Introduction to the Material Point Method

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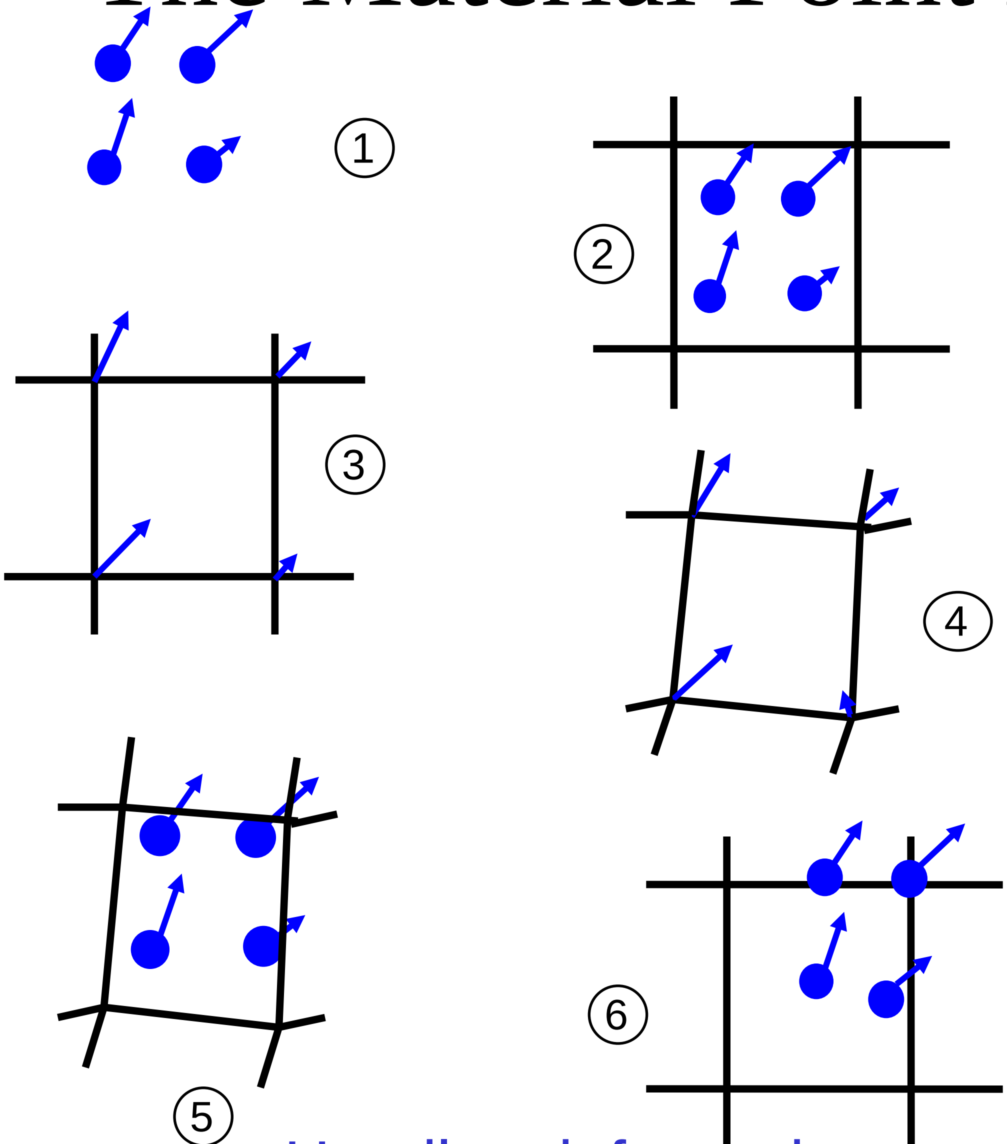
Material Point Method (MPM) Outline History
See Brackbill IJNMF 2005 47 693-705

- *1963 Harlow PIC methods then CiC ViC methods*
- *1980s -90s Flip methods Brackbill et al.*
- *1990s Sulsky Brackbill MPM-PIC*
- *2000+ Sulsky et al. + GIMP Bardenhagen et al*

Since then proved effective on difficult problems involving large deformations fracture e.g CSAFE [Guilkey et al.] + [Sulsky et al] + [Brydon] etc etc

The Material Point Method (MPM)



Particles with properties (velocity, mass etc) defined on a mesh

Particle properties mapped onto mesh points

Forces, accelerations, velocities calculated on mesh points

Mesh point motion calculated but only the particles moved by mapping velocities back to particles

Handles deformation, contact, high strain

Explicit MPM Algorithm

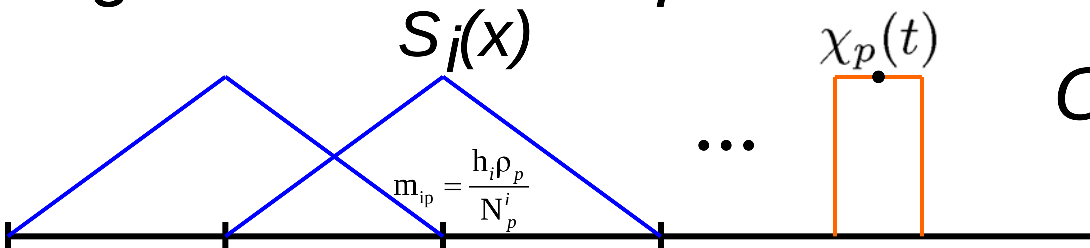
- Particle to grid projection
- Interpolation from grid back to particles
- Time integration of particles

- Finite element + mass matrix lumping

1D : 4-8 particles per cell is optimal

MPM Basis Overview

grid basis fns *particle basis fns*



Or delta fn ..

point Velocity

point mass $m_{ip} = \dfrac{h_i \rho_p}{N_p^i}$

$$v_i = \sum_{p=1}^{np} \lambda_p v_p, \qquad \lambda_p = \frac{m_{ip} S_{ip}}{\sum_{ip=1}^{np} m_{ip} S_{ip}}, \qquad S_{ip} = \frac{1}{V_p} \int_{\Omega_i} S_i(x) \chi_p(x) dx$$

$$a_i = \sum_{p=1}^{np} \mu_p V_p, \qquad \mu_p = \frac{p_{ip} G_{ip}}{\sum_{ip=1}^{np} m_{ip} S_{ip}}, \quad G_{ip} = \frac{1}{V_p} \int_{\Omega_i} \frac{dS_i}{dx} \chi_p(x) dx$$

Steps in MPM

- From particles masses calculate mass at nodes
- From particle velocities calculate velocity at nodes
- Calculate forces at nodes
- Using F = m a   calculate acceleration at nodes
- Calculate updated velocity at nodes
- Calculate displacement at nodes
- Map velocites back to particles and move them

Forward Euler time integration

$$\mathrm{v}_i^{n+1} = \mathrm{v}_i^n + \delta t\, \mathrm{a}_\mathrm{i}, \quad \mathrm{v}_p^{n+1} = \mathrm{v}_p^n + \delta t \sum_{i=1}^{nv} S_{ip}\, \mathrm{a}_\mathrm{i},$$

$$x_p^{n+1} = x_p^n + \delta t \sum_{i=1}^{nv} S_{ip}\, \mathrm{v}_\mathrm{i}^{\mathrm{n+1}}$$

V^{n+1}

n.b. semi-implicit

MPM Applications

- **Angiogenesis**
- **Vocal modeling**
- **Bullet-torso impact**
- **Foam properties**