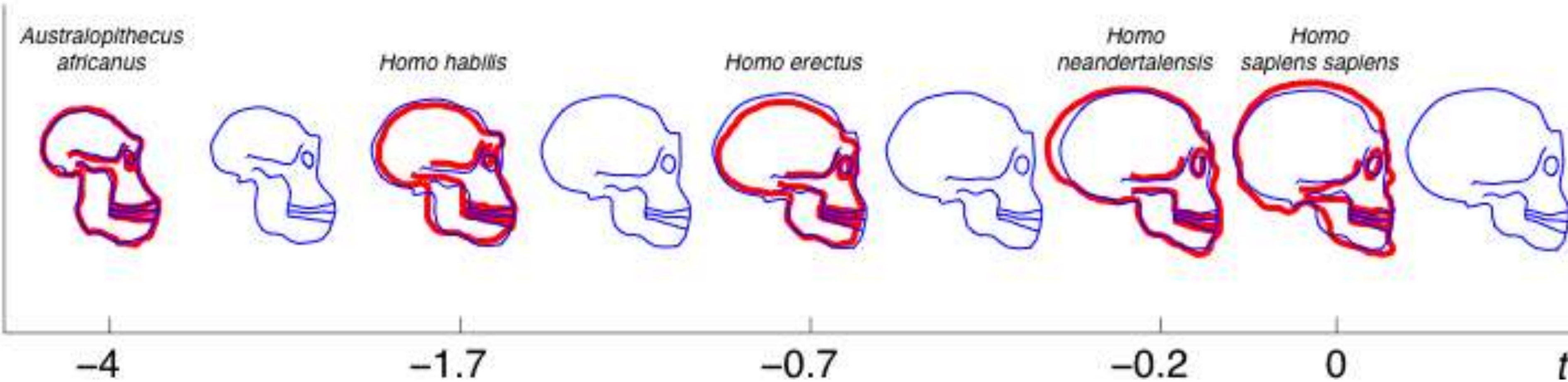
Durrleman et al. MICCAI'09, IJCV'13, Fishbaugh et al. IPMI'13, GSI'13, ISBI'14

Regression of time-series shape data

Geodesic regression: fixed baseline



Geodesic regression: estimated baseline
Output:



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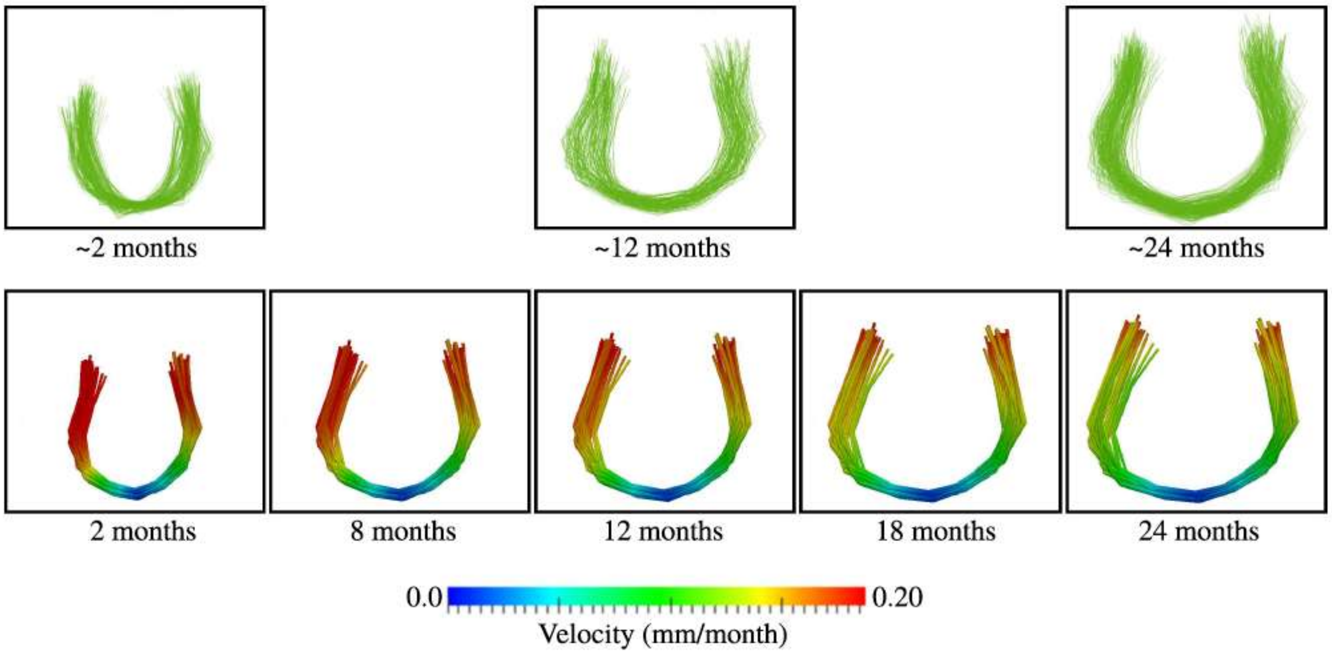
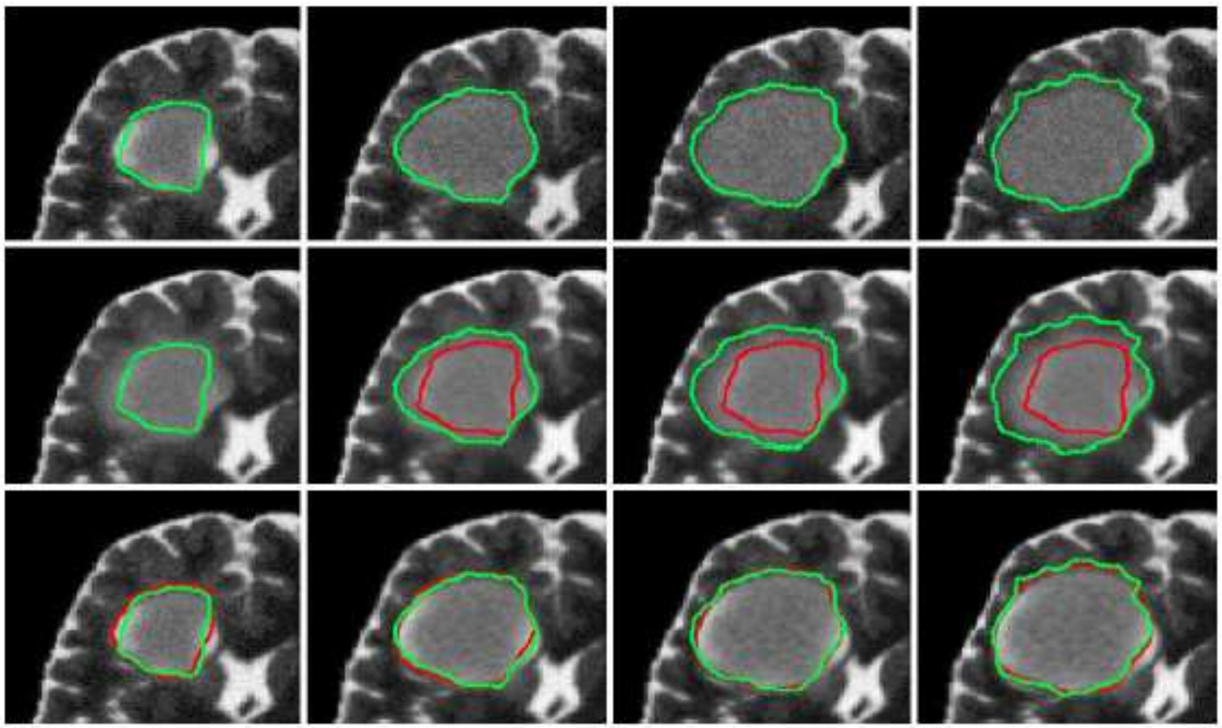
Regression of time-series shape data



Growth of the genu fiber tract [Fishbaugh et al. IPMI'13]

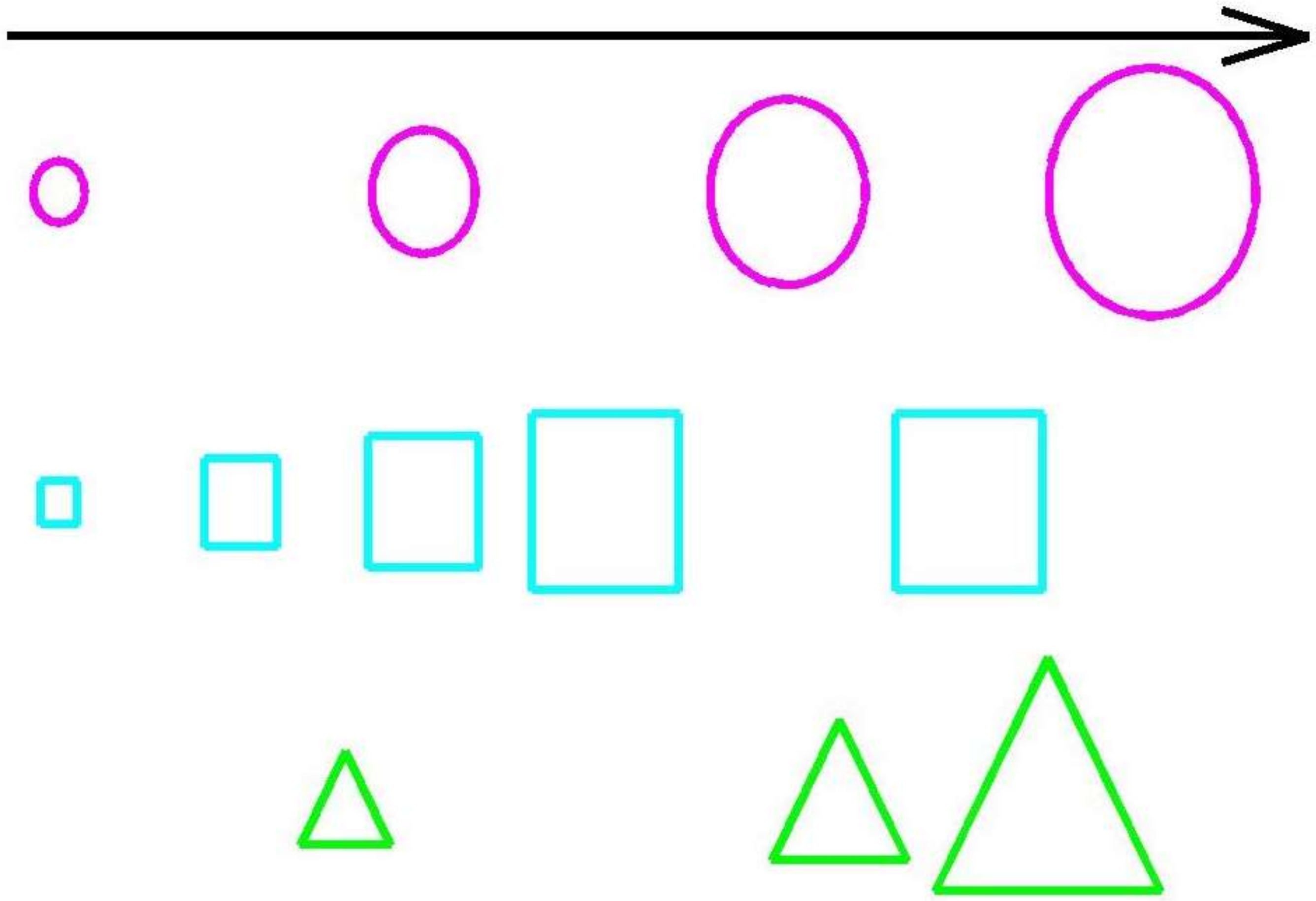
Regression of time-series shape data



Geodesic regression of join image and surface data [Fishbaugh et al. MICCAI'13]

Longitudinal Data Analysis

Longitudinal Data Analysis

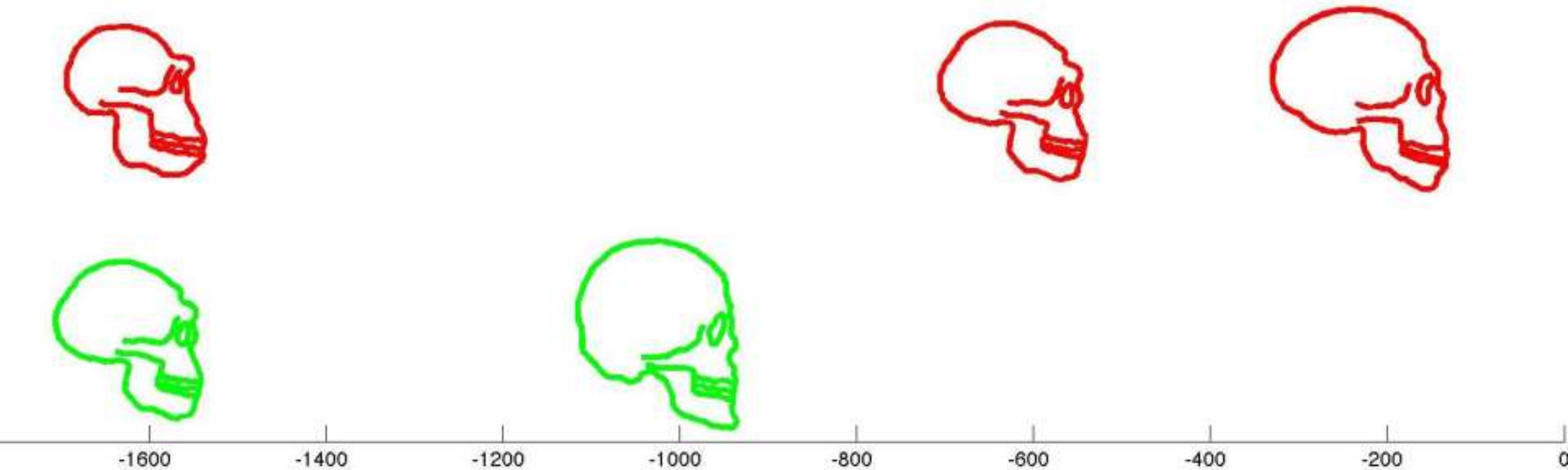


Repeated measurements of a series of subjects. Subjects differ in:

- Shape
- Pace of development

Longitudinal Data Analysis

- Comparison between two lineages (toy example)



Longitudinal Data Analysis

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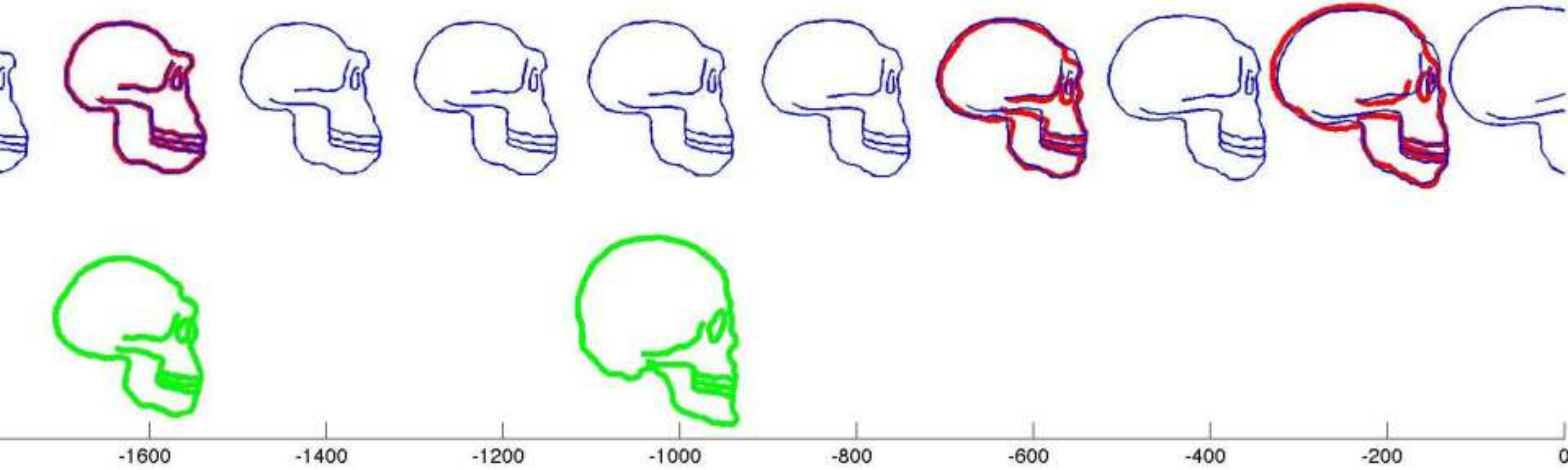

Longitudinal Data Analysis

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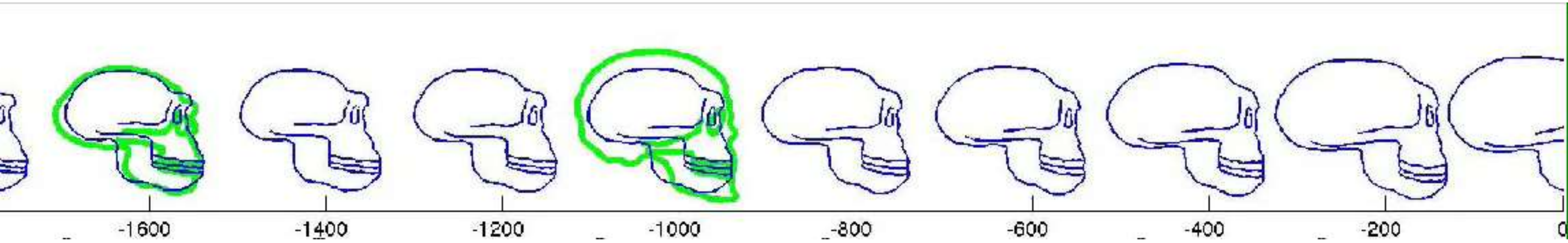
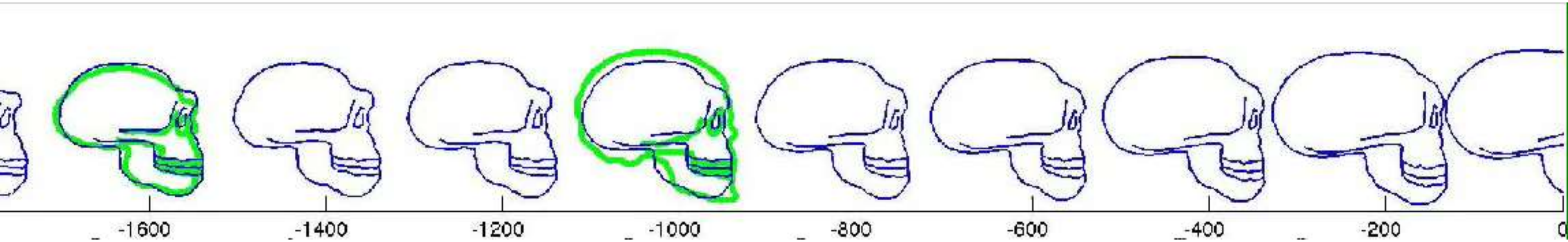


Longitudinal Data Analysis

- Comparison between two lineages (toy example)



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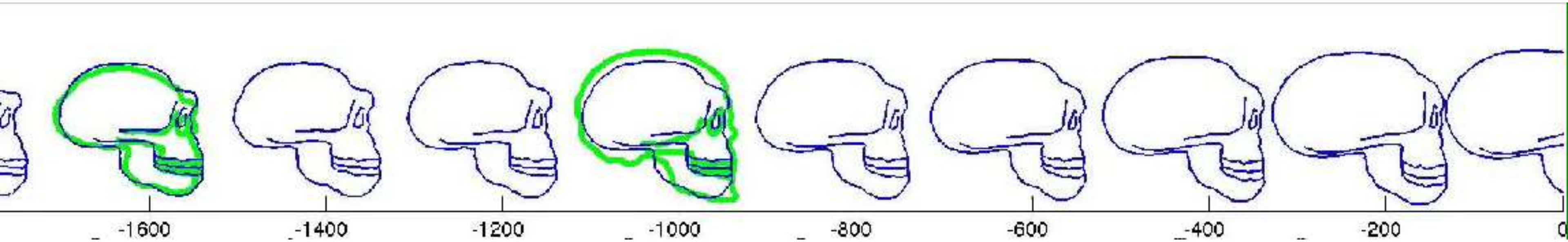

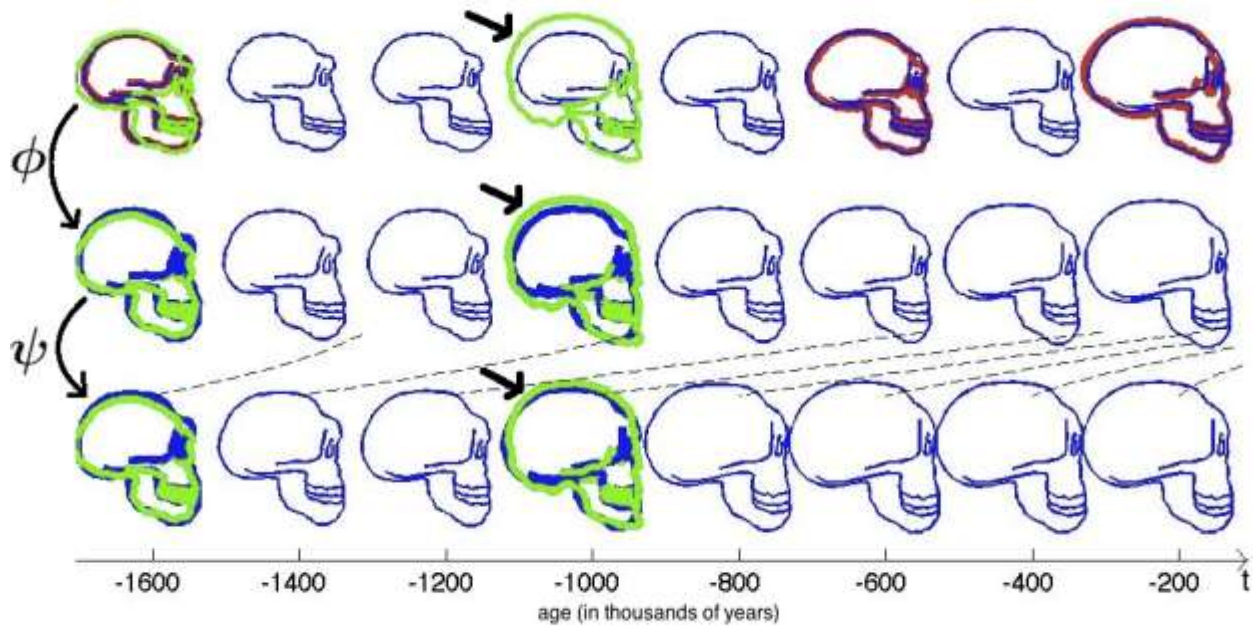

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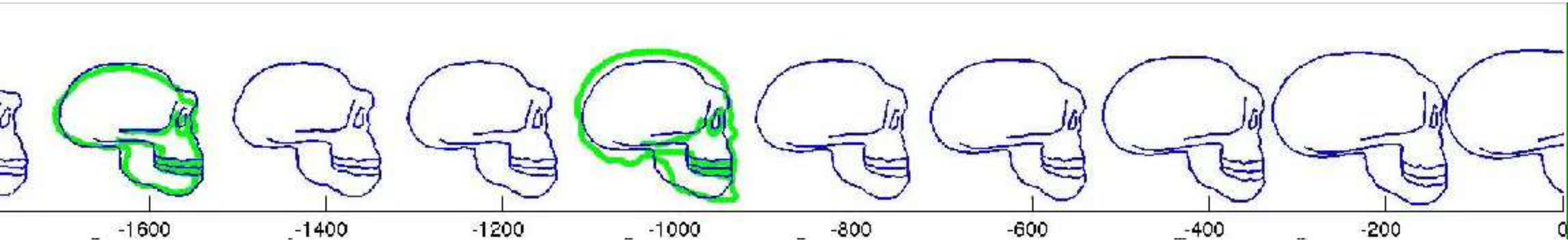

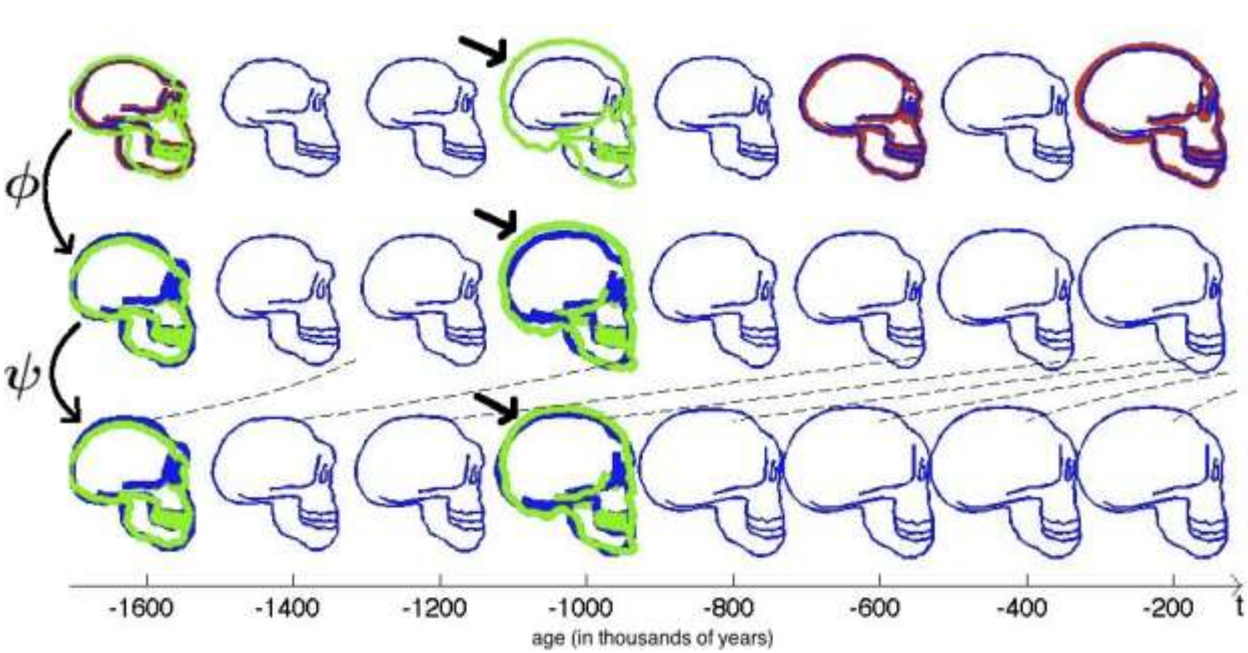

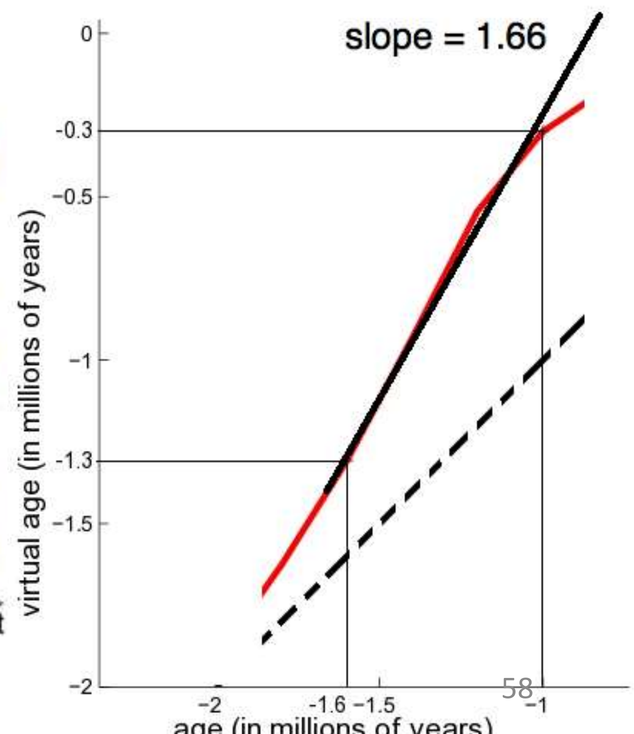

Longitudinal Data Analysis

- Compare regression between subjects [Durrleman et al. JHE’11, IJCV’13]

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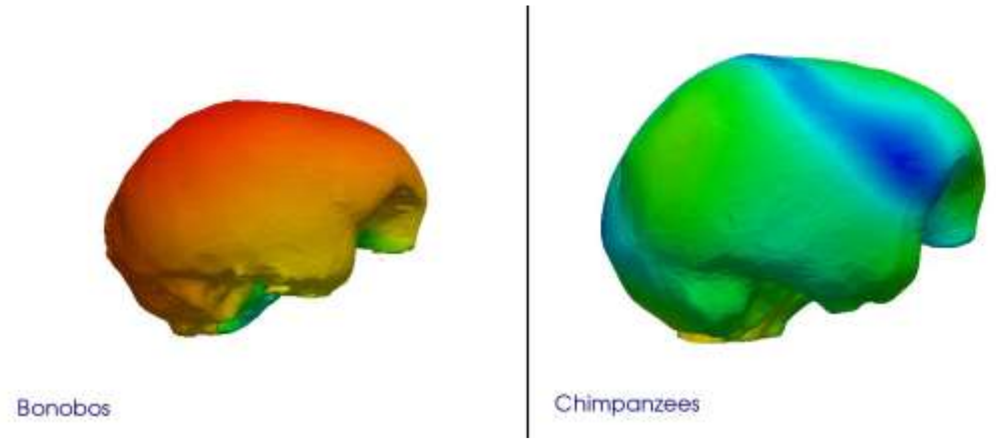

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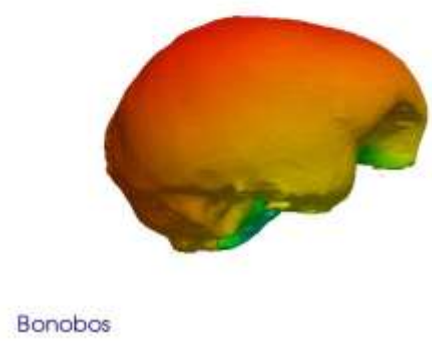

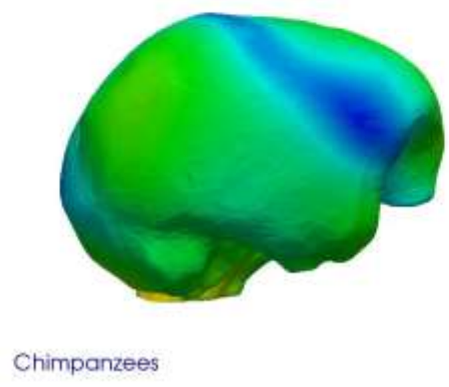

Morphological changes

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Longitudinal Data Analysis

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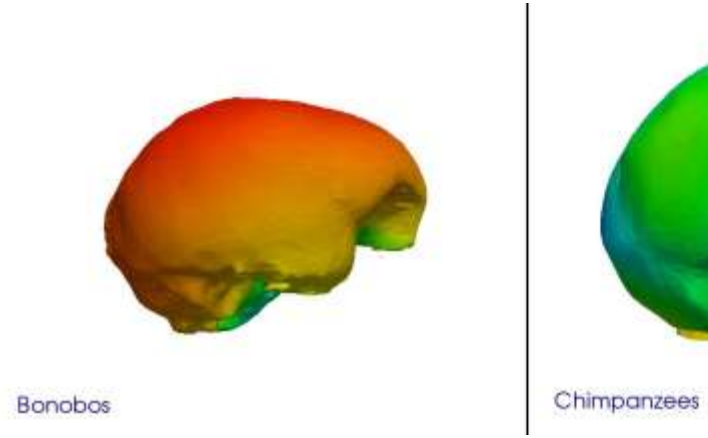

Time warp

Morphological changes

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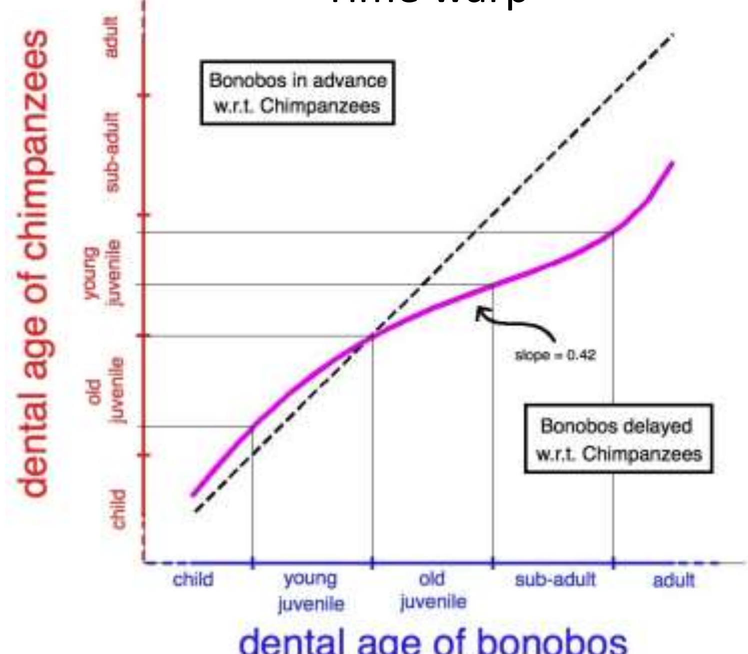

Longitudinal Data Analysis

- Multiple subjects comparison: [Durrleman et al. IJCV'13]
  - Construction of an average growth scenario
  - Spatiotemporal deformation of the average scenario to each subject

Longitudinal Data Analysis

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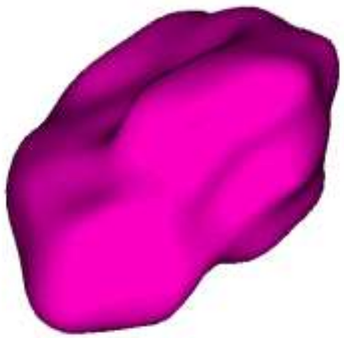
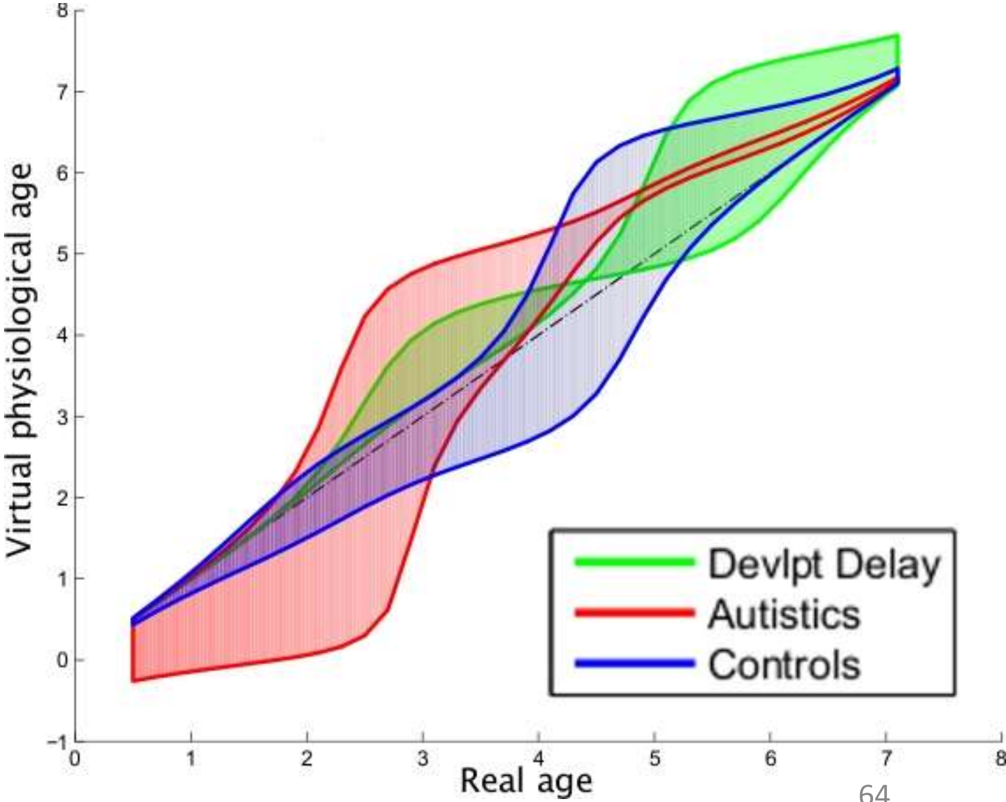
- Developmental delays in autistic children:
  - 2 scans (initial age 2-3 years, follow-up 4-5 years)
  - 12 subjects (4 autistics, 4 developmental delays, 4 controls)



Average growth scenario

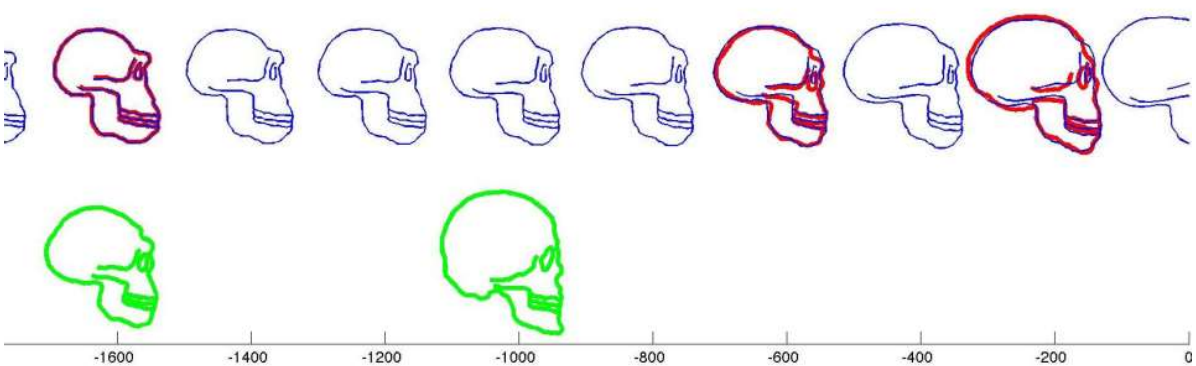


Time warps

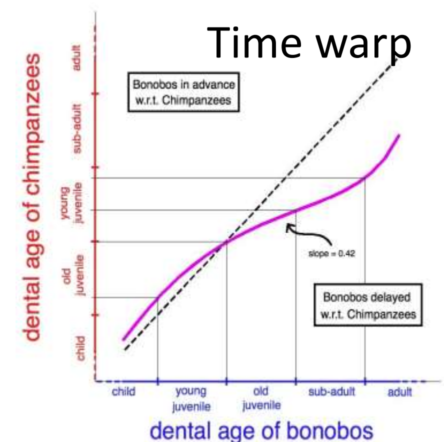
Conclusion

- **An approach to biological shape analysis based on deformations**
- **Regression of time series shape data:**
  - Piecewise geodesic regression
  - Geodesic regression
  - Other alternatives include:
    - Acceleration-controlled (continuously differentiable trajectories) [Fishbaugh'11]
    - Riemannian splines (perturbation of Hamiltonian equations) [Vialard'10]
- **Statistics of longitudinal data sets:**
  - Morphological deformation
  - Time-warp

- **Joint work with:** J. Fishbaugh, G. Gerig, X. Pennec, M. Prastawa, A. Trouvé



Time warp



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