An Efficient Bottom-Up Image Segmentation Method Based on Region Growing, Region Competition and the Mumford Shah Functional

Yongsheng Pan J. Douglas Birdwell Seddik M. Djouadi
Department of Electrical and Computer Engineering
University of Tennessee, Knoxville, TN, 37916
Email: ypan@lit.net birdwell@lit.net djouadi@ece.utk.edu

***Abstract*— Curve evolution implementations [3][17] [18] of the Mumford-Shah functional [11] are of broad interest in image segmentation. These implementations, however, have initialization problems [4]. A mathematical analysis of the initialization problem for the bi-modal Chan-Vese model [3] is provided in this paper. The initialization problem is a result of the non-convexity of the Mumford-Shah functional and the top-down hierarchy of the model's use of global region information in the image. An efficient image segmentation method is proposed that alleviates the initialization problem, based on region growing, region competition and the Mumford Shah functional [11]. This algorithm is able to automatically and efficiently segment objects in complicated images. Using a bottom-up hierarchy, the method avoids the initialization problem in the Chan-Vese model and works for images with multiple junctions and color images. It can be extended to textured images. Experimental results show that the proposed method is robust to the effects of noise.**

I. Introduction

Curve evolution methods [1] [2] [3] [4] [5] [7] [8] [9] [10] [14] [15] [17] [18] [19] [20] are widely used in image segmentation problems. These methods drive one or more initial curve(s), based on image gradient and/or region information , to the boundaries of objects in an image. These methods are derived using variational methods, and are implemented using finite difference approximations to PDEs and level sets [6] [12].

In curve evolution methods, region-based geometric methods [3] [4] [5] [9] [14] [17] [18] [19] have several advantages. They can deal with topological changes automatically, outperforming parametric methods such as [7] and [20]. Utilization of global region information stabilizes their responses to local variations (such as weak edges and noise) in comparison to gradient-based geometric methods [1] [2] [8] [10] [15].

Region-based geometric methods, however, have some limitations. First, most have initialization problems [4]: different initial curves produce different segmentations. Second, these methods have difficulty with complicated images with multiple junctions. Top-down hierarchical methods [9] [13] [17] or multiple coupled evolving curves [18] [19] have been used to segment multiple objects. Top-down hierarchical methods are time-consuming. In the worst case, n curve evolutions must be performed to segment an image with n objects. Coupled evolving curves usually introduce high computational loads, and techniques must be used to ensure that no pixels are left over or segmented twice.

A mathematical analysis of the initialization problem for the Chan-Vese model [3] is provided in this paper. From that analysis, provided in section II, the initialization problem is shown to originate from both the non-convexity of the Mumford-Shah functional and the top-down hierarchical way that region information is utilized.

A bottom-up hierarchical algorithm may therefore be helpful. The region competition method proposed in [20] is a good example. The method of [20] utilizes region competition to combine snakes, region growing and MDL(Minimum Description Length)/Bayes methods. Although this has been shown to work for color and textured images, the proper manual selection of seed points for region growing at the first stage is required, limiting its applicability. Tek and Kimia [16] proposed another bottom-up segmentation method using reaction-diffusion bubbles. These bubbles are hypothesized as fourth order shocks and are randomly initialized in homogeneous areas of the image. These bubbles grow, shrink, split and disappear to capture objects in the image. The method, however, has difficulty with multiple junctions.

In this paper, an efficient bottom-up image segmentation method is proposed that uses region growing and the Mumford-Shah functional. The method avoids the initialization problem of the Chan-Vese model. It works for complicated images and is efficient. It is robust to the effects of noise. Furthermore, the efficiency of the proposed method can be enhanced using multi-scale methods and parallelization.

The paper is organized as follows. In section II, the Chan-Vese model is introduced, and its initialization problem is analyzed. A novel image segmentation method based on region growing and the Mumford-Shah functional is proposed in section III. In section IV, implementation issues are discussed. An analysis of experimental results is provided in section V. Section VI provides summary with conclusions and future work.

II. The Initialization Problem of The Chan-Vese Model

The Chan-Vese model is introduced first in this section, followed by the analysis of its initialization problem.

A. The Chan-Vese Model

The Chan-Vese model [3] [18] is the curve evolution implementation of a special case of the Mumford-Shah model [11]. The bi-modal Chan-Vese model [3] minimizes the following energy functional:

$$F(c_1, c_2, C) = \mu \cdot Length(C) + \lambda_1 \iint_{inside(C)} |I(x,y) - c_1|^2 dxdy + \lambda_2 \iint_{outside(C)} |I(x,y) - c_2|^2 dxdy \tag{1}$$

where I is the original image, C is the evolving curve, and c_1 and c_2 are selected as the average values of pixels inside and outside C, respectively. μ, λ_1 and λ_2 are positive constants. Both λ_1 and λ_2 are usually taken as 1. These two parameters, therefore, are neglected in the following derivations.

The energy functional (1) is minimized by solving the following PDE:

$$\psi_t = \delta_\epsilon(\psi)[\mu \cdot \kappa - (I - c_1)^2 + (I - c_2)^2] \tag{2}$$

where ψ is the level set representation of the evolving curve C, which means $C = \{(x, y)|\psi(x, y) = 0\}$. κ represents the curvature of the evolving curve. $\delta_\epsilon(\psi) = \epsilon/(\pi(\epsilon^2 + \psi^2))$ and ϵ is a positive constant.

From (2), the evolution of the curve is influenced by two terms. The curvature term κ regularizes the curve and makes it smooth during evolution. The region term $-(I-c_1)^2+(I-c_2)^2$ affects the motion of the curve. The initialization of the curve affects curve evolution through this term.

B. The Initialization Problem of The Chan-Vese Model

The Chan-Vese model introduced above is a curve evolution implementation of the minimization of a well-posed case of the Mumford-Shah functional [3]. The Mumford-Shah functional, however, is non-convex and thus may have multiple minima [11]. Furthermore, the Chan-Vese model minimizes the functional using the Euler-Lagrange equation, which is equivalent to gradient descent. The selected initial condition determines the local optimum to which the solution of (2) converges.

Consider a piecewise constant bi-modal image. Suppose there are n_1 pixels in the background of the image, among which $m_1 (0 \leq m_1 \leq n_1)$ pixels lie inside the initial curve. Suppose there are n_2 pixels in the foreground of the image, among which $m_2 (0 \leq m_2 \leq n_2)$ lie inside the initial curve. All the pixels in the background (foreground) take $u_1 (u_2)$ as their intensity values. Obviously, $m_1 + m_2 > 0$ for all initializations.

The the average intensity inside the evolving curve is

$$c_1 = (m_1 u_1 + m_2 u_2)/(m_1 + m_2), \tag{3}$$

and the average intensity outside the evolving curve is

$$c_2 = ((n_1 - m_1)u_1 + (n_2 - m_2)u_2)/((n_1 - m_1) + (n_2 - m_2)). \tag{4}$$

The region terms $-(I - c_1)^2 + (I - c_2)^2$ for points on the evolving curve in the foreground and the background are

$$(u_2 - c_2)^2 - (u_2 - c_1)^2 = K_0 K_2 (m_2 n_1 - m_1 n_2)(u_1 - u_2)^2 \tag{5}$$

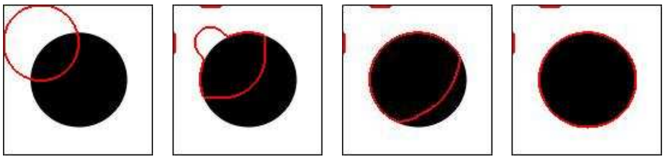
$$(u_1 - c_2)^2 - (u_1 - c_1)^2 = -K_0 K_1 (m_2 n_1 - m_1 n_2)(u_1 - u_2)^2, \tag{6}$$

where $K_0 = 1/\{(n_1 - m_1 + n_2 - m_2)(m_1 + m_2)\}$, $K_1 = (n_2 - m_2)/(n_1 - m_1 + n_2 - m_2) + m_2/(m_1 + m_2)$ and $K_2 = (n_1 - m_1)/(n_1 - m_1 + n_2 - m_2) + m_1/(m_1 + m_2)$, respectively. K_0, K_1 and K_2 are positive for any initialization.

From (5) and (6), it can be seen that the region term for points on the foreground part of the evolving curve is opposite in sign to that on the background part of the evolving curve. Therefore, if one part of the evolving curve expands, the remaining part will have to shrink and vice versa. Without loss of generality, set $\psi > 0$ inside the evolving curve and $\psi < 0$ outside the evolving curve. Then three cases may occur, depending upon different initializations. If $m_2 n_1 - m_1 n_2 > 0$ ($m_2/n_2 > m_1/n_1$), then the foreground part of the curve expands, and the background part shrinks. The curve evolves into the foreground and segments the object from the foreground, as shown in Fig. 1. If $m_2 n_1 - m_1 n_2 < 0$ ($m_2/n_2 < m_1/n_1$), then the foreground part of the curve shrinks and the background part expands. The curve evolves into the background and segment the object from the background, as shown in Fig. 2,. Finally, if $m_2 n_1 - m_1 n_2 \approx 0$, then the influence of the region term on the curve evolution is small at first. The curve is expected to evolve very slowly and may segment nothing.

It can be seen from the above analysis that initialization affects curve evolution in the Chan-Vese model. The results from Fig. 1 and Fig. 2 illustrate this point. Although in Fig. 1 and Fig. 2, the same segmentation results are achieved, in complicated cases, different initializations may generate different segmentation results, as shown in Fig. 3. Both initializations (A)(C) in Fig. 3 satisfy $m_2/n_2 > m_1/n_1$, making the foregound part of the evolving curve expand and the background part shrink. Since the upper object is not included in the initialization in (C), it is not segmented using that initialization. This suggests that every object in the image should have at least one pixel included in the initial curve for good segmentation. For the Chan-Vese model, good choices for the initialization would be the boundaries or multiple bubbles. Multi-modal images can exacerbate the initialization problem. This holds even for the multiphase Chan-Vese model [18], as shown in [4]. In the worst case, the Chan-Vese model can fail to segment any object if the total influence of the region information on the initial curve is zero, as is shown in Fig. 4 (B1) and (B2).

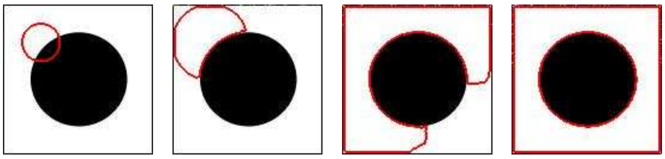
The Chan-Vese model uses a top-down hierarchical method for segmentation, and global region information is utilized only on the evolving curve in the model. The initialization problem is a consequence of this. A bottom-up hierarchical method, which makes use of local information, may reduce the initialization problem and allow its application to more complex images. A new image segmentation method, which is based on region growing and the Mumford-Shah model, is proposed in the next section.



(A) (B) (C) (D)

Fig. 1

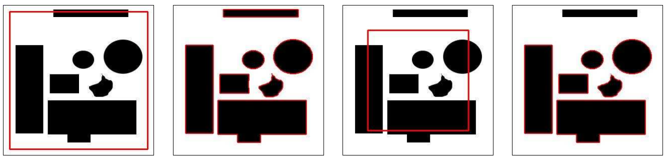
EXAMPLE OF THE INITIALIZATION PROBLEM, WHERE $m_2/n_2 = 0.21$ IS LARGER THAN $m_1/n_1 = 0.16$. (A)(B)(C)(D) SHOW THE CURVE EVOLUTION AFTER 0, 10, 60 AND 234 ITERATIONS SEPARATELY.



(A) (B) (C) (D)

Fig. 2

EXAMPLE OF THE INITIALIZATION PROBLEM, WHERE $m_2/n_2 = 0.036$ IS SMALLER THAN $m_1/n_1 = 0.065$. (A)(B)(C)(D) SHOW THE CURVE EVOLUTION AFTER 0, 10, 600 AND 1182 ITERATIONS SEPARATELY.



(A) (B) (C) (D)

Fig. 3

DIFFERENT INITIALIZATION IN THE CHAN-VESE MODEL MAY GENERATE DIFFERENT SEGMENTATION RESULTS. (A) AND (C) REPRESENT IMAGES WITH DIFFERENT INITIALIZATIONS. (B) AND (D) ARE THE SEGMENTATION RESULTS CORRESPONDING TO (A) AND (C) RESPECTIVELY. THE TOP RECTANGLE IS NOT SEGMENTED IN (D).

III. IMAGE SEGMENTATION USING REGION GROWING AND THE MUMFORD-SHAH FUNCTIONAL

In this section, an image segmentation method is proposed based on region growing, region competition and the Mumford-Shah model. First, an explanation of the Chan-Vese models [3] [18] is provided based on the concept of region competition. The proposed segmentation method is then described. Finally, an extension to color images is developed.

A. Region Competition in the Chan-Vese Models

The Chan-Vese models [3] [18] minimize an energy functional by evolving an initialized curve. Curve evolution can be interpreted as the result of competition between the foreground and the background. This idea is similar to the method proposed in [20]. Curve evolution stops when the competition is in equilibrium.

Consider, for example, the bi-modal case in section II-B, and let $\psi > 0$ inside the evolving curve. The curve evolves according to the competition between the foreground region and the background region. When the evolving curve reaches the boundary of the object, $m_1 = 0$, $m_2 = n_2$, $c_1 = u_2$, and $c_2 = u_1$. The region terms calculated using the boundary points in the foreground are $(u_2 - c_2)^2 - (u_2 - c_1)^2 = (u_2 - u_1)^2 > 0$, and the region terms for the boundary points in the background are $(u_1 - c_2)^2 - (u_1 - c_1)^2 = -(u_2 - u_1)^2 < 0$. These region terms are equal in magnitude and opposite in sign. The competition is balanced, and the evolving curve stops at the boundary of the object.

B. Image Segmentation Based on Region Growing and the Mumford-Shah Functional

The proposed image segmentation method is designed to minimize the well-posed case of the Mumford-Shah functional [11] using bottom-up region growing and region competition. The energy functional takes the form:

$$E(\Gamma) = \sum_i \iint_{R_i} (I - c_i)^2 dxdy + \nu \cdot \Gamma \tag{7}$$

where Γ represents the length of object boundaries, I represents the image to be segmented, c_i represents the average intensity of the ith region R_i and ν is a constant parameter.

As mentioned in [11], the energy functional (7) tends to segment images into piecewise constant regions, which gives an opportunity for minimization using region growing. The proposed method works as follows: Every pixel in the image is initially its own region. A region is merged with a neighboring region if this action will decrease the energy functional (7). In this way, the neighboring regions of any selected current region are competing with each other to reduce the energy functional. After two regions are merged, the intensity of each pixel in the merged region is set to the average intensity of the regions. The process is repeated until no region merging occurs and no further reduction of the value of the energy functional is possible. A mathematical description of region merging is given below.

Consider two neighboring regions Ω_1 and Ω_2 in the image. Suppose Ω_1 and Ω_2 contain n_1 and n_2 pixels respectively, with c_1 and c_2 as their average intensities. These two regions have Γ pixels in common as their boundaries. If the regions are merged, the average intensity would be $c = (n_1 c_1 + n_2 c_2)/(n_1 + n_2)$. The energy functionals before and after region merging can be evaluated:

$$E_{prev} = \iint_{\Omega_1} (I - c_1)^2 dxdy + \iint_{\Omega_2} (I - c_2)^2 dxdy + \nu \cdot \Gamma \tag{8}$$

$$E_{after} = \iint_{\Omega_1} (I-c)^2 dxdy + \iint_{\Omega_2} (I-c)^2 dxdy \quad (9)$$

. The energy difference, therefore, is

$$E_{after} - E_{prev} = 2V_1(c_1 - c) + 2V_2(c_2 - c) + n_1(c^2 - c_1^2) \quad (10)$$
$$+ n_2(c^2 - c_2^2) - \nu \cdot \Gamma$$

where $V_1 = \iint_{\Omega_1} Idxdy$ and $V_2 = \iint_{\Omega_2} Idxdy$. Region merging is performed only if the energy difference is smaller than zero, reducing the value of the energy functional.

During region growing, irregular boundaries may be generated, especially in images with strong noise. Regularization of region boundaries is, therefore, necessary. Gaussian smoothing is utilized, and is performed only when the region becomes larger than a specified threshold.

In the proposed method, region information is used in a manner that is similar to the Chan-Vese model [3], but initialization problem can be avoided and complicated cases such as multiple regions and triple junctions are automatically handled, because of the bottom-up hierarchical approach of the proposed method.

C. Extension to Color Images

The proposed method can be extended in a straightforward manner to color or multi-spectral images. Several color models, such as the one used in [20], can be chosen for the extension. The RGB color model is utilized here for simplicity. The energy functional is taken as the summation of energy functionals for each of three channels of the image, and has the form

$$E(\Gamma) = \sum_{j=1}^{3} \sum_{i} \iint_{R_i} (I_j - c_{ij})^2 dxdy + \nu \cdot \Gamma \quad (11)$$

where j represents the index of the color channel.

The implementation of the proposed method for color images is similar to that for intensity images. The only difference lies in the calculation of the region information, which is straightforward.

IV. Implementation Issues

The key issue for the implementation of the proposed method is how to select an appropriate value for ν for a specified image. Intuitively, ν should be large enough to suppress noise and small enough not to merge regions separated by edges with high gradients. We show that the value of ν should also be related to the size of competing regions.

Consider two extreme cases for the region growing problem of section III-B. First, if $V_1 = n_1c_1$, $V_2 = n_2c_2$ and $n_1 = n_2 = n$, then $c = (c_1 + c_2)/2$, and

$$E_{after} - E_{prev} = 1.5 \cdot n(c_1 - c_2)^2 - \nu \cdot \Gamma \quad (12)$$

If, on the other hand, $V_1 = n_1c_1$, $V_2 = n_2c_2$ and $n_1 >> n_2$, then $c \approx c_1$, and

$$E_{after} - E_{prev} \approx n_2(c_1 - c_2)^2 - \nu \cdot \Gamma \quad (13)$$

In both cases, the energy change as regions grow is highly related to the size of competing regions (n in (12) or n_2 in (13)). It is very difficult to select the proper value of ν if many regions of different sizes are competing at the same time.

Since the proposed method assigns a region to each pixel at the beginning, the above problem can be solved by restricting the maximum size of a region after each iteration. For example, the largest region after the first traverse of the whole image is set to 2 pixels. The image is then segmented into numerous regions containing two pixels after the first traverse. The largest region after the second traverse of the image is then set to 4 pixels. Now almost all competing regions contain 2 pixels and the image will be segmented into regions of 4 pixels, and so on. In this way, most competing regions have the same size, and the selection of the value of ν may be less affected by the sizes of the regions.

Suppose the variance of noise in an intensity image is σ^2, and the gradient of the region boundaries is expected to be g_0 ($g_0 > \sigma$); then the value of ν can be selected such that $\sigma^2 < \nu < g_0^2$. For color images, the value of ν can be selected such that $3\sigma^2 < \nu < 3g_0^2$. It is usually acceptable to choose $\nu = 1000$ for intensity images and $\nu = 3000$ for color images.

V. Experimental Results

Experimental results from the proposed method are shown in this section. The proposed method is implemented on a computer which has two Intel(R) Pentium(R) 3.2GHz CPUs, 2G bytes RAM, and runs the Red Hat Enterprise Linux operating system. The CPU times given in this paper are the sums of system CPU times and user CPU times. The system CPU time is usually very small, typically 0.01 - 0.08 seconds.

Fig. 4 represents the comparison of the proposed method and the Chan-Vese model. (A1) shows the image to be segmented. The image contains one background region (intensity 128) and 4 foreground regions (intensity 32, 64, 192, 224 counterclockwise) of equal size. The segmentation result in (A2) shows that the proposed is very efficient (1.9s). The Chan-Vese model fails for the initialization shown in (B1). Since the effects of the region information on the curve are zero, the curve evolves very slowly, driven by the curvature. After more than 13 seconds, the initial curve evolves into (B2) and will shrink to a point in the end.

Fig. 5 demonstrates the ability of the proposed method to deal with images with multiple junctions. The initialization problem can occur happen for the Chan-Vese model, as shown in [4]. The proposed method generates very good segmentation results with high efficiency.

Fig. 6 shows the stability of the proposed method with respect to noise. It can be seen that the proposed method works very well for images with strong noise. The results also shows that a larger ν is required, and the segmentation process becomes longer for images with stronger noise.

In Fig. 7, the extension of the proposed method to color images is tested. Images in Fig. 7 (A1)(A2) are designed to have the same intensity so that they can not be segmented just using intensity. By means of the color information, the proposed method successfully segments objects with different colors.

Fig. 8 shows the effects of ν on the segmentation results. The image has four regions. The pixels are randomly chosen and independent, with Gaussian distribution $N(60, 40^2)$, $N(110, 40^2)$, $N(160, 40^2)$, and $N(210, 40^2)$. By the discussion in section IV, the choice of ν should satisfy $40^2 < \nu < 50^2$. The results in Fig. 8 show that the proposed method works for a wider range of ν. From the results in Fig. 8 (B)(C)(D), it can be seen that the segmentation time becomes shorter with increasing ν, while at the same time the object boundaries become coarser. This can be explained by the fact that regions are more likely to be merged with larger ν values. In practice, a compromise has to be made between efficiency and accuracy.

Fig. 9 illustrates the utility of the proposed method for images with weak edges. From the results in Fig. 9 (B)(C)(D), we can see that the proposed method over-segments the image (A). This can be explained by the fact that gradient information is utilized in the proposed region growing method to control the segmentation process. This also shows that the proposed method uses more local information than global information. Post-processing may be necessary for images with weak edges.

Experimental results for complicated real images are provided in Fig. 10. Gaussian smoothing is not utilized here since these images are of good quality. It can be seen that the proposed method is very efficient, even for complex images. As in Fig. 9, post-processing may be necessary for better results.

VI. Conclusions and Future Work

In this paper, a mathematical analysis of the initialization problem of the Chan-Vese model is provided. This analysis shows that the initialization problem is caused by the top-down manner in which region information is used. A new bottom-up image segmentation method is proposed to solve this problem. It is based on region growing, region competition, and the Mumford-Shah functional. This method works well for complex images. It is very efficient, easy to implement and robust to noise. Experimental results show this method is able to quickly segment complex images. Future research will be focused on the combination of bottom-up and top-down hierarchical methods.

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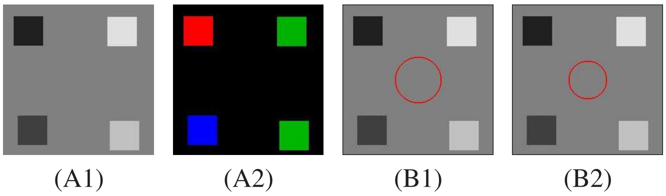

Fig. 4

Comparison of the proposed method and the Chan-Vese model for images with multiple regions. (a1) A gray image (256 by 256) with average intensity 128. (a2) Segmentation of the proposed method, ν=1000, CPU =1.9s. (b1) A gray image with the initialized curve. (b2) Segmentation using the Chan-Vese model, CPU = 13.03s.

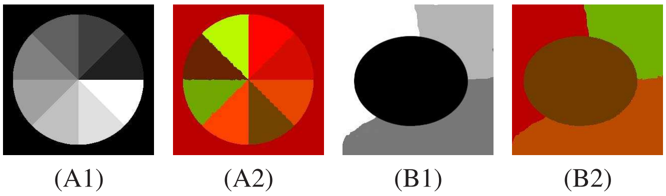

Fig. 5

More results of the proposed method for complicated images. (a1) A biscuit image (300 * 300). (a2) Segmentation of (a1), ν=300, CPU =4.08s. (b1) An image with multiple junctions (300 * 300). (b2) Segmentation of (b1), ν = 2000, CPU = 5.02s.

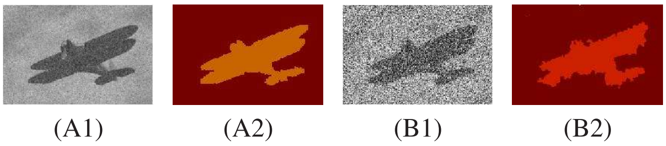

Fig. 6

Stability of the proposed method w.r.t. noise. (a1)(b1) are original images (200 * 133), with noise becoming stronger. (a2) Segmentation of (a1), ν=1000, CPU =1.08s. (b2) Segmentation of (b1), ν = 3000, CPU = 8.85s.

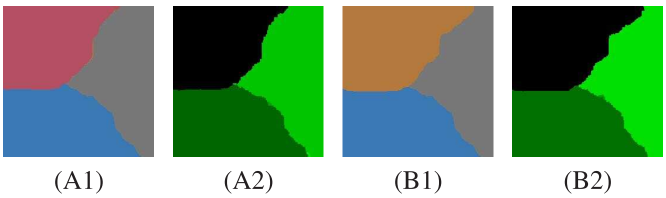

Fig. 7

Extension of the proposed method to color images. (a1)(b1) Color images (256 * 256). (a2) Segmentation of (a1), ν=1050, CPU =2.19s. (b2) Segmentation of (b1), ν = 1200, CPU = 2.05s.

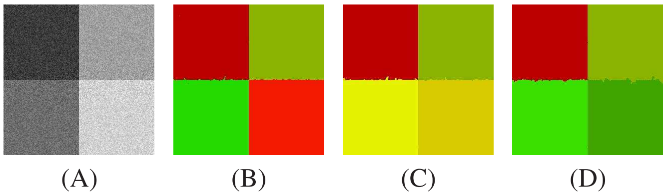

Fig. 8

Effects of ν on segmentation results. (a) A gray image with different distributions (300 * 300). (b)Segmentation of (a), $\nu = 800$, CPU = 24.79s. (c) Segmentation of (a), $\nu = 1200$, CPU = 10.29s. (d) Segmentation of (a), ν = 2000, CPU = 3.91s.

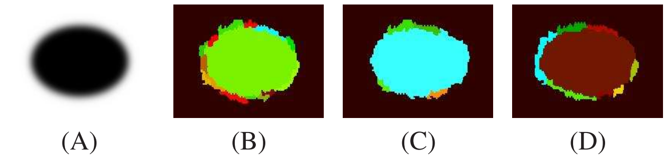

Fig. 9

The ability of the proposed method for weak edges. (a) A gray image with weak edges (200 * 150). (b)Segmentation of (a), $\nu = 5000$, CPU = 1.34s. (c) Segmentation of (a), $\nu = 10000$, CPU = 1.11s. (d) Segmentation of (a), ν = 20000, CPU = 1.12s.

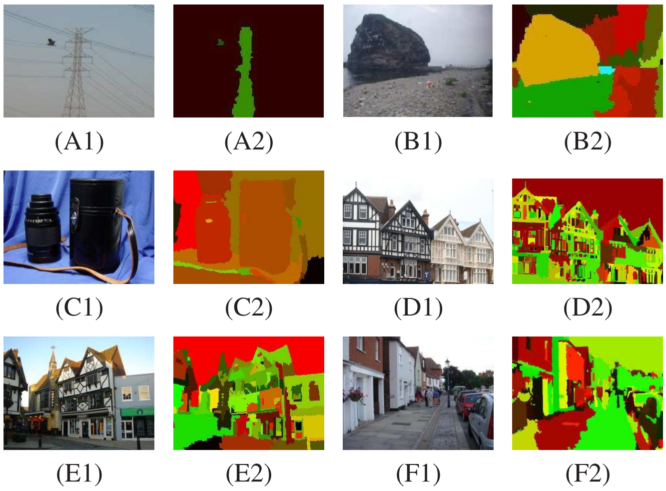

Fig. 10

Experimental results for real images. (a1)-(f1) are original real images. The sizes are: (a1) 200 * 150, (b1) 200 * 150, (c1) 300 * 225, (d1) 400 * 278,(e1) 300 * 250, (f1) 500 * 375. (a2)-(f2) are segmentation results of images (a1)-(f1). Corresponding parameters are: (a2) ν = 1500, CPU = 1.09s, (b2) ν = 2500, CPU = 1.06s, (c2) ν = 15000, CPU = 2.66s, (d2) ν = 3000, CPU = 8.04s, (e2) ν = 5000, CPU = 3.33s, (f2) ν = 3000, CPU = 9.47s.