

> with(linalg);

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian]

> u:=T*(c3*R^3+c2*R^2+c1*R);

$$u := T (c3 R^3 + c2 R^2 + c1 R)$$

> uv:=<u*cos(H), u*sin(H), 0>;

$$uv := \begin{bmatrix} T (c3 R^3 + c2 R^2 + c1 R) \cos(H) \\ T (c3 R^3 + c2 R^2 + c1 R) \sin(H) \\ 0 \end{bmatrix}$$

> Gu:=evalm(matrix([[diff(uv[1],R)*cos(H)-diff(uv[1],H)*sin(H)/R, diff(uv[1],R)*sin(H)+diff(uv[1],H)*cos(H)/R, 0], [diff(uv[2],R)*cos(H)-diff(uv[2],H)*sin(H)/R, diff(uv[2],R)*sin(H)+diff(uv[2],H)*cos(H)/R, 0], [0,0,0]]));

Gu :=

$$\begin{bmatrix} T (3 c3 R^2 + 2 c2 R + c1) \cos(H)^2 + \frac{T (c3 R^3 + c2 R^2 + c1 R) \sin(H)^2}{R}, \\ T (3 c3 R^2 + 2 c2 R + c1) \cos(H) \sin(H) - \frac{T (c3 R^3 + c2 R^2 + c1 R) \sin(H) \cos(H)}{R}, 0 \\ T (3 c3 R^2 + 2 c2 R + c1) \cos(H) \sin(H) - \frac{T (c3 R^3 + c2 R^2 + c1 R) \sin(H) \cos(H)}{R}, \\ T (3 c3 R^2 + 2 c2 R + c1) \sin(H)^2 + \frac{T (c3 R^3 + c2 R^2 + c1 R) \cos(H)^2}{R}, 0 \\ [0, 0, 0] \end{bmatrix}$$

> I3:=matrix([[1,0,0],[0,1,0],[0,0,1]]);

$$I3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

> F:=simplify(evalm(I3+Gu));

F :=

$$\begin{bmatrix} [1 + 2 T \cos(H)^2 c3 R^2 + T \cos(H)^2 c2 R + T c3 R^2 + T c2 R + T c1, \\ T \cos(H) \sin(H) R (2 c3 R + c2), 0] \\ [T \cos(H) \sin(H) R (2 c3 R + c2), \\ 1 + 3 T c3 R^2 - 2 T \cos(H)^2 c3 R^2 + 2 T c2 R - T \cos(H)^2 c2 R + T c1, 0] \\ [0, 0, 1] \end{bmatrix}$$

> P:=simplify(evalm(E/2*inverse(F)&*(F&*transpose(F)-I3)));

P :=

$$\begin{bmatrix} \frac{1}{2} T E (6 T^2 \cos(H)^2 c3^3 R^6 + 2 c2 R + 10 T^2 \cos(H)^2 c3 R^3 c2 c1 + 2 T^2 \cos(H)^2 c3 R^2 c1^2 \\ + 13 T^2 \cos(H)^2 c3^2 R^5 c2 + 8 T^2 \cos(H)^2 c3^2 R^4 c1 + 8 T \cos(H)^2 c3^2 R^4 + 4 T \cos(H)^2 c3 R^2 c1 \\ + 2 T \cos(H)^2 c2 R c1 + 3 T \cos(H)^2 c2^2 R^2 + T^2 \cos(H)^2 c2 R c1^2 + 3 T^2 \cos(H)^2 c2^2 R^2 c1 \\ + 9 T^2 \cos(H)^2 c3 R^4 c2^2 + 2 \cos(H)^2 c2 R + 4 \cos(H)^2 c3 R^2 + 3 T c1^2 + 2 c3 R^2 \\ + 2 T^2 \cos(H)^2 c2^3 R^3 + 8 T c2 R c1 + 10 T c1 c3 R^2 + 12 T c3 R^3 c2 + 10 T \cos(H)^2 c3 R^3 c2 \\ + 7 T^2 c3^2 R^4 c1 + 8 T^2 c3^2 R^5 c2 + 5 T^2 c3 R^2 c1^2 + 5 T^2 c2^2 R^2 c1 + 4 T^2 c2 R c1^2 + 2 c1 \\ + 7 T^2 c3 R^4 c2^2 + T^2 c1^3 + 12 T^2 c3 R^3 c2 c1 + 7 T c3^2 R^4 + 5 T c2^2 R^2 + 2 T^2 c2^3 R^3 + 3 T^2 c3^3 R^6 \\) / (1 + 2 T c1 + 3 T^2 c2 R c1 + 5 T^2 c3 R^3 c2 + 3 T c2 R + 2 T^2 c2^2 R^2 + 4 T^2 c1 c3 R^2 + 4 T c3 R^2 \\ + T^2 c1^2 + 3 T^2 c3^2 R^4), \frac{1}{2} E T \cos(H) \sin(H) R (2 c3 R + c2) (2 + 3 T c2 R + 2 T^2 c2^2 R^2 \\ + 4 T c3 R^2 + 3 T^2 c3^2 R^4 + T^2 c1^2 + 2 T c1 + 3 T^2 c2 R c1 + 5 T^2 c3 R^3 c2 + 4 T^2 c1 c3 R^2) / (1 \\ + 2 T c1 + 3 T^2 c2 R c1 + 5 T^2 c3 R^3 c2 + 3 T c2 R + 2 T^2 c2^2 R^2 + 4 T^2 c1 c3 R^2 + 4 T c3 R^2 \\ + T^2 c1^2 + 3 T^2 c3^2 R^4), 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} E T \cos(H) \sin(H) R (2 c3 R + c2) (2 + 3 T c2 R + 2 T^2 c2^2 R^2 + 4 T c3 R^2 + 3 T^2 c3^2 R^4 \\ + T^2 c1^2 + 2 T c1 + 3 T^2 c2 R c1 + 5 T^2 c3 R^3 c2 + 4 T^2 c1 c3 R^2) / (1 + 2 T c1 + 3 T^2 c2 R c1 \\ + 5 T^2 c3 R^3 c2 + 3 T c2 R + 2 T^2 c2^2 R^2 + 4 T^2 c1 c3 R^2 + 4 T c3 R^2 + T^2 c1^2 + 3 T^2 c3^2 R^4), -\frac{1}{2} E \\ T (6 T^2 \cos(H)^2 c3^3 R^6 - 4 c2 R + 10 T^2 \cos(H)^2 c3 R^3 c2 c1 + 2 T^2 \cos(H)^2 c3 R^2 c1^2 \\ + 13 T^2 \cos(H)^2 c3^2 R^5 c2 + 8 T^2 \cos(H)^2 c3^2 R^4 c1 + 8 T \cos(H)^2 c3^2 R^4 + 4 T \cos(H)^2 c3 R^2 c1 \end{bmatrix}$$

$$\begin{aligned}
& + 2 T \cos(H)^2 c2 R c1 + 3 T \cos(H)^2 c2^2 R^2 + T^2 \cos(H)^2 c2 R c1^2 + 3 T^2 \cos(H)^2 c2^2 R^2 c1 \\
& + 9 T^2 \cos(H)^2 c3 R^4 c2^2 + 2 \cos(H)^2 c2 R + 4 \cos(H)^2 c3 R^2 - 3 T c1^2 - 6 c3 R^2 \\
& + 2 T^2 \cos(H)^2 c2^3 R^3 - 10 T c2 R c1 - 14 T c1 c3 R^2 - 22 T c3 R^3 c2 + 10 T \cos(H)^2 c3 R^3 c2 \\
& - 15 T^2 c3^2 R^4 c1 - 21 T^2 c3^2 R^5 c2 - 7 T^2 c3 R^2 c1^2 - 8 T^2 c2^2 R^2 c1 - 5 T^2 c2 R c1^2 - 2 c1 \\
& - 16 T^2 c3 R^4 c2^2 - T^2 c1^3 - 22 T^2 c3 R^3 c2 c1 - 15 T c3^2 R^4 - 8 T c2^2 R^2 - 4 T^2 c2^3 R^3 \\
& - 9 T^2 c3^3 R^6) / (1 + 2 T c1 + 3 T^2 c2 R c1 + 5 T^2 c3 R^3 c2 + 3 T c2 R + 2 T^2 c2^2 R^2 \\
& + 4 T^2 c1 c3 R^2 + 4 T c3 R^2 + T^2 c1^2 + 3 T^2 c3^2 R^4), 0]
\end{aligned}$$

[0, 0, 0]

```
> Q:=matrix([[ cos(H),sin(H),0],
             [-sin(H),cos(H),0],
             [0,0,1]]);
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$$Q := \begin{bmatrix} \cos(H) & \sin(H) & 0 \\ -\sin(H) & \cos(H) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> PQ:=simplify(evalm(Q&*P&*transpose(Q)));
```

PQ :=

$$\begin{aligned}
& \left[\frac{1}{2} (9 T c3^2 R^4 + 12 T c3 R^3 c2 + 6 T c1 c3 R^2 + 4 T c2^2 R^2 + 6 c3 R^2 + 4 T c2 R c1 + 4 c2 R \right. \\
& \left. + T c1^2 + 2 c1) T E / (T c1 + 1 + 2 T c2 R + 3 T c3 R^2), 0, 0 \right]
\end{aligned}$$

$\left[0, \frac{1}{2} \right.$

$$\begin{aligned}
& (T c3^2 R^4 + 2 T c3 R^3 c2 + 2 T c1 c3 R^2 + T c2^2 R^2 + 2 c3 R^2 + 2 T c2 R c1 + 2 c2 R + T c1^2 + 2 c1) \\
& \left. T E / (T c3 R^2 + T c2 R + T c1 + 1), 0 \right]
\end{aligned}$$

[0, 0, 0]

```
> Pb:=unapply(PQ[1,1],R);
```

$$\begin{aligned}
Pb := R \rightarrow & \frac{1}{2} (9 T c3^2 R^4 + 12 T c3 R^3 c2 + 6 T c1 c3 R^2 + 4 T c2^2 R^2 + 6 c3 R^2 + 4 T c2 R c1 \\
& + 4 c2 R + T c1^2 + 2 c1) T E / (T c1 + 1 + 2 T c2 R + 3 T c3 R^2)
\end{aligned}$$

```
> ub:=unapply(u,R);
```

$$ub := R \rightarrow T (c3 R^3 + c2 R^2 + c1 R)$$

```
> assume(Ri>0);
```

```
> assume(Ro>Ri);
```

```
> interface(showassumed=2);
```

> solve({Pb(Ro)=0,Pb(Ri)=0,ub(Ro)=T},{c1,c2,c3});

$$\{c3 = -2 \frac{1}{Ro^2 (Ro - 3 Ri)}, c2 = 3 \frac{Ro + Ri}{Ro^2 (Ro - 3 Ri)}, c1 = -6 \frac{Ri}{(Ro - 3 Ri) Ro}\}, \{$$

$$c3 = -2 \frac{-T Ri + Ro^2 + T Ro}{T Ro^2 (3 Ri^2 - 4 Ri Ro + Ro^2)}, c1 = -2 \frac{3 T Ri Ro + Ro^3 - 3 T Ri^2}{T Ro (3 Ri^2 - 4 Ri Ro + Ro^2)},$$

$$c2 = \frac{4 Ro^3 + 3 T Ro^2 - 3 T Ri^2}{T Ro^2 (3 Ri^2 - 4 Ri Ro + Ro^2)}\}, \{$$

$$c3 = -2 \frac{2 Ro + T}{T Ro^2 (Ro - 3 Ri)}, c2 = 3 \frac{2 Ro^2 + T Ro + 2 Ri Ro + T Ri}{T Ro^2 (Ro - 3 Ri)}, c1 = -2 \frac{Ro^2 + 3 Ri Ro + 3 T Ri}{T Ro (Ro - 3 Ri)}$$

$$\}, \{c2 = \frac{2 Ro^3 + 3 T Ro^2 - 6 Ri^2 Ro - 3 T Ri^2}{T Ro^2 (3 Ri^2 - 4 Ri Ro + Ro^2)}, c1 = -2 \frac{Ri (2 Ro^2 + 3 T Ro - 3 Ri Ro - 3 T Ri)}{T (3 Ri^2 - 4 Ri Ro + Ro^2) Ro},$$

$$c3 = -2 \frac{Ro^2 + T Ro - 2 Ri Ro - T Ri}{T Ro^2 (3 Ri^2 - 4 Ri Ro + Ro^2)}\}$$

with assumptions on Ro and Ri

> dP:=simplify(<diff(P[1,1],R)*cos(H)-diff(P[1,1],H)*sin(H)/R
+diff(P[2,1],R)*sin(H)+diff(P[2,1],H)*cos(H)/R,
diff(P[1,2],R)*cos(H)-diff(P[1,2],H)*sin(H)/R
+diff(P[2,2],R)*sin(H)+diff(P[2,2],H)*cos(H)/R,
0>);

dP :=

$$\left[\frac{1}{2} \cos(H) T E (12 T c2 c1 + 19 T c2^2 R + 72 T^3 c3^4 R^7 + 24 T^2 c2^3 R^2 + 9 T^2 c2 c1^2 + 68 T c3^2 R^3 + 3 T^3 c2 c1^3 + 12 T^3 c2^4 R^3 + 120 c3^3 R^5 T^2 + 16 c3 R + 6 c2 + 32 T c3 R c1 + 112 c3^2 R^3 T^2 c1 + 24 c3 R T^2 c1^2 + 76 T c3 R^2 c2 + 30 T^2 c2^2 R c1 + 221 T^2 c3^2 R^4 c2 + 130 T^2 c3 R^3 c2^2 + 56 T^3 c3^2 R^3 c1^2 + 8 T^3 c3 R c1^3 + 120 T^3 c3^3 R^5 c1 + 191 T^3 c3^2 R^5 c2^2 + 195 T^3 c3^3 R^6 c2 + 80 T^3 c3 R^4 c2^3 + 15 T^3 c2^2 c1^2 R + 24 T^3 c2^3 c1 R^2 + 122 T^2 c3 R^2 c2 c1 + 61 T^3 c3 R^2 c1^2 c2 + 221 T^3 c3^2 R^4 c1 c2 + 130 T^3 c3 R^3 c1 c2^2) / ((T c1 + 1 + 2 T c2 R + 3 T c3 R^2) (1 + 2 T c1 + 3 T^2 c2 R c1 + 5 T^2 c3 R^3 c2 + 3 T c2 R + 2 T^2 c2^2 R^2 + 4 T^2 c1 c3 R^2 + 4 T c3 R^2 + T^2 c1^2 + 3 T^2 c3^2 R^4)) \right]$$

$$\left[\frac{1}{2} (12 T c2 c1 + 19 T c2^2 R + 72 T^3 c3^4 R^7 + 24 T^2 c2^3 R^2 + 9 T^2 c2 c1^2 + 68 T c3^2 R^3 + 3 T^3 c2 c1^3 + 12 T^3 c2^4 R^3 + 120 c3^3 R^5 T^2 + 16 c3 R + 6 c2 + 32 T c3 R c1 + 112 c3^2 R^3 T^2 c1 + 24 c3 R T^2 c1^2 + 76 T c3 R^2 c2 + 30 T^2 c2^2 R c1 + 221 T^2 c3^2 R^4 c2 + 130 T^2 c3 R^3 c2^2 + 56 T^3 c3^2 R^3 c1^2 + 8 T^3 c3 R c1^3 + 120 T^3 c3^3 R^5 c1 + 191 T^3 c3^2 R^5 c2^2 + 195 T^3 c3^3 R^6 c2$$

$$\begin{aligned}
& + 80 T^3 c^3 R^4 c^2 + 15 T^3 c^2 c l^2 R + 24 T^3 c^2 c^3 c l R^2 + 122 T^2 c^3 R^2 c^2 c l + 61 T^3 c^3 R^2 c l^2 c^2 \\
& + 221 T^3 c^3 R^4 c l c^2 + 130 T^3 c^3 R^3 c l c^2) E T \sin(H) / ((T c l + 1 + 2 T c^2 R + 3 T c^3 R^2) (1 \\
& + 2 T c l + 3 T^2 c^2 R c l + 5 T^2 c^3 R^3 c^2 + 3 T c^2 R + 2 T^2 c^2 R^2 + 4 T^2 c l c^3 R^2 + 4 T c^3 R^2 \\
& + T^2 c l^2 + 3 T^2 c^3 R^4)) \Big]
\end{aligned}$$

[0]

> **b:=evalm(-pi^2*E/rho*uv-1/rho*dP);**

$$\begin{aligned}
b := & \left[-\frac{\pi^2 E T (c^3 R^3 + c^2 R^2 + c l R) \cos(H)}{\rho} - \frac{1}{2} \cos(H) T E (12 T c^2 c l + 19 T c^2 R \right. \\
& + 72 T^3 c^3 R^7 + 24 T^2 c^2 c^3 R^2 + 9 T^2 c^2 c l^2 + 68 T c^3 R^3 + 3 T^3 c^2 c l^3 + 12 T^3 c^2 R^3 \\
& + 120 c^3 R^5 T^2 + 16 c^3 R + 6 c^2 + 32 T c^3 R c l + 112 c^3 R^3 T^2 c l + 24 c^3 R T^2 c l^2 \\
& + 76 T c^3 R^2 c^2 + 30 T^2 c^2 R c l + 221 T^2 c^3 R^4 c^2 + 130 T^2 c^3 R^3 c^2 + 56 T^3 c^3 R^3 c l^2 \\
& + 8 T^3 c^3 R c l^3 + 120 T^3 c^3 R^5 c l + 191 T^3 c^3 R^5 c^2 + 195 T^3 c^3 R^6 c^2 + 80 T^3 c^3 R^4 c^2^3 \\
& + 15 T^3 c^2 c l^2 R + 24 T^3 c^2 c^3 c l R^2 + 122 T^2 c^3 R^2 c^2 c l + 61 T^3 c^3 R^2 c l^2 c^2 \\
& + 221 T^3 c^3 R^4 c l c^2 + 130 T^3 c^3 R^3 c l c^2) / (\rho (T c l + 1 + 2 T c^2 R + 3 T c^3 R^2) (1 + 2 T c l \\
& + 3 T^2 c^2 R c l + 5 T^2 c^3 R^3 c^2 + 3 T c^2 R + 2 T^2 c^2 R^2 + 4 T^2 c l c^3 R^2 + 4 T c^3 R^2 + T^2 c l^2 \\
& + 3 T^2 c^3 R^4)), -\frac{\pi^2 E T (c^3 R^3 + c^2 R^2 + c l R) \sin(H)}{\rho} - \frac{1}{2} (12 T c^2 c l + 19 T c^2 R \\
& + 72 T^3 c^3 R^7 + 24 T^2 c^2 c^3 R^2 + 9 T^2 c^2 c l^2 + 68 T c^3 R^3 + 3 T^3 c^2 c l^3 + 12 T^3 c^2 R^3 \\
& + 120 c^3 R^5 T^2 + 16 c^3 R + 6 c^2 + 32 T c^3 R c l + 112 c^3 R^3 T^2 c l + 24 c^3 R T^2 c l^2 \\
& + 76 T c^3 R^2 c^2 + 30 T^2 c^2 R c l + 221 T^2 c^3 R^4 c^2 + 130 T^2 c^3 R^3 c^2 + 56 T^3 c^3 R^3 c l^2 \\
& + 8 T^3 c^3 R c l^3 + 120 T^3 c^3 R^5 c l + 191 T^3 c^3 R^5 c^2 + 195 T^3 c^3 R^6 c^2 + 80 T^3 c^3 R^4 c^2^3 \\
& + 15 T^3 c^2 c l^2 R + 24 T^3 c^2 c^3 c l R^2 + 122 T^2 c^3 R^2 c^2 c l + 61 T^3 c^3 R^2 c l^2 c^2 \\
& + 221 T^3 c^3 R^4 c l c^2 + 130 T^3 c^3 R^3 c l c^2) E T \sin(H) / (\rho (T c l + 1 + 2 T c^2 R + 3 T c^3 R^2) (\\
& 1 + 2 T c l + 3 T^2 c^2 R c l + 5 T^2 c^3 R^3 c^2 + 3 T c^2 R + 2 T^2 c^2 R^2 + 4 T^2 c l c^3 R^2 + 4 T c^3 R^2 \\
& + T^2 c l^2 + 3 T^2 c^3 R^4)), 0 \Big]
\end{aligned}$$

> **with(codegen,C);**

[C]

> **C(b,optimized,mode=double);**

```

t1 = pi*pi;
t3 = 1/rho;
t4 = t1*E*t3;
t5 = R*R;
t6 = t5*R;
t8 = c2*t5;
t9 = c1*R;

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t11 = T*(c3*t6+t8+t9);
t12 = cos(H);
t18 = c2*c2;
t22 = T*c2;
t25 = c3*R;
t27 = T*T;
t28 = t27*T;
t29 = t28*c3;
t34 = c3*c3;
t35 = t28*t34;
t36 = t5*t5;
t41 = c1*c1;
t46 = t27*c3;
t50 = t18*c2;
t54 = t34*t34;
t59 = t27*c2;
t62 = t34*c3;
t63 = t36*R;
t67 = t18*t18;
t72 = t41*c1;
t78 = 6.0*c2+19.0*T*t18*R+12.0*t22*c1+16.0*t25+130.0*t29*t6*c1*t18+221.0*
t35*t36*c1*c2+61.0*t29*t5*t41*c2+122.0*t46*t8*c1+24.0*t27*t50*t5+72.0*t28*t54*
t36*t6+9.0*t59*t41+120.0*t62*t63*t27+12.0*t28*t67*t6+3.0*t28*c2*t72+68.0*T*t34*
t6;
t79 = t27*t41;
t83 = t27*c1;
t86 = T*c3;
t89 = t27*t18;
t94 = t27*t34;
t101 = t28*t62;
t129 = 24.0*t25*t79+112.0*t34*t6*t83+32.0*t86*t9+30.0*t89*t9+76.0*t86*t8+
221.0*t94*t36*c2+191.0*t35*t63*t18+120.0*t101*t63*c1+8.0*t29*R*t72+56.0*t35*t6*
t41+130.0*t46*t6*t18+15.0*t28*t18*t41*R+80.0*t29*t36*t50+195.0*t101*t36*t5*c2+
24.0*t28*t50*c1*t5;
t130 = t78+t129;
t132 = T*c1;
t133 = t22*R;
t135 = t86*t5;
t156 = 1/(t132+1.0+2.0*t133+3.0*t135)/(1.0+2.0*t132+3.0*t59*t9+5.0*t46*t6
*c2+3.0*t133+2.0*t89*t5+4.0*t83*c3*t5+4.0*t135+t79+3.0*t94*t36);
t161 = sin(H);
b[0] = -t4*t11*t12-t3*t12*T*E*t130*t156/2.0;
b[1] = -t4*t11*t161-t3*t130*E*T*t161*t156/2.0;
b[2] = 0.0;

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