Generalized Data Depth and Applications

Mukund Raj, Mahsa Mirzargar, Robert M. Kirby and Ross T. Whitaker

**MOTIVATION**

- Ensembles of data objects are common in many areas.
- Parametric methods of analysis require prior knowledge of the underlying distribution.
- Data depth is a nonparametric approach for characterizing ensembles.

**DATA DEPTH BACKGROUND**

- Salient Features of Data Depth:
  - Nonparametric
  - Robust
  - Descriptive Statistical Method
  - Derived order statistics can be used for visualization based on the classical boxplot.
- Data Depth Formulations for Multivariate Points:
  - Simplicial depth
  - Location depth
  - And many others..
- Data Depth Formulations for Complex Data:
  - Functions [1] and multivariate curves [3,4]
  - Sets and Isocontours [2]
- Definition of Band for Paths on a Graph
  Let graph \( G = (V, E, W) \) be a set of vertices, edges and weights on edges. We denote a path \( p \) as \( p : I \rightarrow V \) over an index set \( I = \{1, 2, \ldots, m\} \).
  The convex hull of a set of \( j \) vertices \( V_j \) is the smallest geodesic-convex set that contains \( V_j \) and is denoted as \( \text{Hull}(V_j) \).
  Then, band formed by \( j \) paths is defined as follows:
  \[
  p \in B[j] \quad \text{iff} \quad p(i) \in \text{Hull}(V_{p(i)}), \ldots, p(j) \quad \forall i \in I.
  \]
- Path Band Depth (pBD) for Paths on a Graph
  pBD for a path \( p \) is defined as follows:
  \[
  \text{pBD}(p) = E \left[ x \left( p \in B[p] \right) \right]
  \]

**EVALUATING ALIGNMENT OF SHAPES**

- Evaluating alignment of shapes is important in many areas.
- Ensemble visualization through contour boxplot [2] can be an effective method to evaluate alignment of shapes.

**CONTRIBUTIONS**

- Generalization of contour boxplot method for 3D shapes.
- Application of contour boxplot to evaluate alignment of shapes.
- Formulation of method to calculate data depth for paths on a graph and corresponding path boxplot visualization scheme.

**REFERENCES**

- [1] Pintado et al. JASA 2009

**ACKNOWLEDGEMENTS**

This work was supported by NSF grant IIS-1212806.