PROBABILISTIC CURVE EVOLUTION USING PARTICLE FILTERS

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ABSTRACT

A probabilistic active contour model is formulated, in which curve evolution is viewed as state estimation for a nonlinear dynamical system. The method is implemented using particle filters in a Bayesian framework. Level set methods are utilized and enable the proposed model to handle topological changes. Experimental results show that the proposed method works well for complicated images, but is, as expected, computationally intense.

1. INTRODUCTION

Active contour models are among the most state-of-art image segmentation methods. In these models, simple closed curve C initialized in an image evolves according to attributes of the image and the curve itself. Image segmentation is achieved by stopping the evolving curve at boundaries between objects.

Numerous active contour models have been proposed. In [1]-[4], gradient information and/or region information from the image are utilized. Although the ways to use the image information are different, these models are all geometric flows.

Image segmentation can also be viewed as a specific tracking problem: tracking of object boundaries using curve evolution from an initial position. In this sense, Kalman snakes [7]-[8] are alternatives for image segmentation. However, Kalman snakes and their extensions are only applicable for systems with Gaussian noise. This assumption fails in curve evolution, which has nonlinear dynamics with unknown noise.

A new probabilistic active contour model is formulated, in which curve evolution is a state prediction and estimation problem for a nonlinear dynamical system. Particle filters [9][10], which are applicable to nonlinear systems with non-Gaussian noise, are used to solve the state prediction and estimation problem in a Bayesian framework. The states in the model are defined as the properties of a narrow band of image pixels around the evolving curve. By means of level set methods, the proposed model automatically deals with topological changes in image segmentation. Both gradient and region information are utilized to guide the curve's evolution.

The paper is organized as follows. The geometric curve evolution model is introduced in section 2, and a probabilistic active contour model is proposed in section 3. In section 4, some background information on particle filters is provided. Several implementation problems are discussed in section 5. Section 6 provides an analysis of experimental results, followed by a brief conclusion in section 7.

2. THE GEOMETRIC ACTIVE CONTOUR MODEL

Geometric active contour models are briefly introduced in this section. Basic concepts and their implementations are discussed, followed by a more complicated model, which makes use of both gradient and region information for curve evolution.

In the geometric curve evolution model, the evolving curve C is usually implicitly represented by the zero level set of a function of two dimensions ψ,

$$C = \{(x, y) : \psi(x, y) = 0\} \tag{1}$$

This formulation allows topological changes as the function ψ evolves, and this is a great advantage over parametric curve evolution methods.

A simple curve evolution model evolves the initial curve at a constant speed along its normal direction, making the curve expand or shrink. The curvature is utilized at each point on the curve to make the evolving curve smooth at all times. This model can be described as

$$\psi_t = (\upsilon + \epsilon\kappa)|\nabla\psi| \tag{2}$$

where υ is the constant speed of evolution, ϵ is a positive constant, and $\kappa = div(\nabla\psi/|\nabla\psi|)$ is the curvature, where div represents the divergence operator.

Sethian's work [11] gives a finite difference algorithm to solve (2):

$$\psi_{k+1} = \psi_k + \triangle t\{max(\upsilon, 0)\nabla^+ + min(\upsilon, 0)\nabla^-\} \quad (3)$$

where

$$\nabla^+ = \{max(D^{-x}, 0)^2 + min(D^{+x}, 0)^2 + max(D^{-y}, 0)^2 + min(D^{+y}, 0)^2\}^{1/2}$$

and

$$\nabla^- = \{max(D^{+x}, 0)^2 + min(D^{-x}, 0)^2 + max(D^{+y}, 0)^2 + min(D^{-y}, 0)^2\}^{1/2}$$

in which D^{-x}, D^{+x}, D^{-y} and D^{+y} are the backward and the forward finite differences of ψ in the x and y directions, respectively.

This algorithm illustrates the fundamental idea of curve evolution; however, no information from the image is utilized. In order to perform image segmentation, both image gradient and region information should be used to guide the evolving curve to boundaries of objects in the image. A new curve evolution model is proposed in [12] that extends (2):

$$\psi_t = \alpha\phi_l(\upsilon+\epsilon\kappa)|\nabla\psi| + (1-\alpha)\gamma\delta_\beta(\psi)\{(I-c_2)^2 - (I-c_1)^2\} \quad (4)$$

Here, ψ is the evolving curve, and I represents the image. The first term in the right hand of (4) makes use of the gradient information in the image, where $\phi_l = 1/\{1 + \|\nabla I\|\}$. In this term, υ is the inflationary term, and κ is the curvature. The second term uses region information. In this term, $\delta_\beta(x) = \beta/(\pi(x^2 + \beta^2))$ approximates a delta function. c_1 and c_2 are the average values of the points inside and outside the evolving curve, respectively, and α, β, γ and ϵ are positive constant coefficients.

Following the approach of Sethian [11], a numerical algorithm for the model (4) can be derived:

$$\psi_{k+1} = \psi_k + \triangle t\{\alpha\{max(\phi_l\upsilon, 0)\nabla^+ + min(\phi_l\upsilon, 0)\nabla^-\} + (1-\alpha)\cdot\gamma\delta_\beta(\psi)\{(I-c_2)^2 - (I-c_1)^2\}\} \quad (5)$$

An illustration of this algorithm's behavior is given in Fig. 1, in which anisotropic diffusion methods [13][14] are incorporated to reduce the effects of noise. In Fig. 1, (a) is the image with the initial curve, (b) and (c) are intermediate results during curve evolution, and (d) is the final result. It can be clearly seen that the algorithm segments the image and converges to the boundaries.

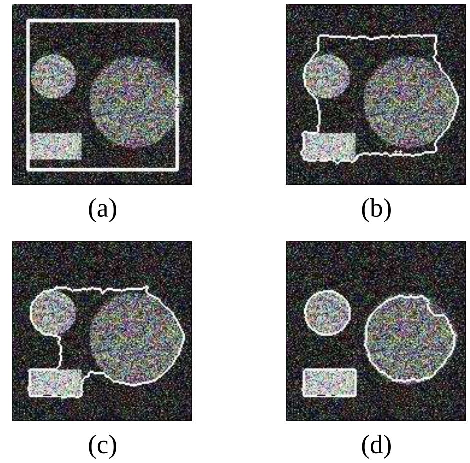

Fig. 1. Boundary detection using curve evolution.

3. PROBABILISTIC CURVE EVOLUTION MODEL

The geometric curve evolution model is clearly nonlinear, and images typically contain noise, either Gaussian or non-Gaussian. A state-space model of the curve evolution can be constructed, embedding traditional geometric curve evolution models in a probabilistic framework.

3.1. The State Model

In the state space model, points selected on and close to the evolving curve $\psi_k^i (i = 1...N)$ form the states of the model. The ideal motions of the states follow a distribution approximating

$$\psi_t = \alpha\phi(\upsilon + \epsilon\kappa)|\nabla\psi| \quad (6)$$

where $\phi = 1/\{1 + \|\nabla I\|\}$.

With the presence of additive noise, a finite difference model has the form:

$$\psi_{k+1} = \psi_k + \triangle\psi_k + n_k \quad (7)$$

where ψ_k is the vector of the values of the selected points in the image, n_k is the noise affecting process dynamics, and $\triangle\psi_k = \triangle t \cdot \alpha\{max(\phi\upsilon, 0)\nabla^+ + min(\phi\upsilon, 0)\nabla^-\}$ measures the difference between the values of the current state and the next state.

The conditional probability density function of ψ^{k+1} given ψ^k is modeled using a Gibbs (or Boltzmann) distribution of the form

$$p(\psi_{k+1}|\psi_k^i) = \frac{1}{P_s} exp\{-|\psi_{k+1} - (\psi_k^i + \triangle\psi_k^i)|\} = \frac{1}{P_s} exp\{-|\psi_{k+1} - \psi_{k+1}^i|\} \quad (8)$$

where $\psi_{k+1} \in [-1, 1]$. $p(\psi_{k+1}|\psi_k^i)$ is 0 when ψ_{k+1} is not in $[-1, 1]$, and P_s is a normalization constant, which takes the following value:

$$P_s = 2 - exp(-1-(\psi_k^i + \triangle\psi_k^i)) - exp(-1+(\psi_k^i + \triangle\psi_k^i)) \quad (9)$$

3.2. The Measurement Model

Region information in the image is utilized in the measurement model. Since region information depends on the evolving curve, which is the zero level set of the states $\psi_i^k(i = 1...N)$, this measurement model is a function of the states. Additive noise is assumed to corrupt the measured image data. The measurement model is postulated to be

$$Z_k = f(\psi_k) + \upsilon_k \tag{10}$$

where $f(\psi_k) = \triangle t \cdot (1-\alpha)\gamma\{(I-c_2)^2 - (I-c_1)^2\}$, and $f(\psi_k)$ is normalized to lie between $-\triangle t$ and $\triangle t$. Comparing the formation of $f(\psi_k)$ with (5), it corresponds to the the region term of the increments, which is chosen to make use of region information in the geometric model.

The conditional probability density function of Z_k given ψ_k is assumed to be a Gibbs distribution:

$$p(Z_k|\psi_k^i) = \begin{cases} \frac{1}{P_z} exp\{-|Z_k - f(\psi_k^i)|\} & \psi_k \in [-1,1] \\ 0 & otherwise \end{cases} \tag{11}$$

where $\psi_k \in [-1, 1]$, and P_z is a normalization constant too, taking the form:

$$P_z = 2 - exp(-1 - f(\psi_k^i)) - exp(-1 + f(\psi_k^i)) \tag{12}$$

3.3. Initialization

To initiate the curve's evolution, the initial curve is represented by the zero level set of a two-dimensional plane ψ. The points inside the initial curve are assigned positive values, the points outside it are assigned negative values, and the points on the curve are given value 0. The values of each point is proportional to its distance to the initial curve.

The initial probabilities of the components of the states for the model are assumed to be exponential functions of their initial values, in which a Gibbs distribution is assumed:

$$p(\psi) = \begin{cases} \frac{1}{P_0} exp\{-|\psi - \psi_0|\} & \psi \in [-1,1] \\ 0 & otherwise \end{cases} \tag{13}$$

where $\psi_0 \in [-1, 1]$, ψ is a component of the initial state and P_0 is a normalization constant:

$$P_0 = 2 - exp(-1 - \psi_0) - exp(-1 + \psi_0) \tag{14}$$

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The state model and the measurement model are designed to simulate the geometric curve evolution model. In this way, the evolving contour, usually initialized to shrink from the image boundary, is forced to stop at boundaries of objects and thus accomplish image segmentation.

4. PARTICLE FILTERS

An overview of the theory of particle filters is given in this section; additional information can be found in [9][10]. A formation of nonlinear Bayesian state prediction and estimation will be described first.

4.1. Nonlinear State Estimation

For a nonlinear system, the state space model can be described as

$$x_{k+1} = f_k(x_k, v_k) \tag{15}$$

$$y_k = h_k(x_k, n_k) \tag{16}$$

where x_k are states of the system, f_k and h_k are possibly nonlinear functions, v_k and n_k are i.i.d process noise sequences, $k \in \mathbb{N}$, the set of natural numbers.

State estimation in a Bayesian framework for the tracking problem of the nonlinear system includes two stages - the prediction stage and the update stage. The prediction stage uses the state model to obtain the pdf of the state at time $k + 1$, prior to receipt of a measurement y_{k+1}, via the Chapman-Kolmogorov equation

$$p(x_{k+1}|y_{1:k}) = \int p(x_{k+1}|x_k)p(x_k|y_{1:k})dx_k \tag{17}$$

where the Markov property of the state space model is used: $p(x_{k+1}|x_k, y_{1:k}) = p(x_{k+1}|x_k)$.

In the update stage, the posterior pdf, given a new measurement y_{k+1}, is calculated using Bayes' rule:

$$p(x_{k+1}|y_{1:k+1}) = \frac{p(y_{k+1}|x_{k+1})p(x_{k+1}|y_{1:k})}{p(y_{k+1}|y_{1:k})} \tag{18}$$

where $p(y_{k+1}|y_{1:k})$ is a normalizing constant.

Analytic solutions of (17-18) are typically not feasible for a nonlinear system with non-Gaussian noise. In these cases, particle filters, which will be introduced in the next subsection, provide an approximate solution.

4.2. Particle Filters

Particle filters [9][10] are sequential Monte Carlo methods for the approximate solution of the nonlinear filtering equations (17-18). A probability density function $p(x)$ is approximated by a discrete random measure $\{x^{(n)}, w^{(n)}\}_{n=1}^N$, defined by particles $\{x^{(n)}\}$ and weights $\{w^{(n)}\}$. The probability distribution is approximated in the weak convergence by

$$p(x) = \sum_{n=1}^{N} w^{(n)}\delta(x - x^{(n)}) \tag{19}$$

where $\delta(.)$ is the delta function. The state estimate problem is converted to the prediction and update of the particles' positions and weights.

When sampling from $p(x)$ is computationally intractable, one can generate particles $x^{(n)}$ from a distribution $\pi(x)$, known as an importance function. Weights are assigned according to

$$w^{*(n)} = \frac{p(x)}{\pi(x)} \tag{20}$$

and then normalized to obtain the weights $w^{(n)}$. using the following formula:

$$w^{(n)} = \frac{w^{*(n)}}{\sum_{k=1}^{N} w^{*(n)}} \tag{21}$$

Suppose the posterior distribution $p(x_{1:k}|y_{1:k})$ is approximated by the discrete random measure $\chi_k = \{x_{0:k}^{(n)}, w_k^{(n)}\}_{n=1}^{N}$. The estimation problem is transformed to: given χ_k and y_{k+1}, find χ_{k+1}. This reduces to finding $x_{k+1}^{(n)}$ and $w_{k+1}^{(n)}$.

If an importance function $\pi(x_{0:k+1}|y_{0:k+1})$ can be factored,

$$\pi(x_{0:k+1}|y_{0:k+1}) = \pi(x_{k+1}|x_{0:k}, y_{0:k+1})\pi(x_{0:k}|y_{0:k}) \tag{22}$$

then the solution to the tracking problem can be accomplished using the update:

$$x_{k+1}^{(n)} \sim \pi(x_{k+1}|x_{0:k}^{(n)}, y_{0:k+1}) \tag{23}$$

$$w_{k+1}^{(n)} \propto \frac{p(y_{k+1}|x_{k+1}^{(n)})p(x_{k+1}^{(n)}|x_k^{(n)})}{\pi(x_{k+1}^{(n)}|x_{0:k}^{(n)}, y_{0:k+1})} w_{0:k}^{(n)} \tag{24}$$

Selection of the importance function remains, which plays a key role in particle filtering. Two frequently used importance functions are the prior and the optimal importance function. If the importance function is selected as the prior importance function, given by $p(x_{k+1}|x_k^{(n)})$, then the weights update as follows [9][10]:

$$w_{k+1}^{(n)} \propto w_k^{(n)} p(y_{k+1}|x_{k+1}^{(n)}) \tag{25}$$

The optimal importance function is designed to minimize the variance of $p(x_{k+1}^i|y_{1:k+1})/\pi(x_{k+1}^i|x_k^i, y_{k+1})$, which is $p(x_{k+1}|x_{0:k}^{(n)}, y_{0:k+1})$. The corresponding weight update is

$$w_{k+1}^{(n)} \propto w_k^{(n)} p(y_{k+1}|x_k^{(n)}) \tag{26}$$

Although the optimal importance function usually provides better performance than the prior importance function, it is more difficult to implement, because both integration and sampling from $p(x_{k+1}|x_{0:k}^{(n)}, y_{0:k+1})$ are required in this case. In the proposed model, the prior model is selected to be the importance function, and the weights are updated using (25). A more complete description for particle filters is provided in [9][10].

5. IMPLEMENTATION

In this section, several implementation issues are discussed. The first problem is sampling for a pdf. The method in [15] is utilized for this problem. Sampling in this method is performed by selection of a piecewise constant function whose integral over any interval approximates the pdf, and a pseudo-random number generator is used to select points according to this approximation density. Emphasis of this section will be placed on the implementations for both prediction and update.

5.1. The Prediction Step

In this probabilistic curve evolution model, evolution is accomplished in the prediction stage, predicting the states of the evolving curve at the next iteration using current state and the observed data (the region information). The prediction stage is implemented (17), from which the prediction is derived:

$$\begin{aligned}
p(x_{k+1}|y_{1:k}) &= \int p(x_{k+1}|x_k)p(x_k|y_{1:k})dx_k \\
&= \int_{-1}^{1} p(x_{k+1}|x_k) \sum_{i=1}^{N} w_k^{(i)} \delta(x_k - x_k^{(i)})dx_k \\
&= \sum_{i=1}^{N} w_k^{(i)} \int_{-1}^{1} p(x_{k+1}|x_k)\delta(x_k - x_k^{(i)})dx_k \\
&= \sum_{i=1}^{N} w_k^{(i)} p(x_{k+1}|x_k^{(i)}) \\
&= \sum_{i=1}^{N} w_k^{(i)} \cdot exp\{-|\psi_{k+1} - \psi_k^{(i)} - \nabla\psi_k^{(i)}|\}
\end{aligned} \tag{27}$$

$p(x_{k+1}|y_{1:k})$ is assumed to take the form of $exp\{-|\psi_{k+1} - \psi_{k+1}^{(i)}|\}$ to be consistent with the state model, in which $\psi_{k+1}^{(j)}$ represents the next state of the evolving contour. For the exponential distribution, most of the particles lie near the peak, and the absolute value of $\psi_{k+1} - \psi_{k+1}^{(i)}$ is very close to zero, in which case

$$exp\{-|\psi_{k+1} - \psi_{k+1}^{(i)}|\} \approx 1 - |\psi_{k+1} - \psi_{k+1}^{(i)}| \tag{28}$$

The following formula is obtained and used to update the states of the evolving curve:

$$\begin{aligned}
p(x_{k+1}|y_{1:k}) &= \frac{1}{P} exp\{-|\psi_{k+1} - \psi_{k+1}^{(i)}|\} \\
&= \frac{1}{P} exp\{-|\psi_{k+1} - \sum_{i=1}^{N} w_k^{(i)}(\psi_k^{(i)} + \nabla\psi_k^{(i)})|\}
\end{aligned} \tag{29}$$

The state at the next iteration is the weighted summation of the updated densities of all the particles, which is intuitive. Experimental results show that this approximation method works.

5.2. The Update Step

In the update step, region information is utilized to direct the evolution of the evolving curve. The update step would normally be implemented using (25):

$$w_{k+1}^{(n)} \propto w_k^{(n)} p(y_{k+1}|x_{k+1}^{(n)})$$

However, a slight revision is made: If the sign of the region information $f(\psi_k) = \triangle t \cdot (1-\alpha)\gamma\{|I-c_2|^2 - |I-c_1|^2\}$ is same as the difference $\nabla\psi_k^{(i)} = \psi_{k+1}^{(i)} - \psi_k^{(i)}$ for a particle, the weight for that particle is multiplied by $exp\{|f(\psi_k) * \phi_l|\}$; otherwise, the weight is multiplied by $exp\{|f(\psi^k) * \phi_l|\}$. In this way, both the gradient and the region information are utilized and affect the evolution of the curve, causing it to stop at object boundaries.

6. EXPERIMENTAL RESULTS

In this section, experimental results are presented and analyzed. In all the experiments shown below, the parameter α is set to be 0.2. The narrow-band method in [11] is utilized.

For the first experiment, the ability of the proposed model to undergo topological change is tested. The number of particles for each state is set to be 1300. An initial simply connected closed curve is used from Fig. 2(a)-(d). The topological change of the evolving curve to four disjoint curves is successful.

Next, the role of the region information in the model is tested. The number of particles for each state is once again 1300. The evolving curve is set to shrink. The region information causes the evolving contour to converge to the boundary of the object. This is illustrated in images (a)- (d) of Fig. 3.

The third experiment applies the proposed model to a noisy real image, as shown in Fig. 4 and 5. As in Fig. 1, anisotropic diffusion methods [13][14] are utilized to reduce the effects of noise. Comparison of Fig. 4 to Fig. 1 shows that the proposed probabilistic model achieves similar results to the geometric model. From Fig. 5(a)-(d), it can be seen that the airplane in the noisy image is successfully segmented.

7. CONCLUSION

A probabilistic curve evolution model is proposed, based on the geometric model and a Gibbs distribution. Both

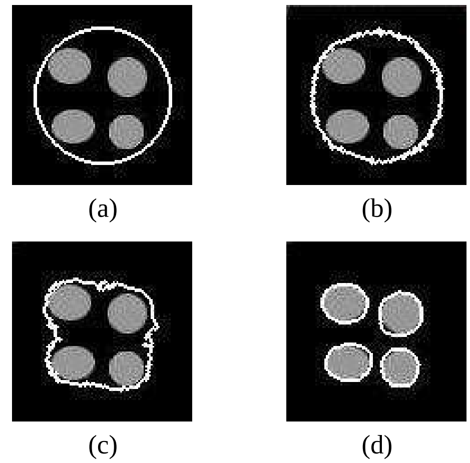

Fig. 2. Topological change of the probabilistic curve evolution.

the gradient and the region information are utilized for curve evolution. Curve evolution is accomplished using particle filters in a Bayesian framework. Experimental results show that this model does work, and good results have been achieved. The results are comparable to those acquired using geometrical models. A potential problem with the proposed model is that it is computationally intense because of the large number of the states. Further research will be focused on the improvement of the model to increase evolution speed.

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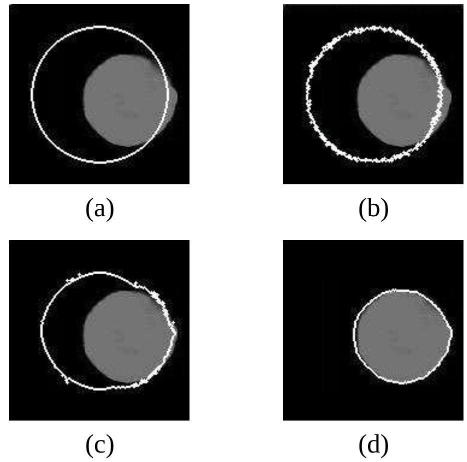

Fig. 3. The role of region information in the probabilistic curve evolution.

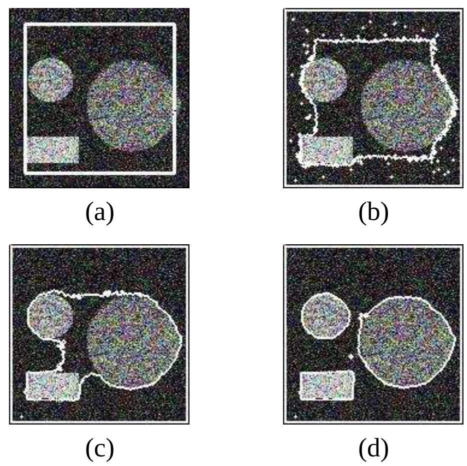

Fig. 4. Segmentation of multiple regions with noise.

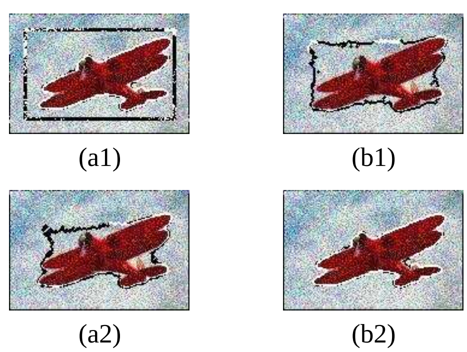

Fig. 5. Segmentation of real noisy images