FFTW:

An Adaptive Software Architecture for the FFT

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The Fastest Fourier Transform in the West

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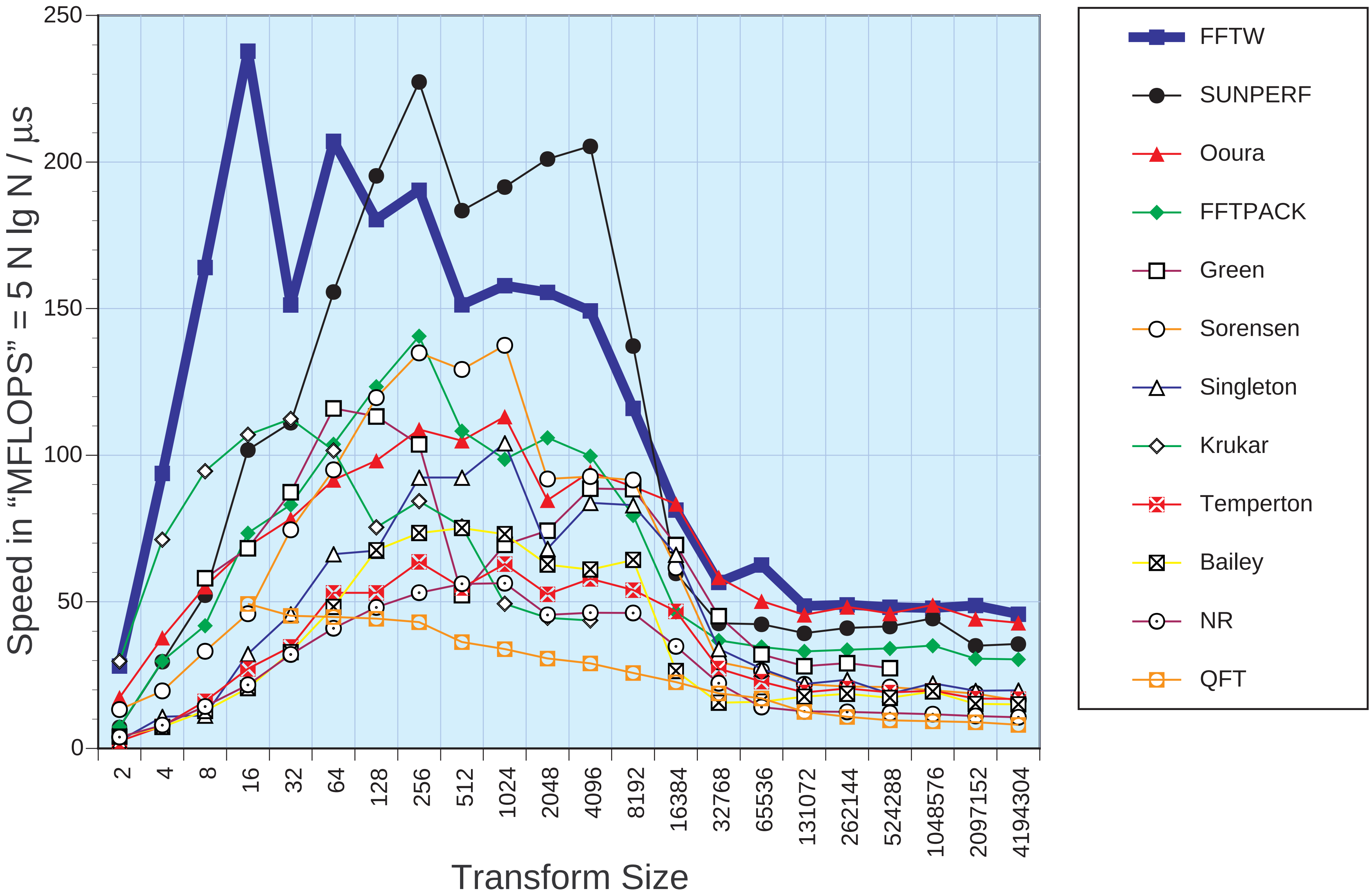
FFTW is:

- A C library for computing the FFT
  - in one or more dimensions
  - includes parallel transforms
- Fast
  - competitive with vendor-tuned libraries
  - FFTW's performance is portable
- Here today (since 3/97)
  - ...with thousands of happy users
  - `http://theory.lcs.mit.edu/~fftw`

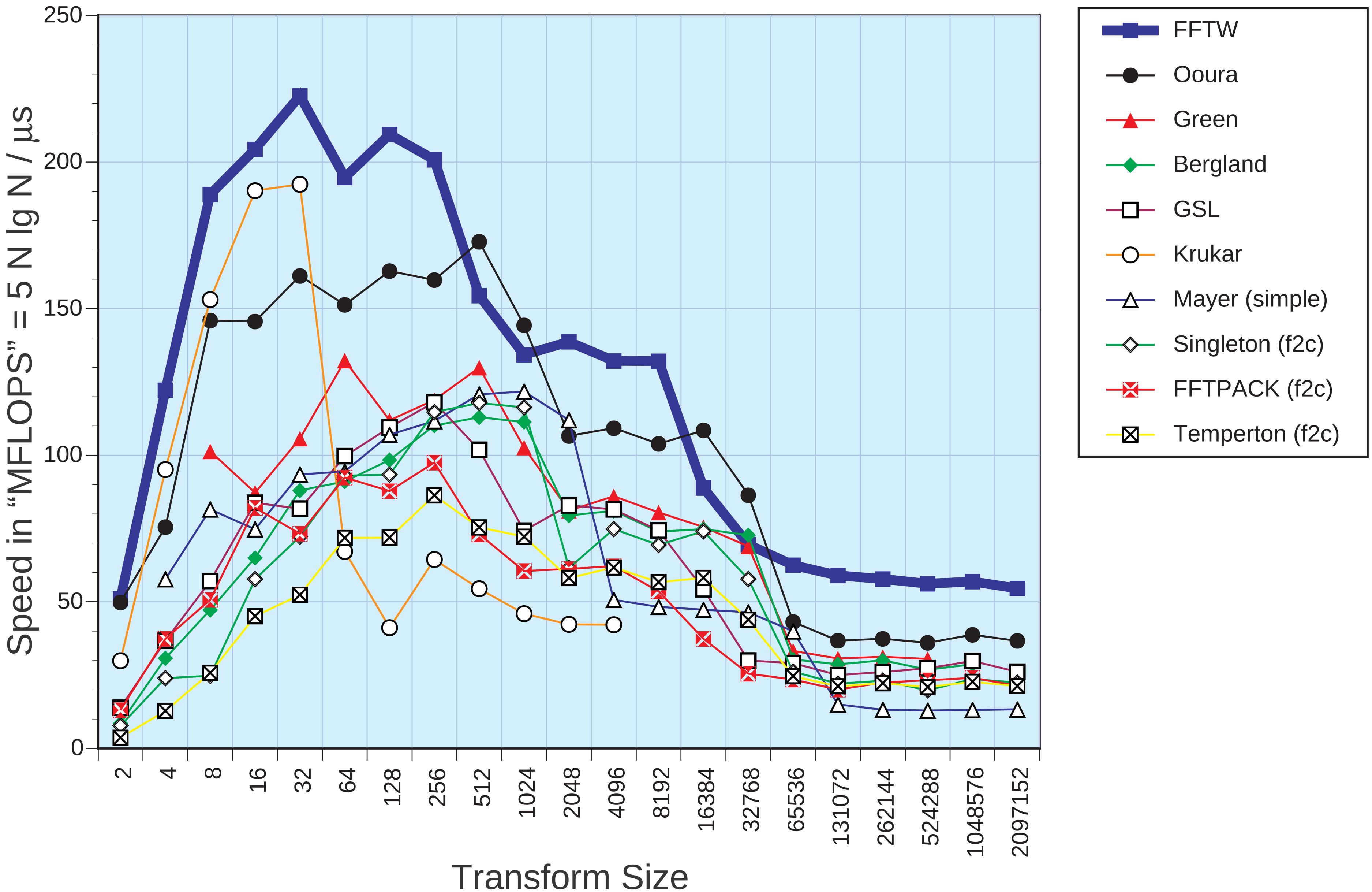
FFTW is Not:

- **Always the fastest code**
  - **but is the fastest more often than any other program**
- **A new FFT algorithm per se**
  - **it is a new way of implementing known algorithms**

FFT Benchmark on a 167MHz UltraSPARC-I (xolas)



FFT Benchmark on a 300MHz Pentium II



We Also Have Results From...

- Over 40 FFT codes
- 19 different machines
- Both 1D and 3D transforms
- Transform sizes not a power of 2

Why is FFTW So Fast?

- **The Runtime Planner**
  - **optimizes FFTW for your CPU, your cache size, etcetera**
- **The Codelets**
  - **composable blocks of optimized code**
  - **computer generated**
- **The Executor**
  - **interprets the plan to compose the codelets**
  - **includes a few tricks of its own**
- **But first, a review of the FFT...**

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The Cooley-Tukey FFT Algorithm

- **Computes a DFT of size N = N1 * N2**
  - **first, does N1 transforms of size N2**
  - **then, multiplies by “twiddle factors”**
  - **finally, does N2 transforms of size N1**
- **Performance is O(N lg N)**
- **Base cases of recursion are optimized small transforms**

The Planner

- For a given N, there are many factorizations
  - not clear a priori which is best
- The planner tries them "all" and picks the best one
  - uses actual runtime timing measurements
  - result is encoded in a "plan"
- Uses dynamic programming to reduce number of possible plans
  - remembers optimal sub-plans for small sizes

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Example Plans

- **N=32768 on Alpha and Pentium II**

Alpha:

```
RADIX 16: 32768 -> 16*2048
RADIX  8:  2048 -> 8*256
RADIX  8:   256 -> 8*32
SOLVE 32
```

Pentium II:

```
RADIX 64: 32768 -> 64*512
RADIX 16:   512 -> 16*32
SOLVE 32
```

The Codelet Generator

- **Generates highly-optimized transforms of small sizes ("codelets")**
  - **with and without twiddle factors**
  - **form the base cases of the FFT recursion**
- **Written in the Caml-Light dialect of ML**
- **Manipulates abstract syntax tree which is unparsed to C**
  - **knows about complex arithmetic, etcetera**

Advantages of Generating Codelets

- Long, unrolled code takes advantage of:
  - optimizing compilers
    - instruction scheduling, etcetera
  - large register sets
- Applies tedious optimizations
- Easy to experiment with different algorithms
  - prime factor, split-radix, Rader
    - express the algorithm once, abstractly
  - various optimization hacks
- You only have to get it right once

The Expression Simplifier

- Here is a fragment that helps simplify multiplications:

```
simplify_times = fun
            (Real a) (Real b) -> (Real (a *. b))
           | a (Real b) -> simplify_times (Real b) a
           | (Uminus a) b -> simplify (Uminus (Times (a,b)))
           | (Real a) (Times ((Real b), c)) ->
               simplify (Times ((Real (a *. b)), c))
           | (Real a) b -> if (almost_equal a 0.0) then (Real 0.0
                           else if (almost_equal a 1.0) then b
                           else if (almost_equal a (-1.0))then
                               simplify (Uminus b)
                           else Times ((Real a), b)
           | ...
```

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Tricky Optimizing Rules

- **Quiz: which of the following is faster?**

```
a = 0.5 * b;
c = 0.5 * d;
e = 1.0 + a;
f = 1.0 - c;
```

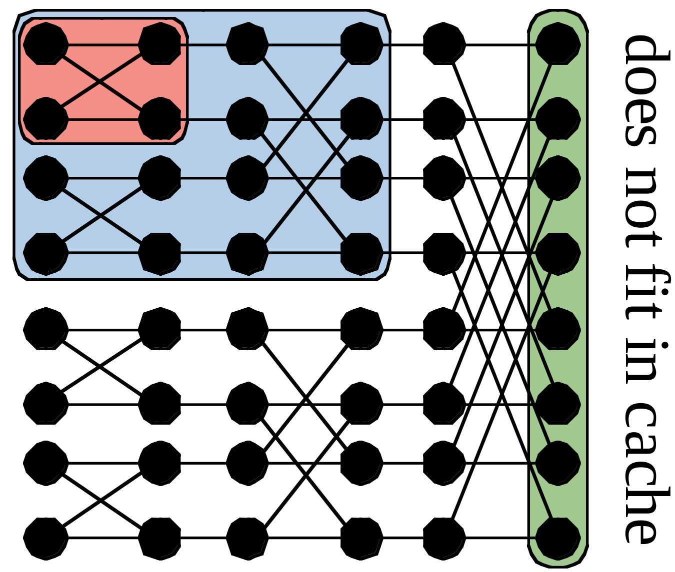
```
a = 0.5 * b;
c = -0.5 * d;
e = 1.0 + a;
f = 1.0 + c;
```

Answer: the fragment on the left.

The number of floating-point constants should be minimized.

The Executor

- **Executes the plan by composing codelets**
- **Explicit recursion**
  - **divide-and-conquer uses all levels of the memory hierarchy**
- **Novel storage for the twiddle factors**
  - **store them in the order they are used**



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FFTW is Easy to Use

- **Complexity is abstracted from the user:**

```
COMPLEX A[n], B[n];
fftw_plan plan;

/* create the plan: */
plan = fftw_create_plan(n);

/* use the plan: */
fftw(plan,A);

/* re-use the plan: */
fftw(plan,B);
```

Parting Thought: This is Ridiculous!

- All this for an FFT?
- Modern architectures are invalidating conventional wisdom about what is fast
  - no new wisdom is emerging
- In the name of performance, designers have sacrificed:
  - predictability
  - repeatability
  - composability
- Hand-optimization of programs is becoming impractical