cs6630 | October 30 2014

GRIDS

Miriah Meyer

University of Utah

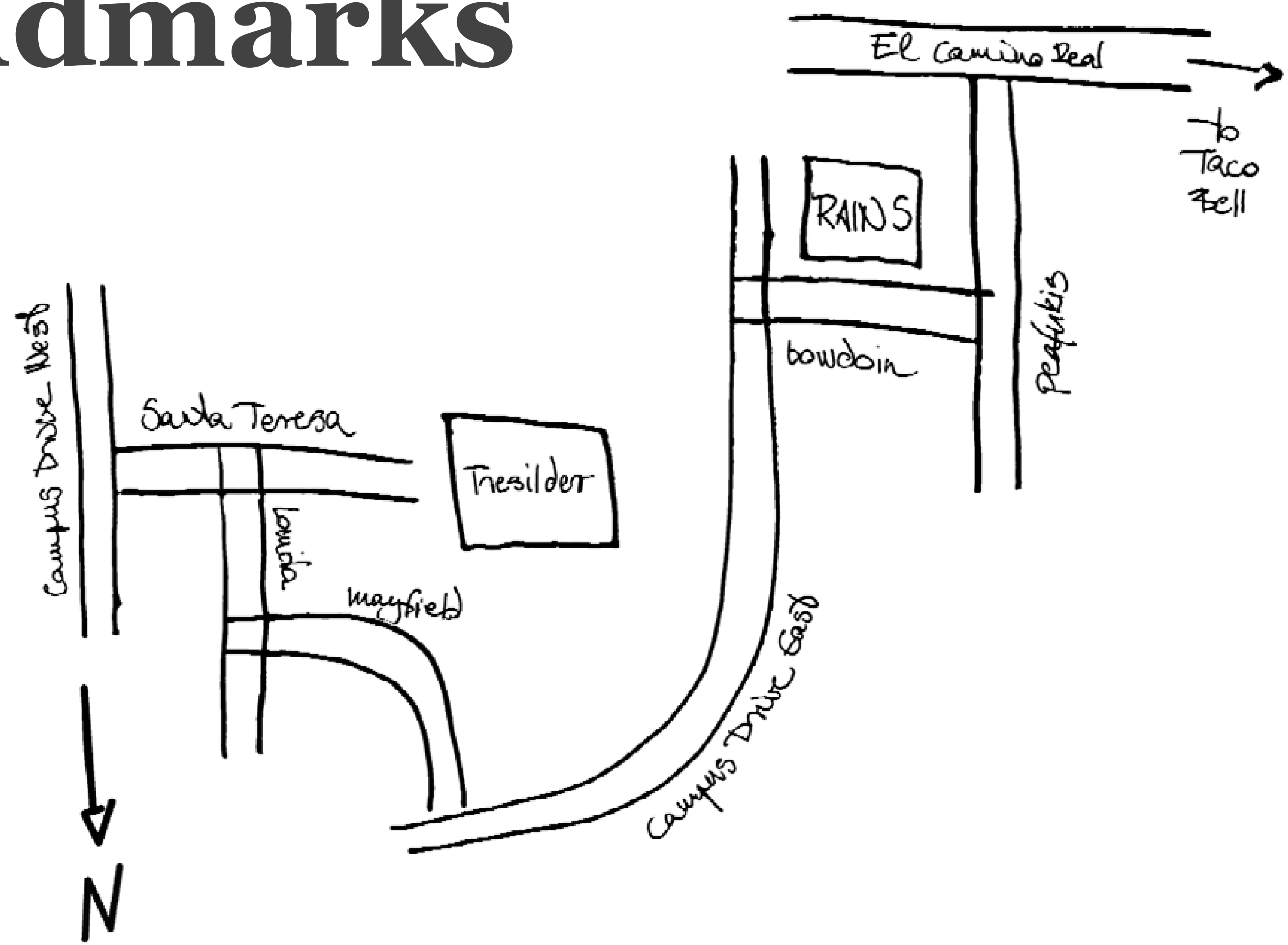
administrivia . . .

- parallel coordinates assignment due tonight
- scalar data assignment out today

last time . . .

MAPS

landmarks

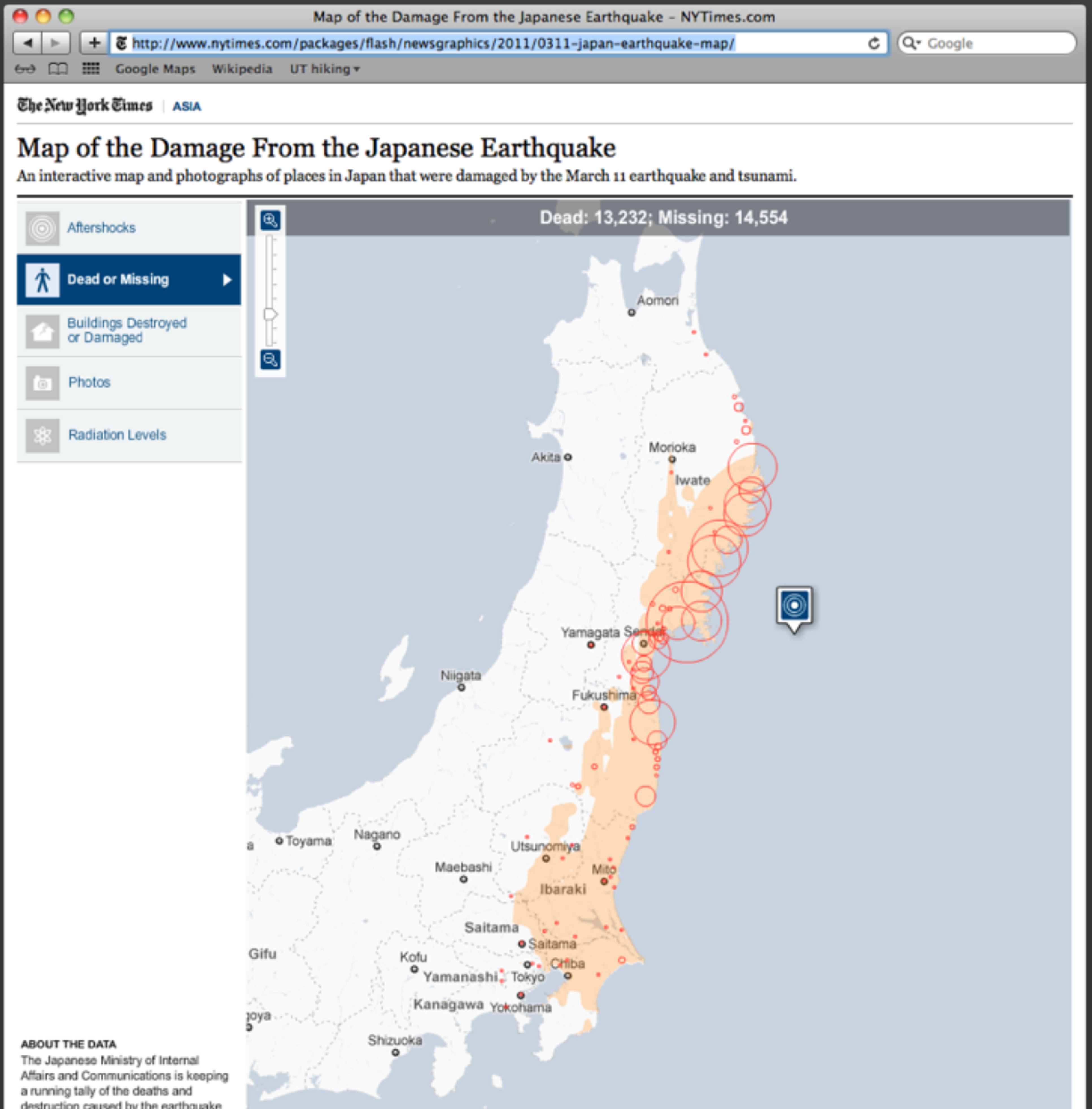

El Camino Real
to Taco Bell
RAINS
bowdoin
Campus Drive West
Santa Teresa
Tresidder
mayfield
Campus Drive East
N

what do they mean?

data as points

data : ordered/ quantitative

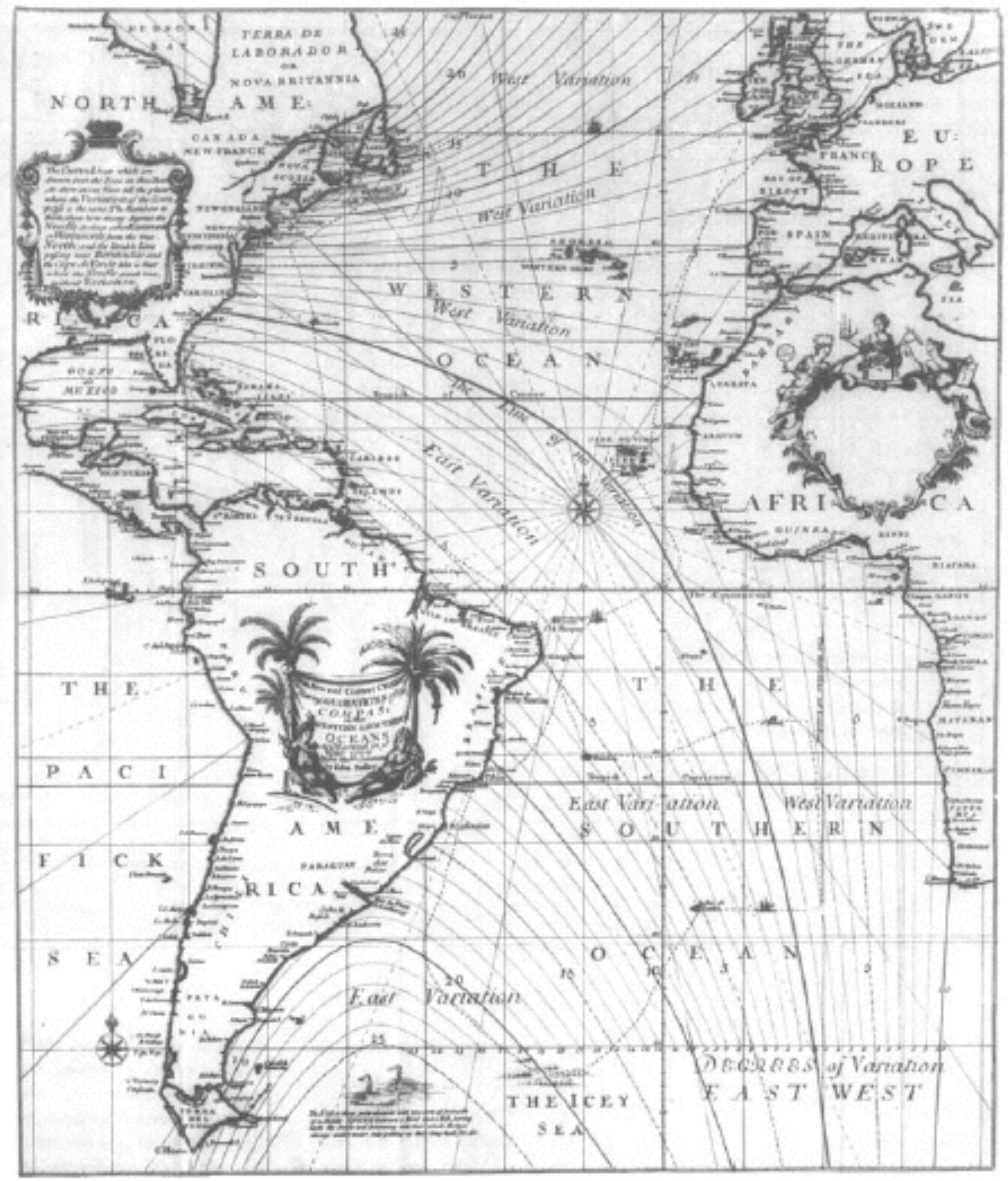
encoding : size



isopleth

map which overlays continuous data using a third encoding channel

Lines of Equal Magnetic Declination
first contour map

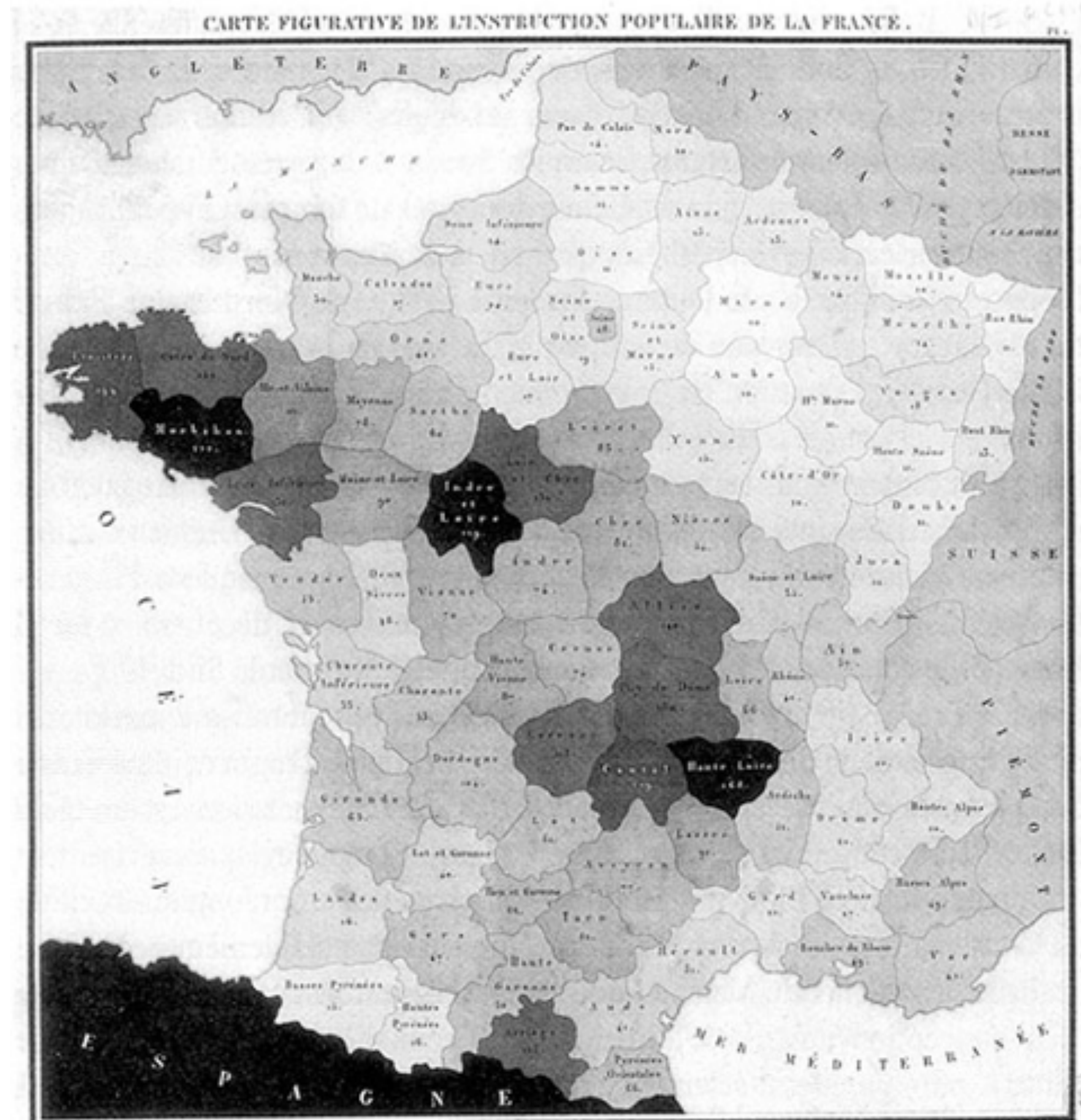


Edmond Halley, 1701

choropleth

map in which areas are shaded, colored, or patterned relative to a data attribute value

Illiteracy in France
first choropleth map

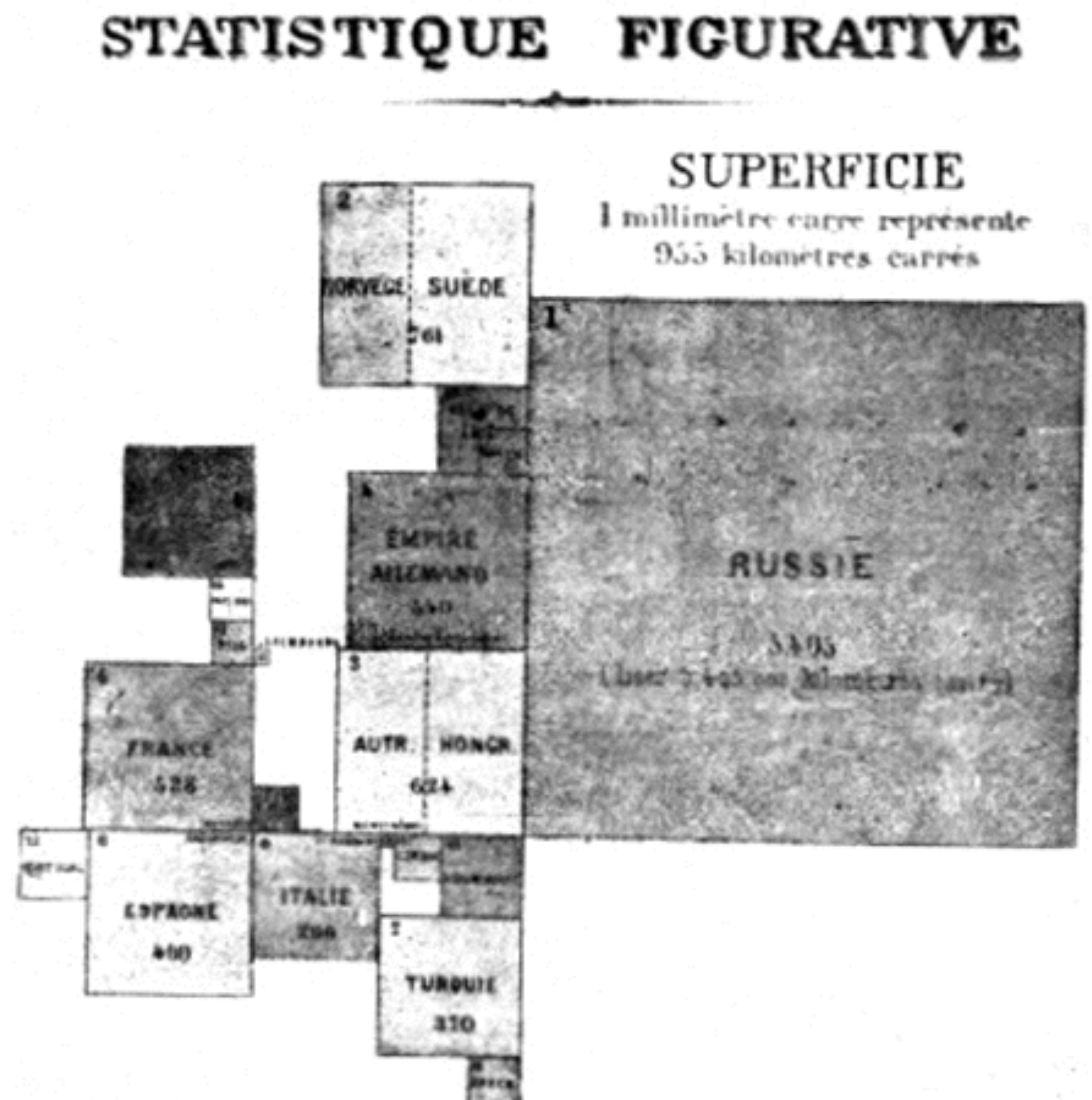


Charles Dupin, 1826

cartogram

map in which areas are scaled and distorted relative to a data attribute value

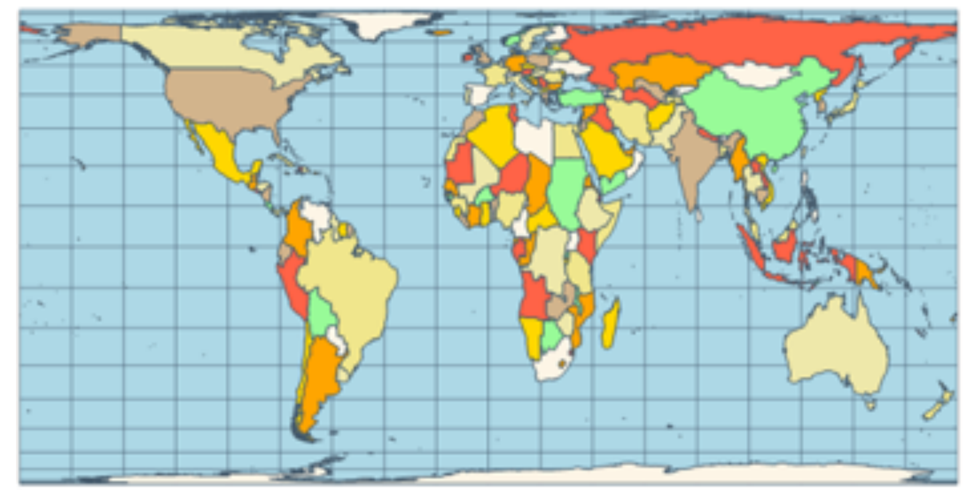
Land Area
first cartogram



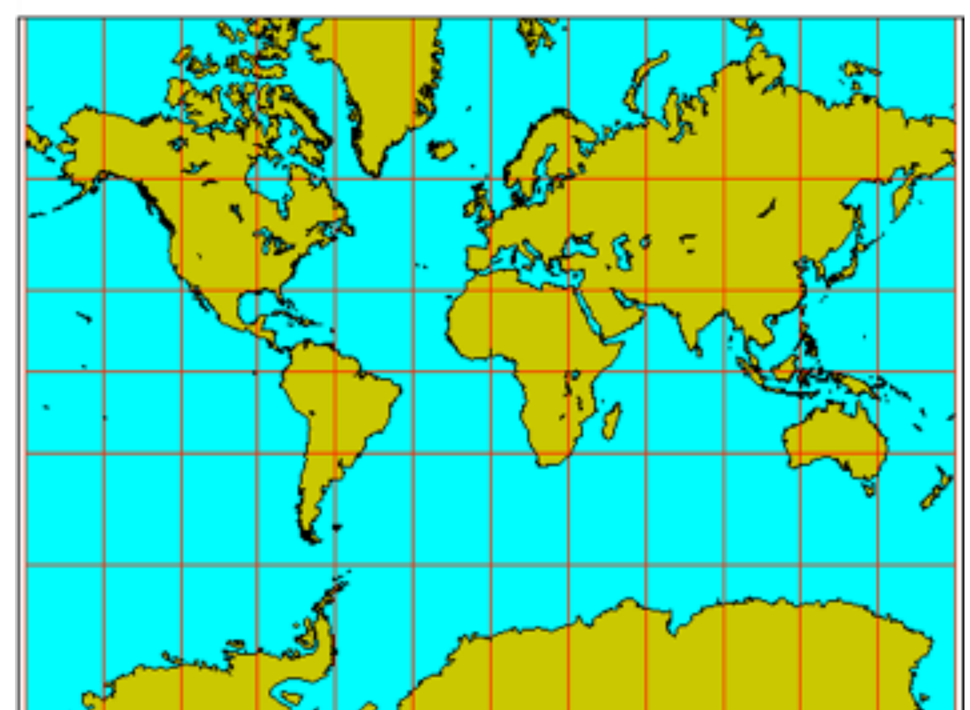
Emile Levasseur, 1868

azimuthal
preserves direction

equal-area
preserves area

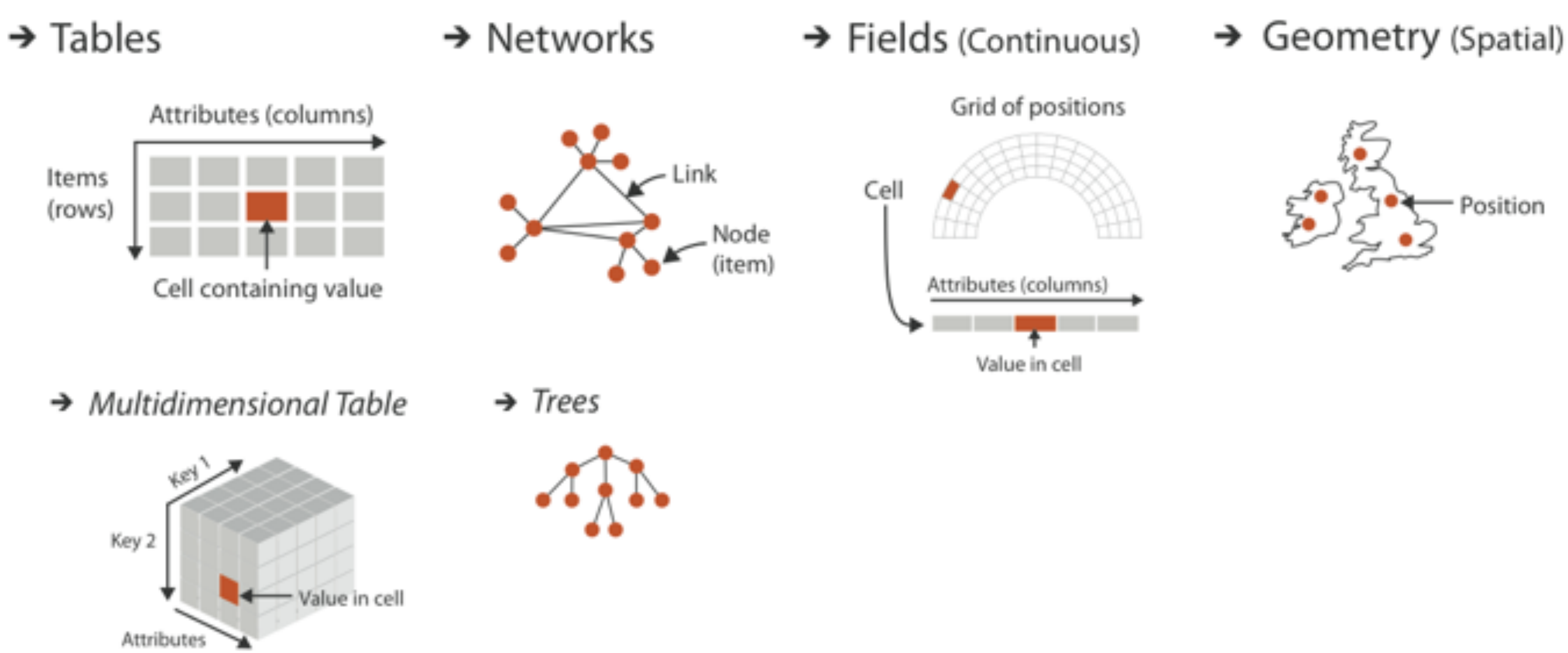


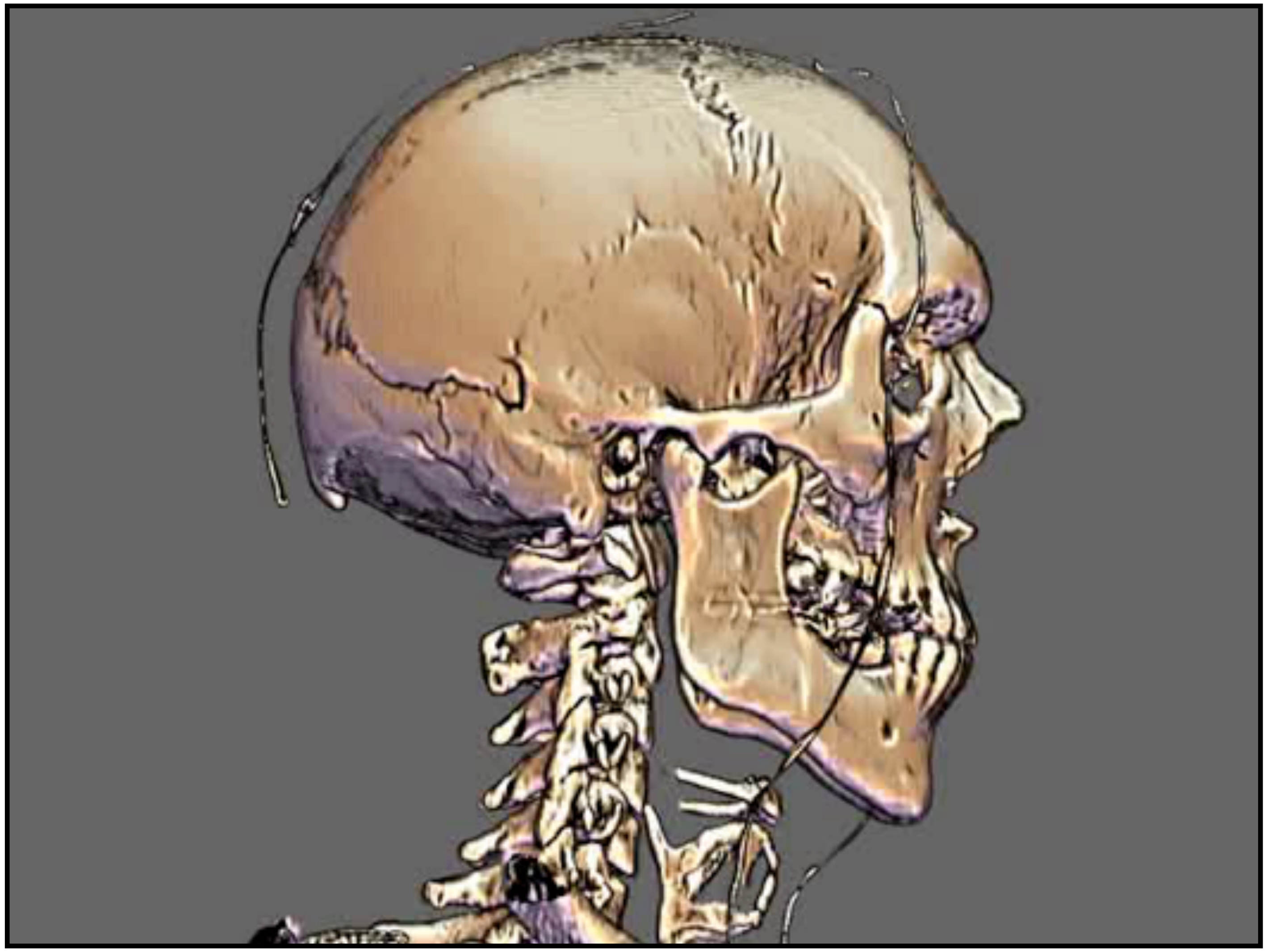
conformal
preserves local shapes

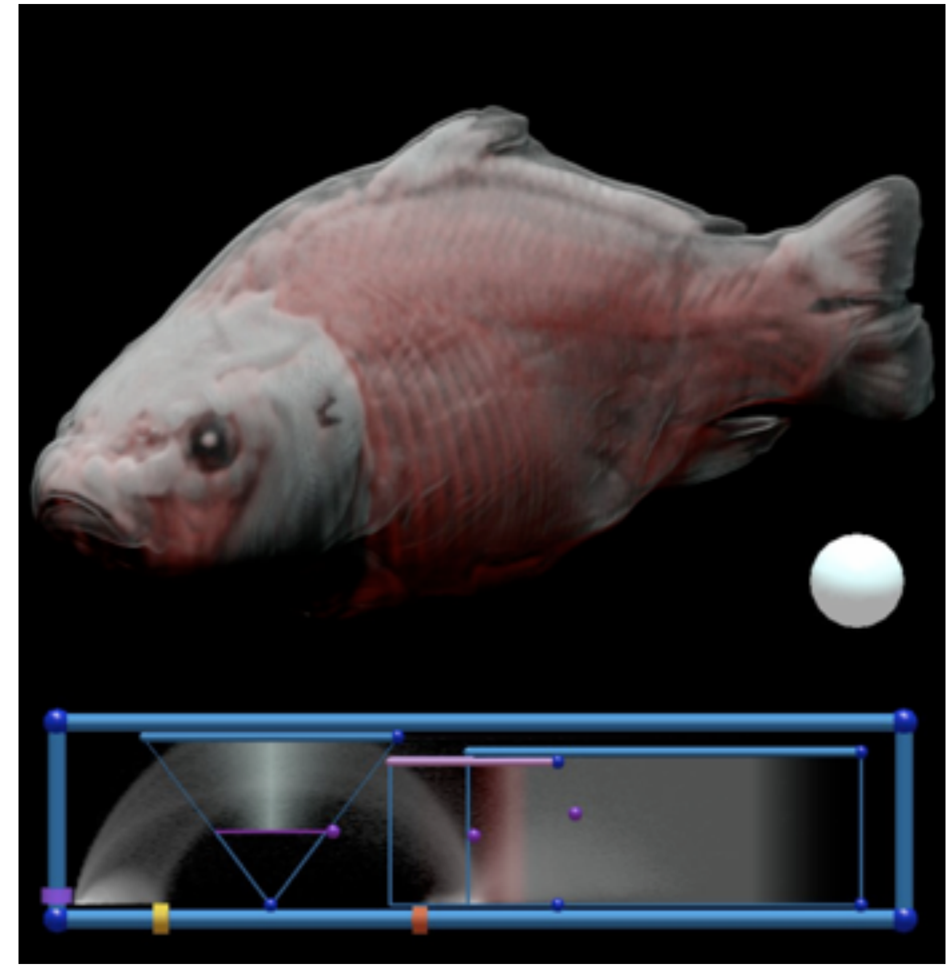


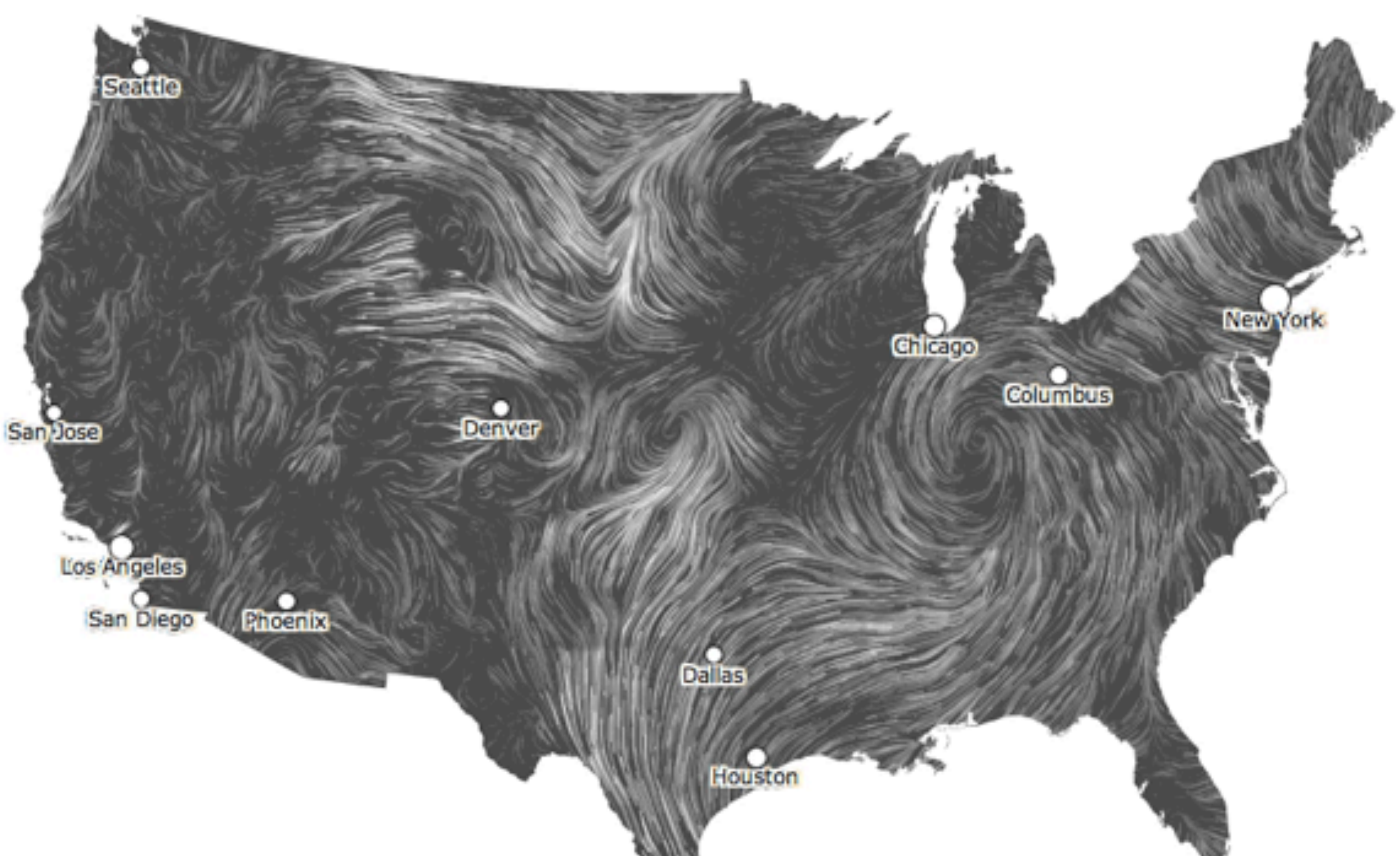
today . . .

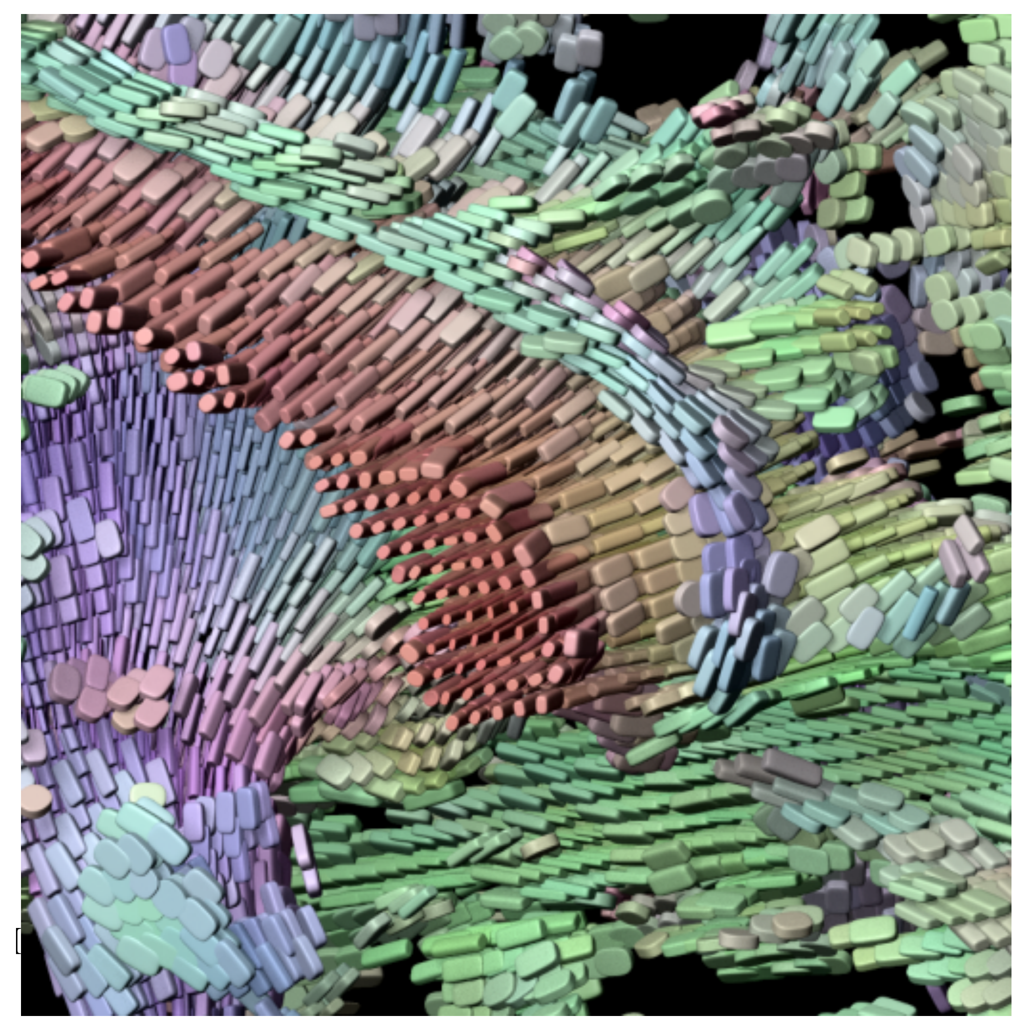
Tables	Networks & Trees	Fields	Geometry	Clusters, Sets, Lists
Items	Items (nodes)	Grids	Items	Items
Attributes	Links	Positions	Positions	
	Attributes	Attributes		

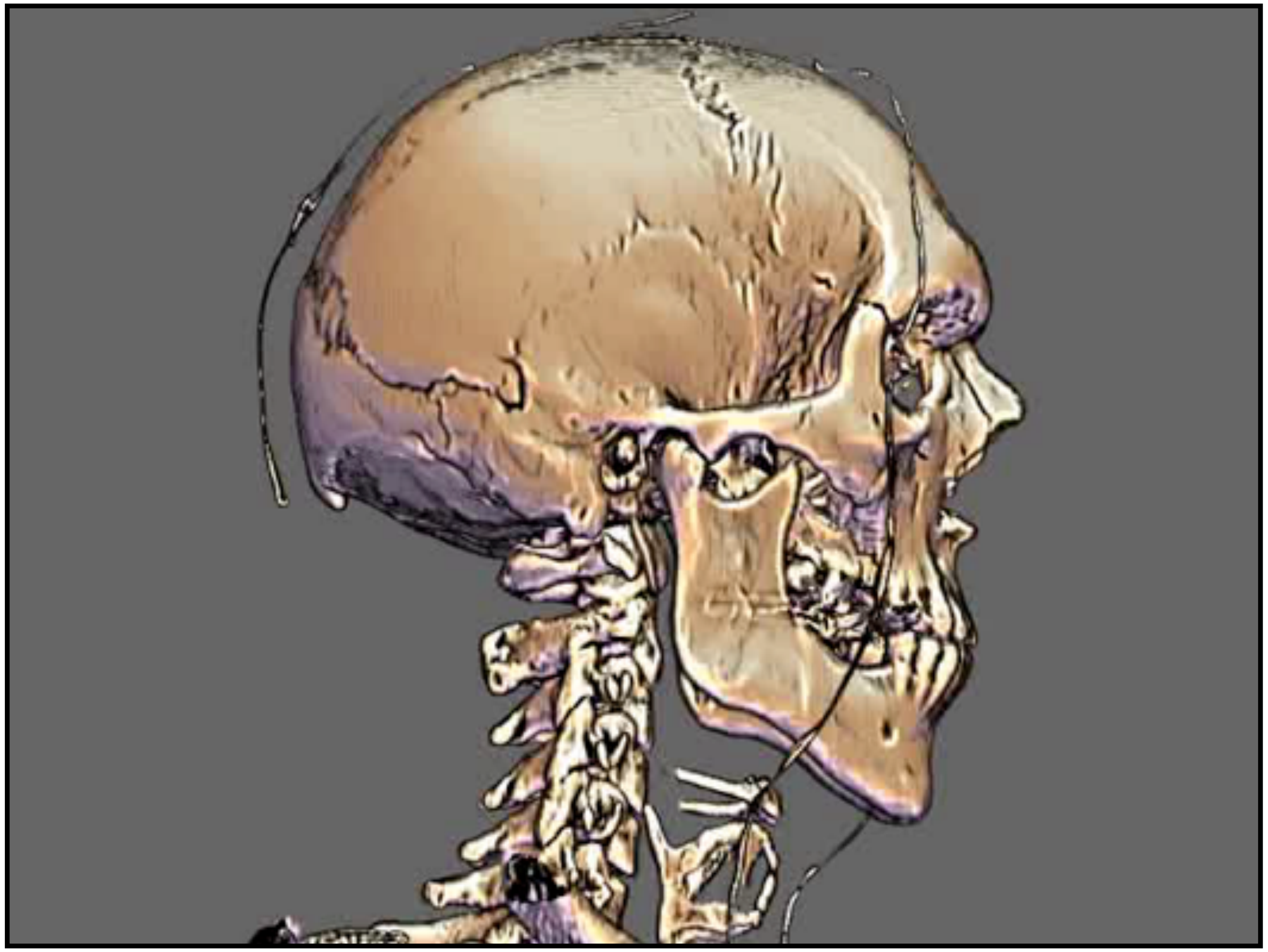


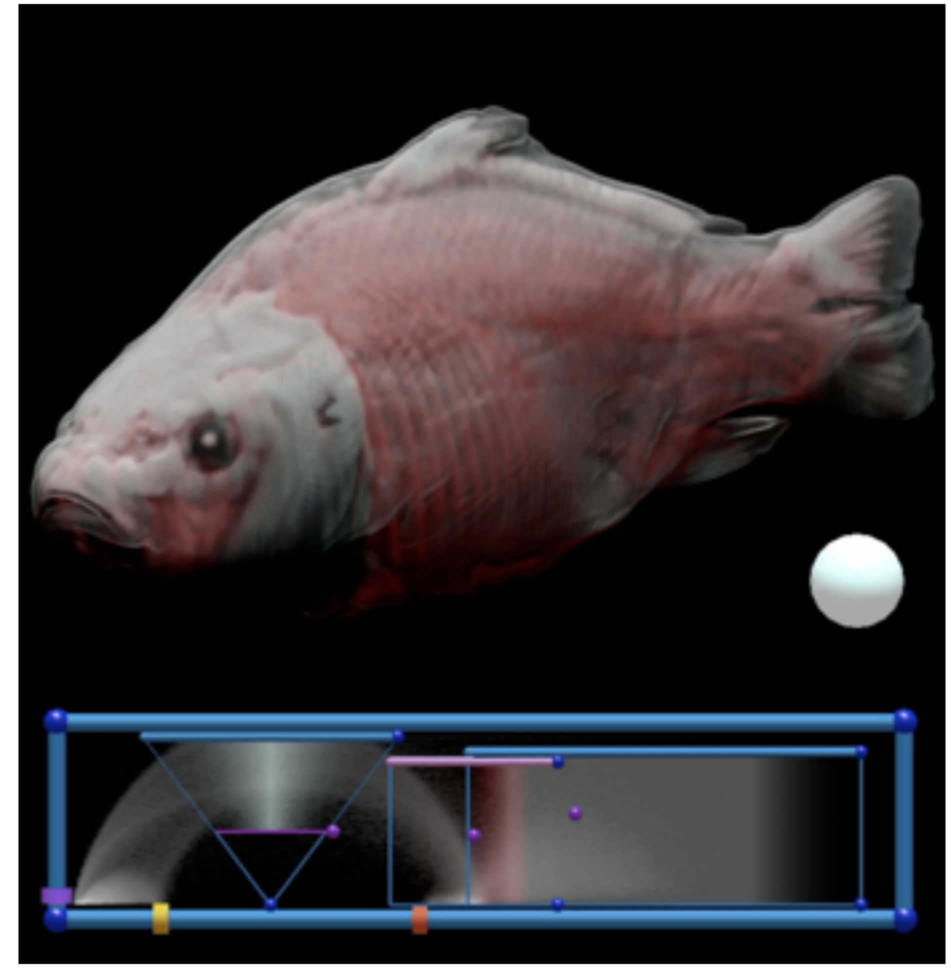


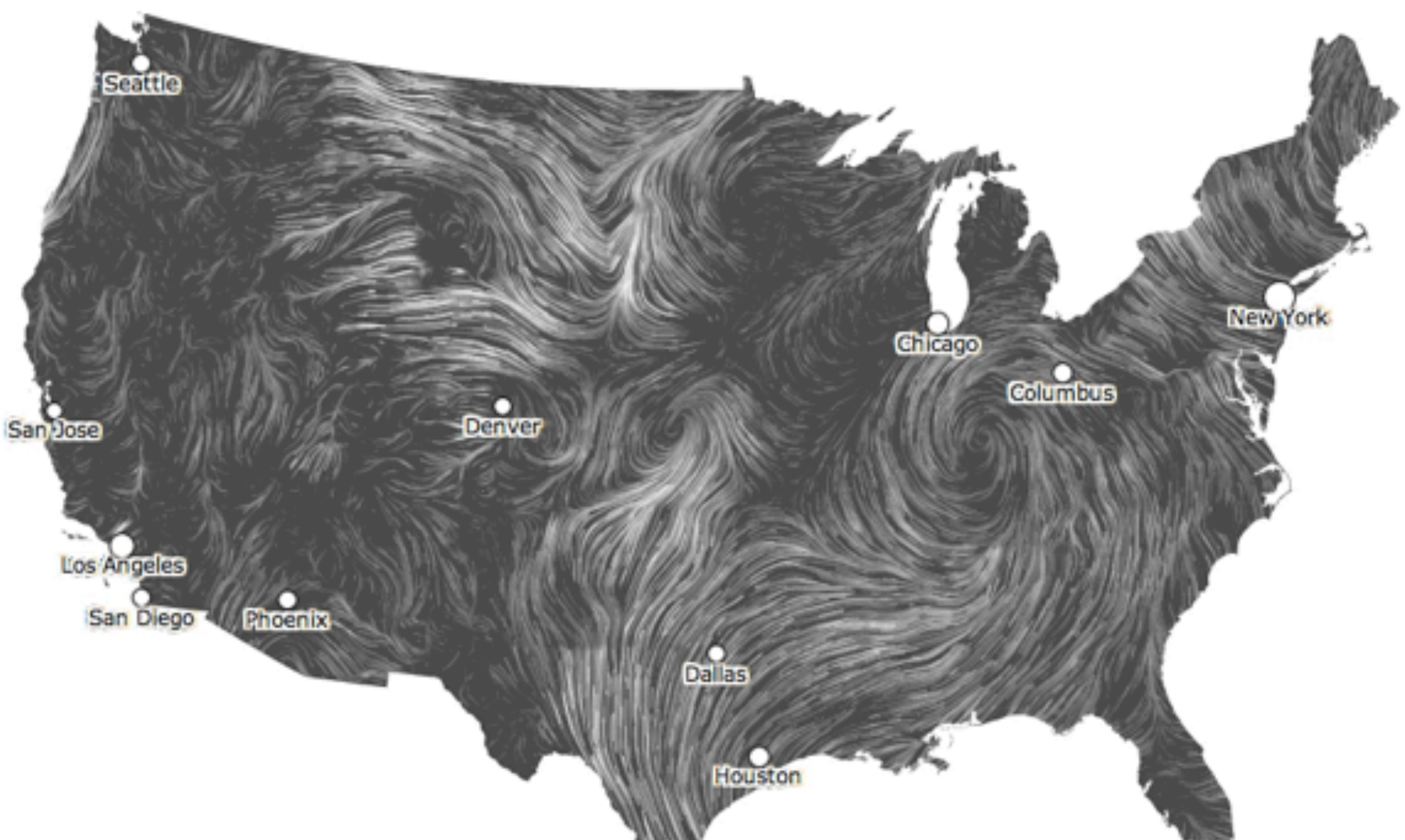



Seattle
San Jose
Denver
Chicago
Columbus
New York
Los Angeles
San Diego
Phoenix
Dallas
Houston








Seattle
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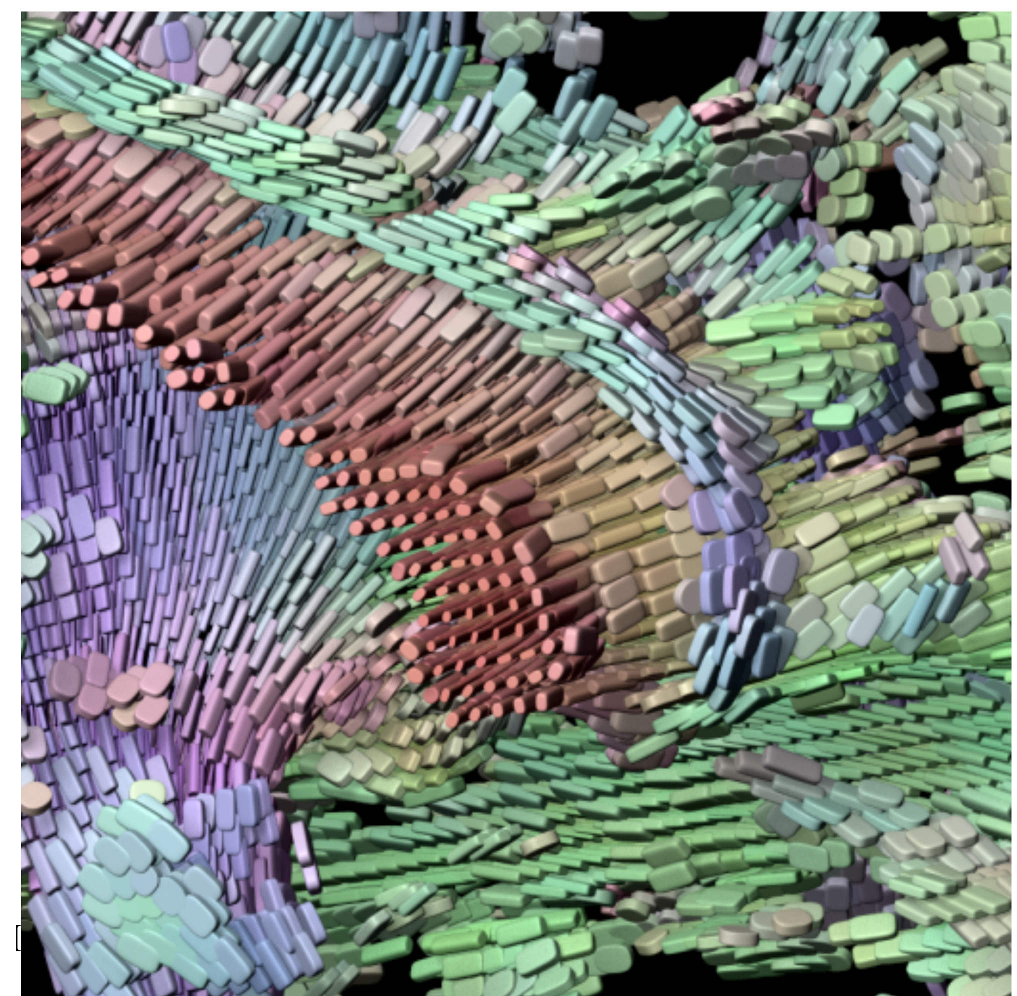
San Diego
Phoenix
Dallas
Houston



- data sources
- data representation
- interpolation

DATA SOURCES

data sources

- Medical Imaging (MRI, CT, PET)
- Geographical information systems (GIS)
- Electron microscopy

MB

- Meteorology and environmental sciences (satellites)
- Seismic data
- Crystallography

GB

- High energy physics
- Astronomy (e.g. Hubble Space Telescope 100MB/day)
- Defense

TB

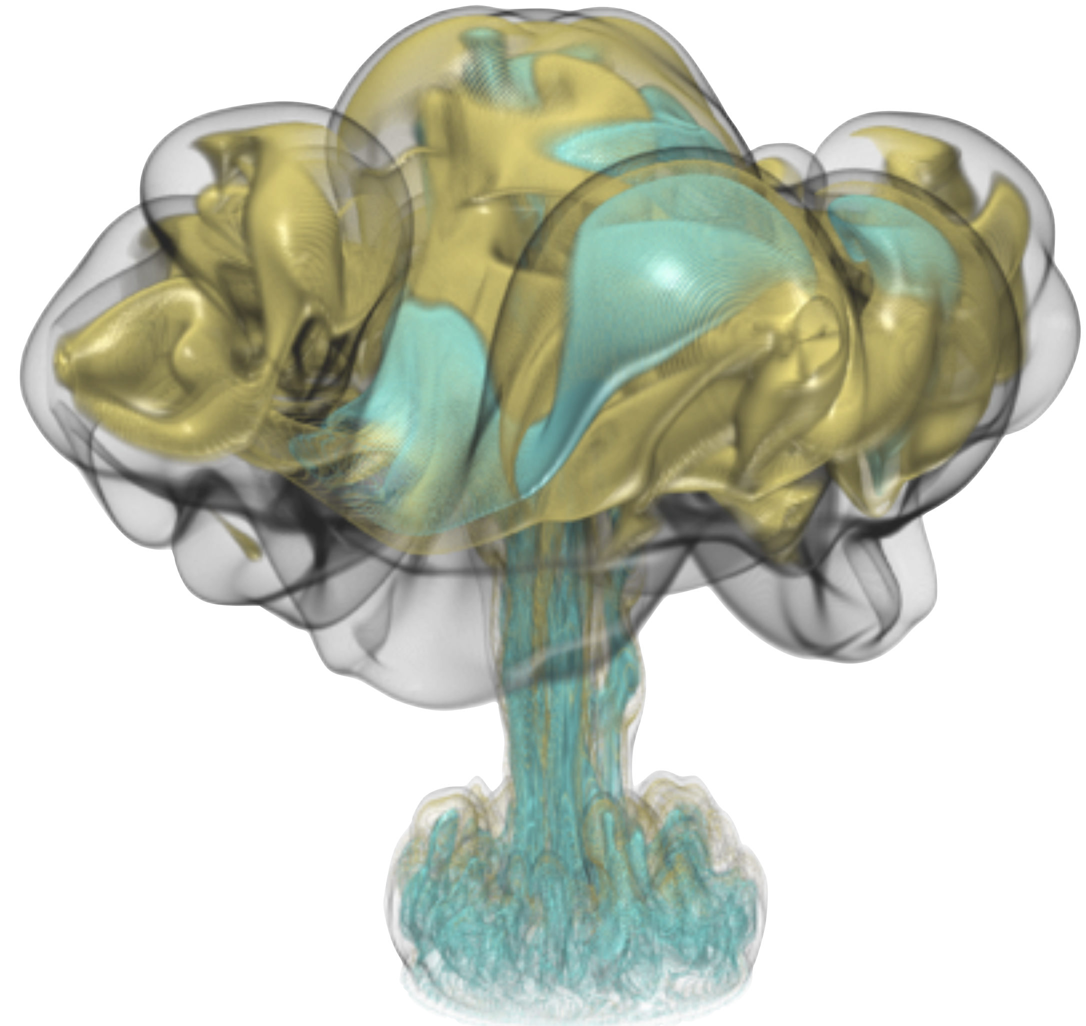
THEORETICAL MEASUREMENTS

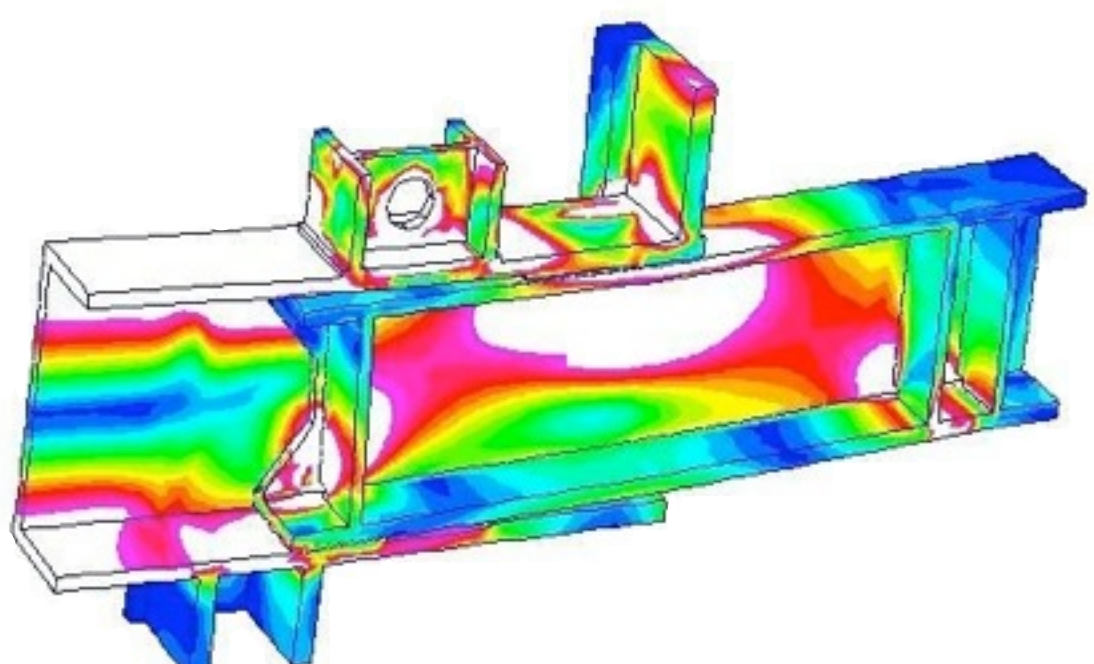
- Sciences
  - Molecular dynamics
  - Quantum chemistry
  - Mathematics

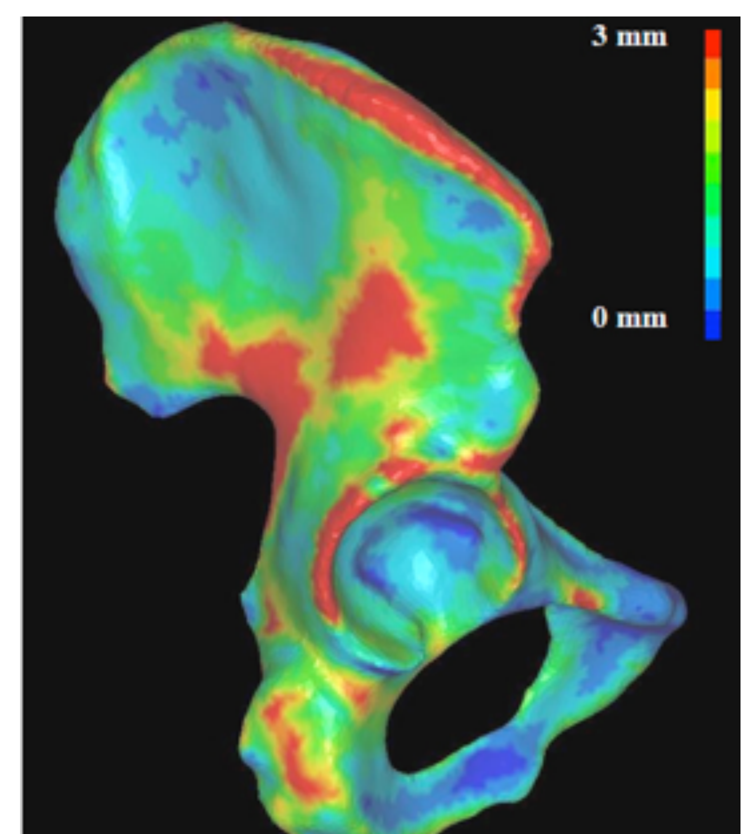
    MB
  - Molecular modeling
  - Computational physics
  - Meteorology
  - Computational fluid mechanics (CFD)

    GB
- Engineering
  - Architectural walk-throughs
  - Structural mechanics
  - Car body design

    GB







DATA REPRESENTATION

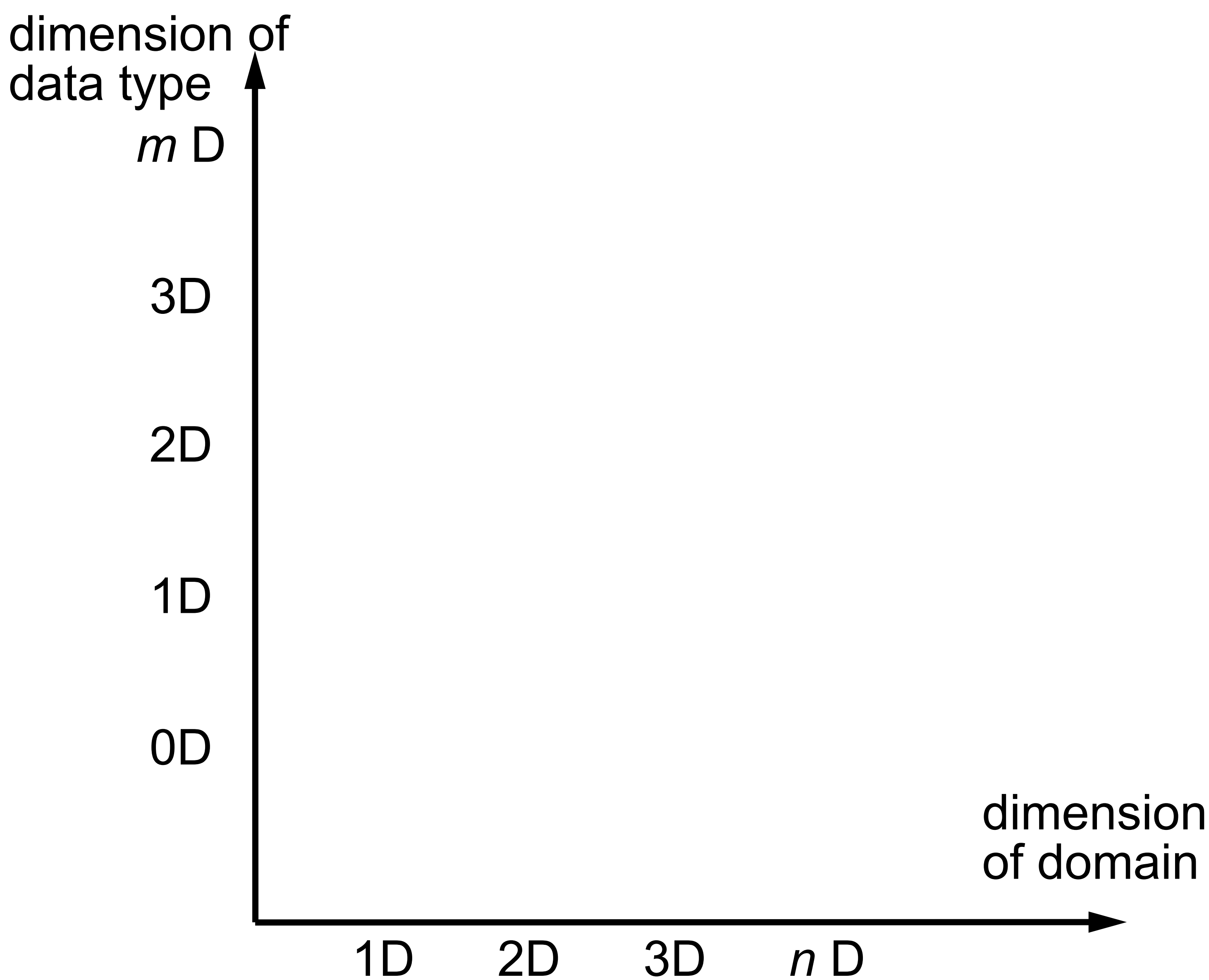
- Discrete representations
  - objects we want to visualize are continuous
  - but, data only given at discrete locations
  - grids (meshes) consist of cells generated from data points

- Primitives in different dimensions

dimension	cell	mesh
0D	points	
1D	lines (edges)	polyline(–gon)
2D	triangles, quadrilaterals (rectangles)	2D mesh
3D	tetrahedra, prisms, hexahedra	3D mesh

Types and Classification of Field Data

- dimension of domain (the field)
- dimension of the data to visualize (the geometry)



Examples:

A: gas station along a road
B: map of cholera in London
C: temperature along a rod
D: height field of a continent
E: 2D air flow
F: 3D air flow in the atmosphere
G: stress tensor in a mechanical part
H: ozone concentration in the atmosphere

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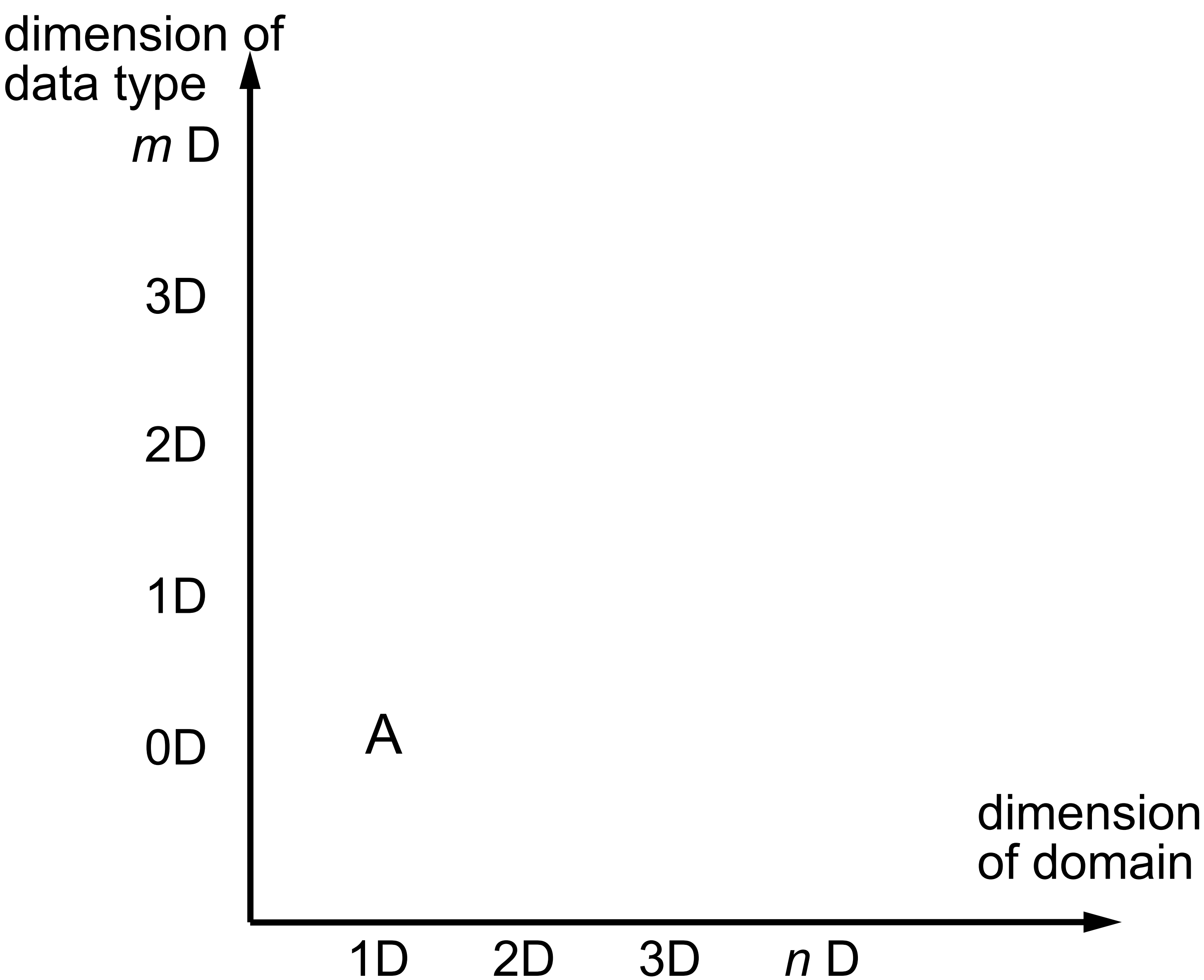

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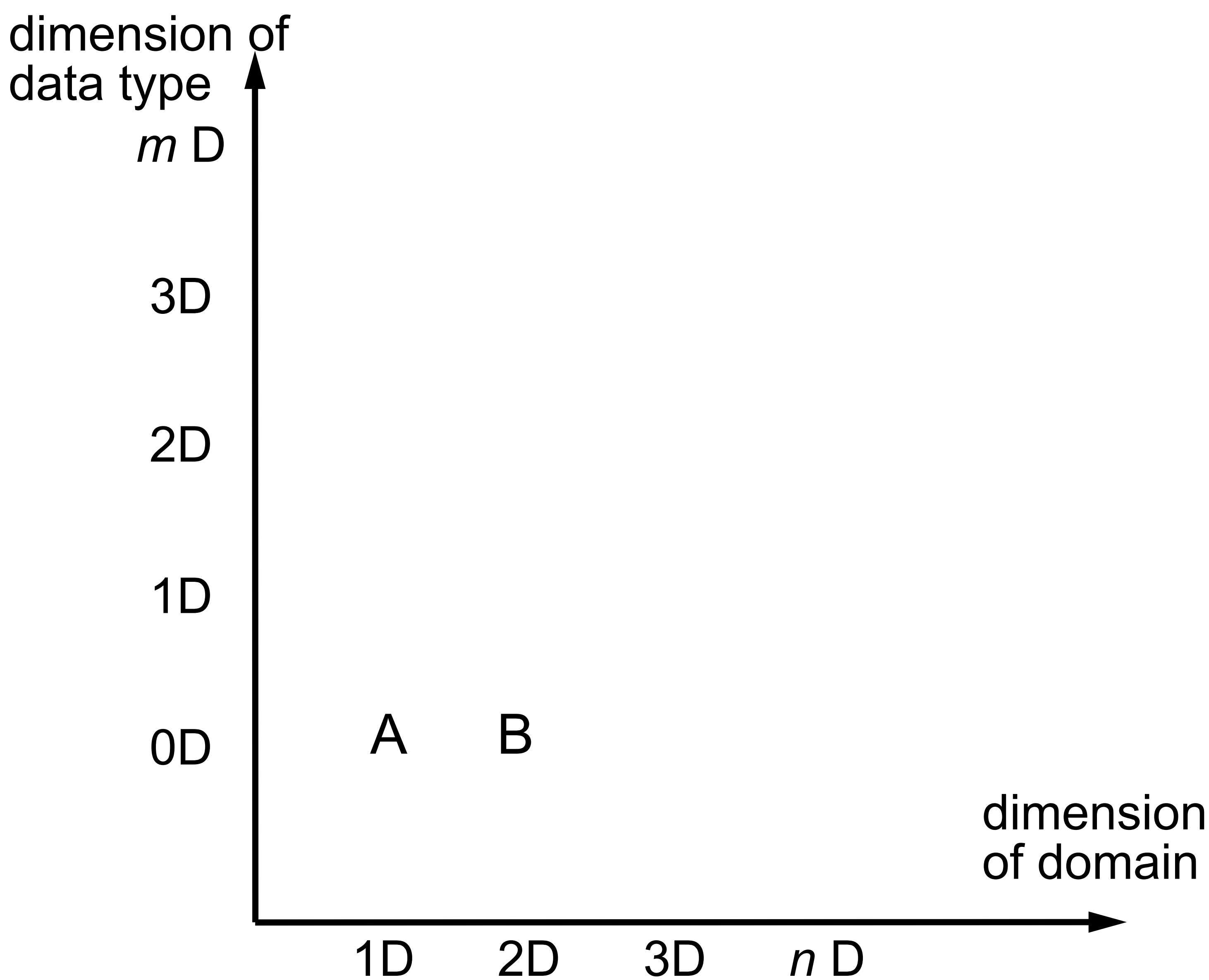

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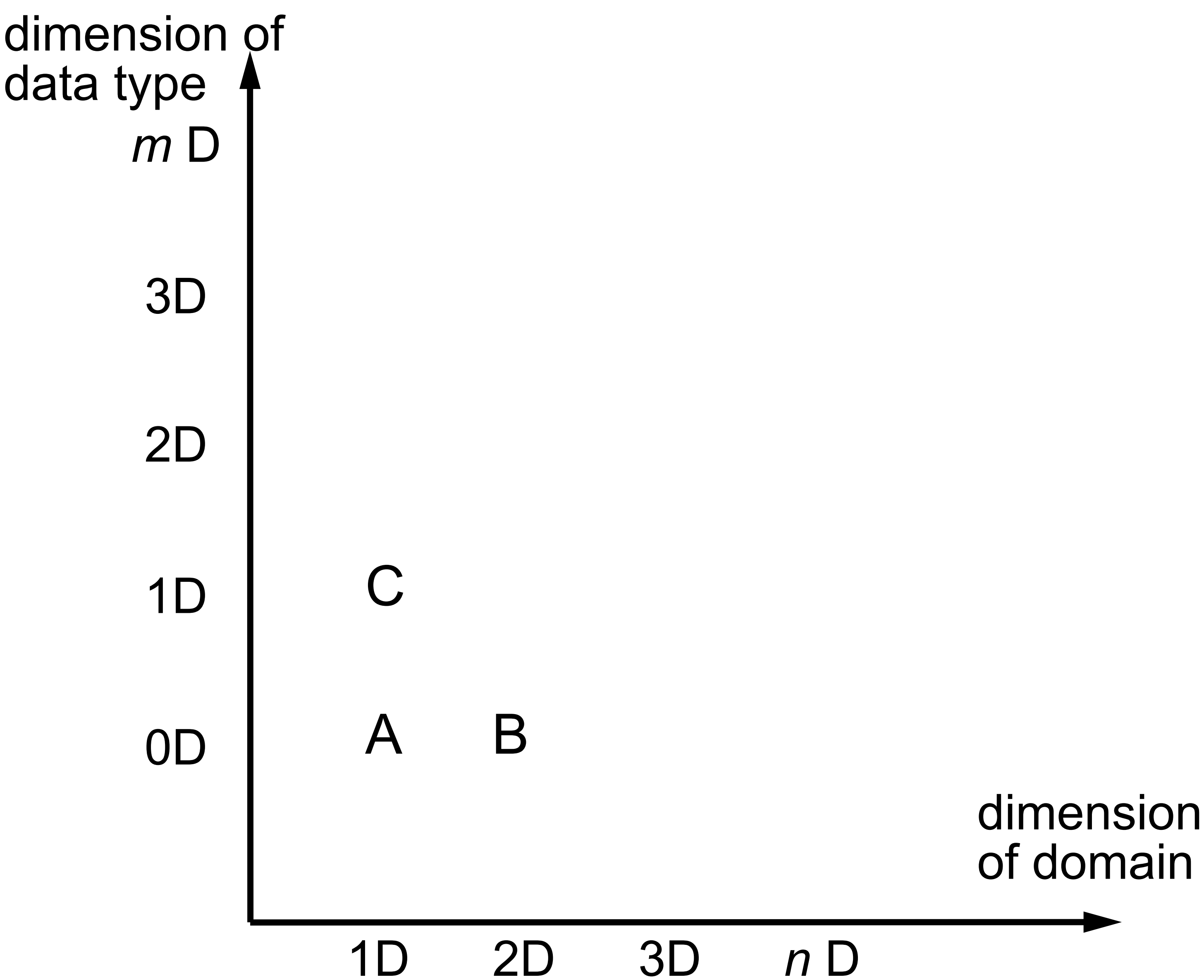

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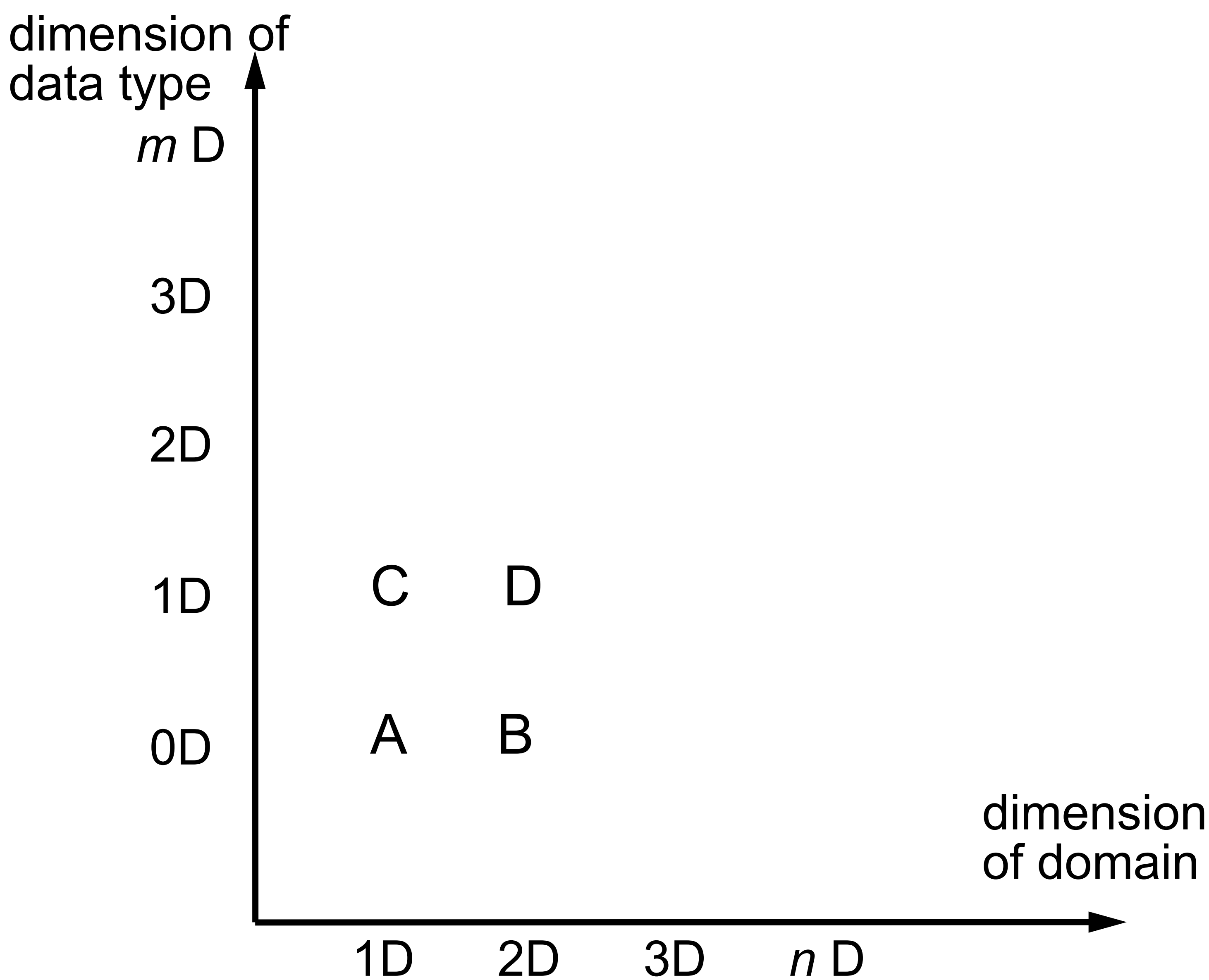

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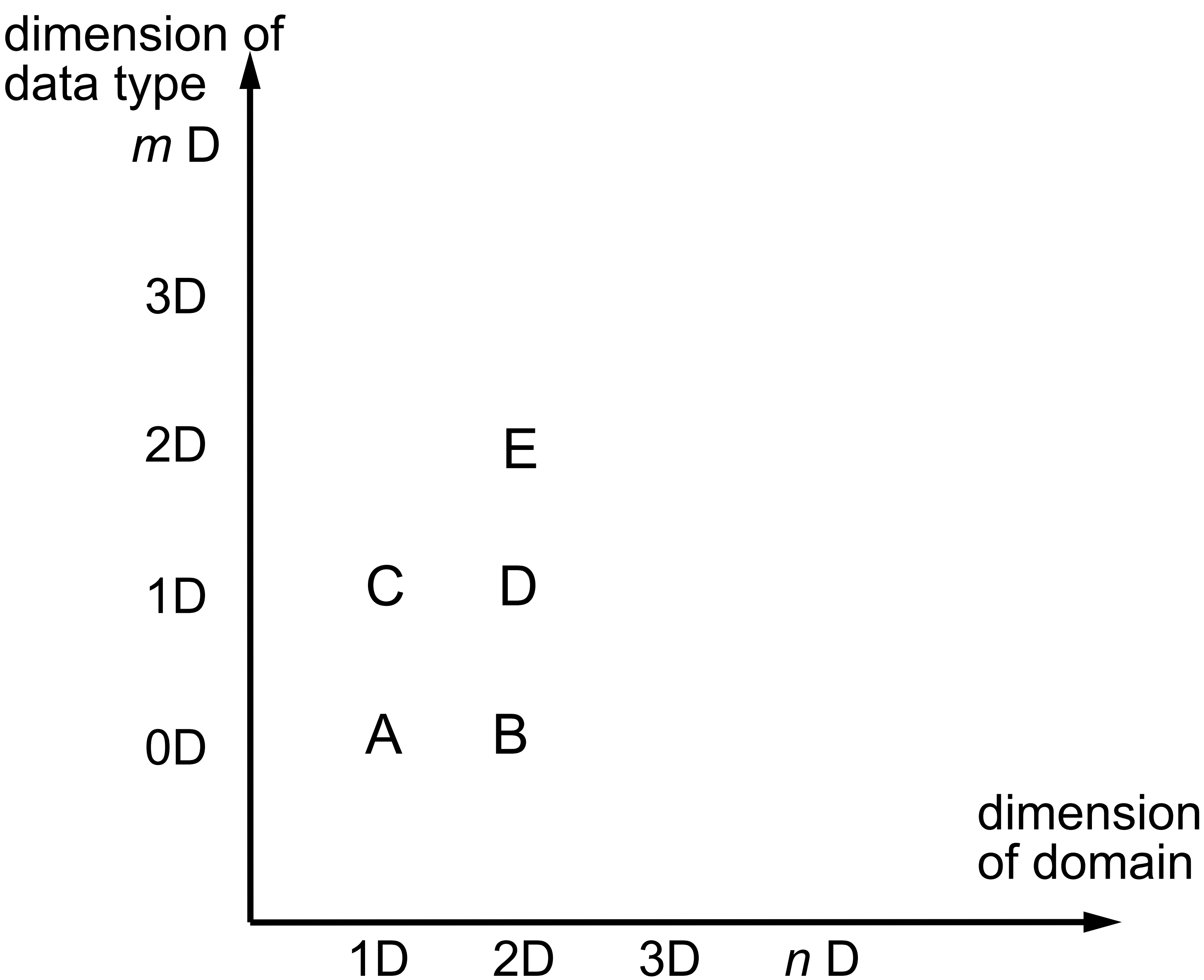

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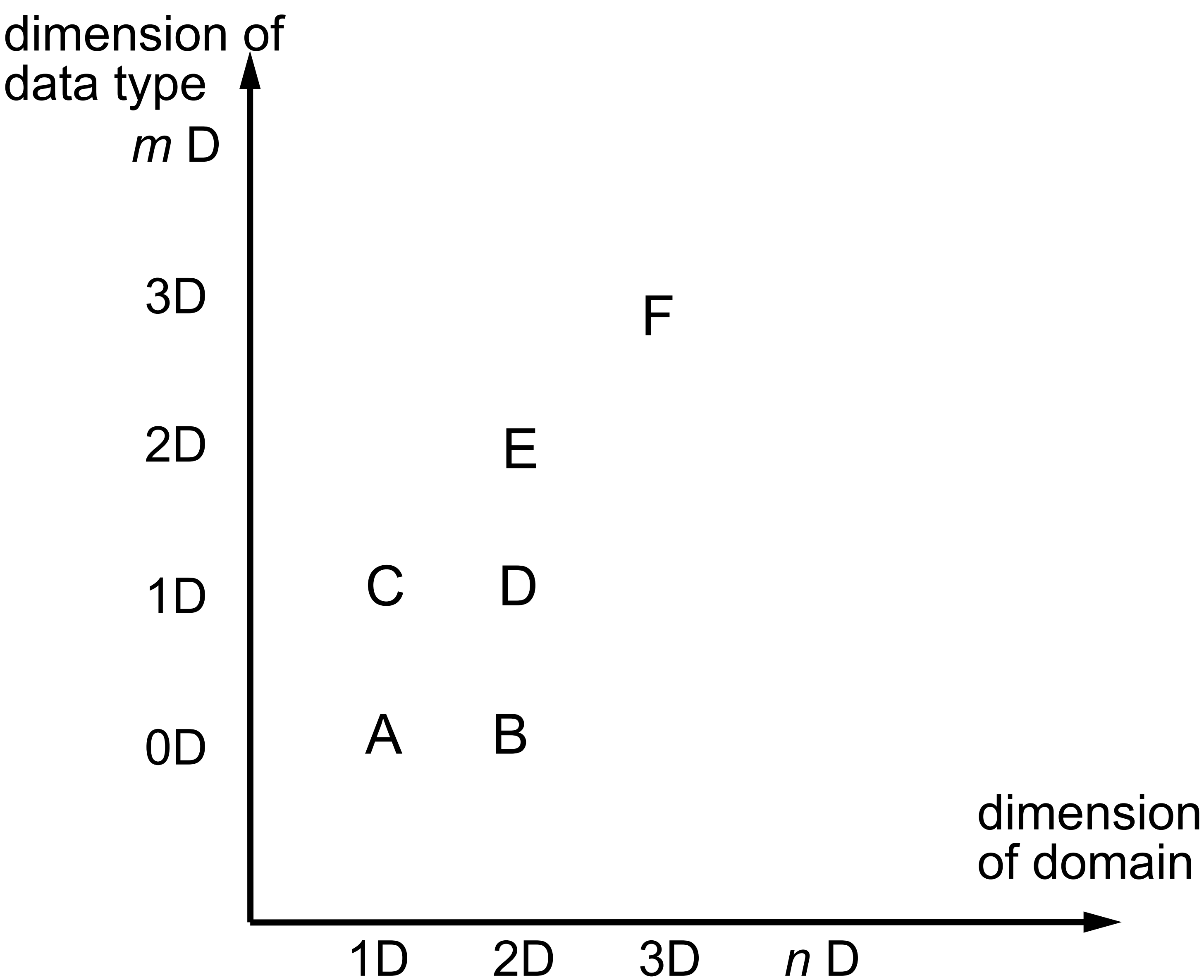

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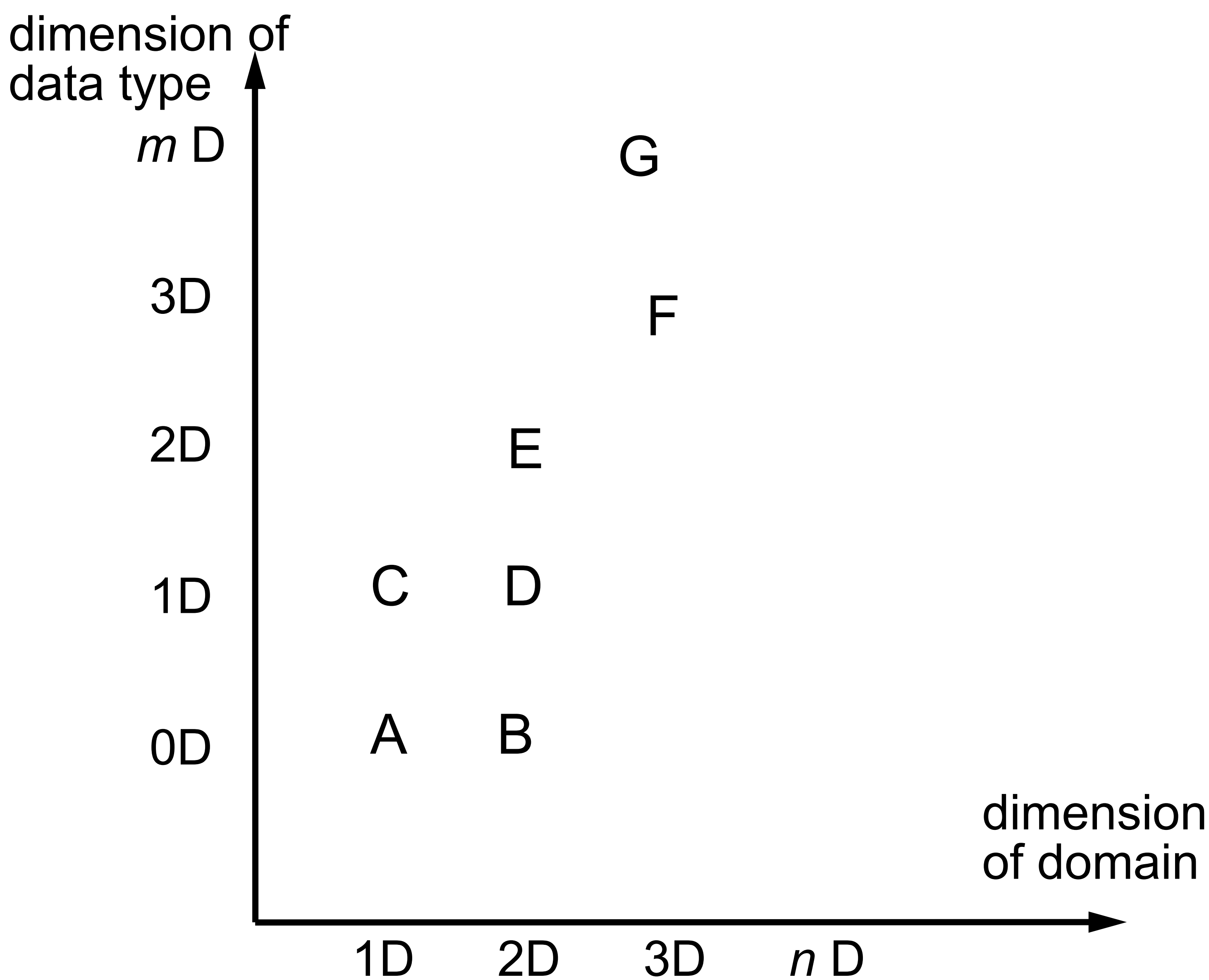

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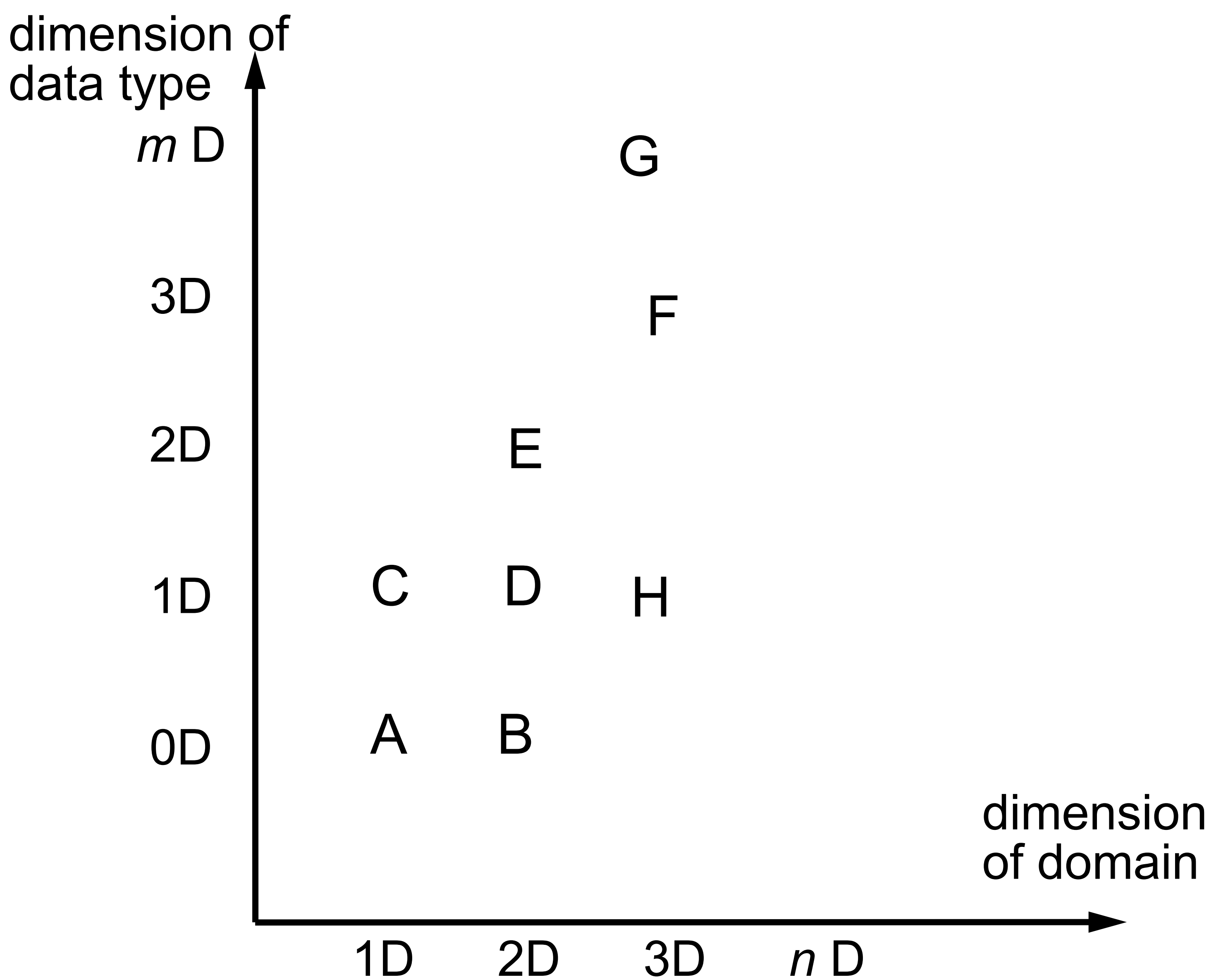

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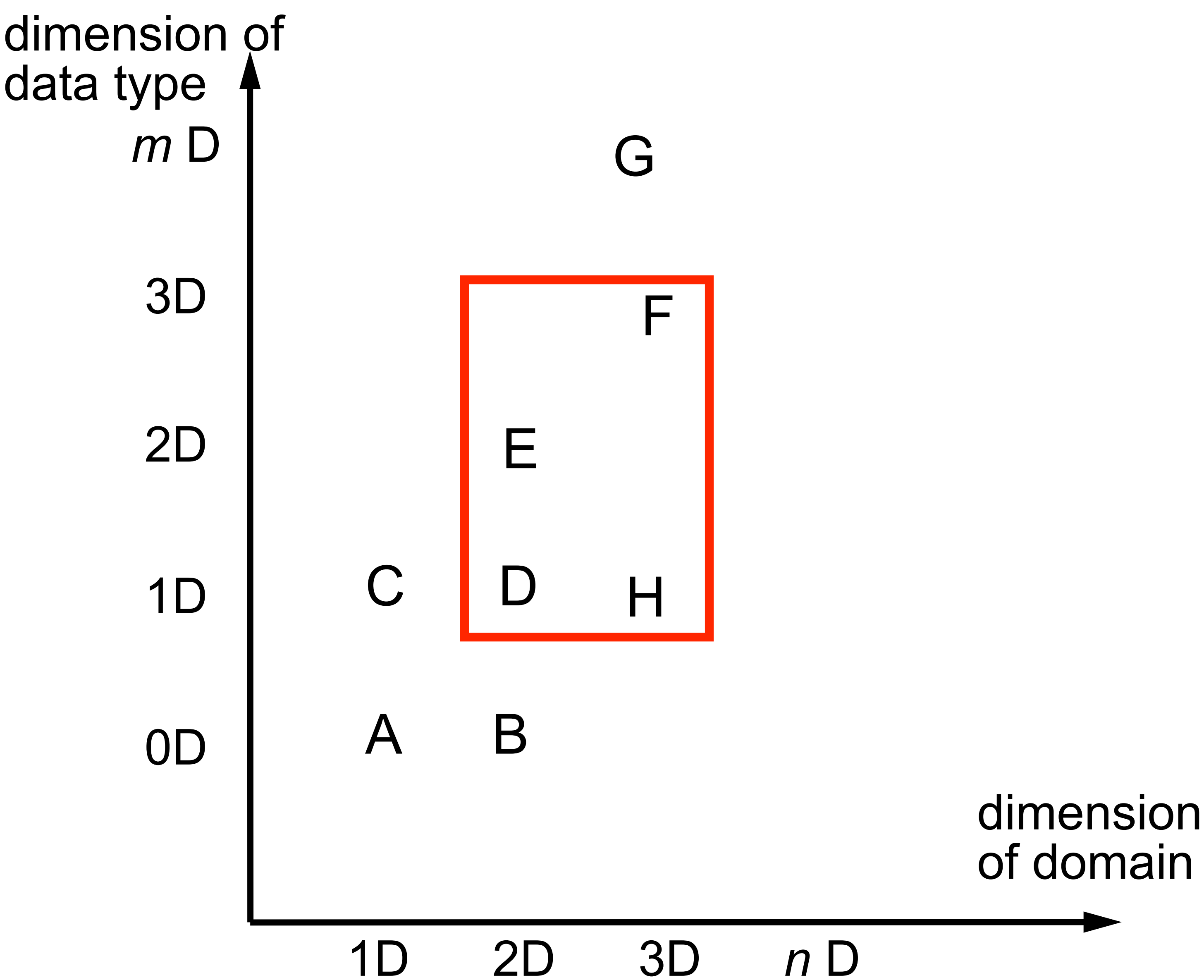

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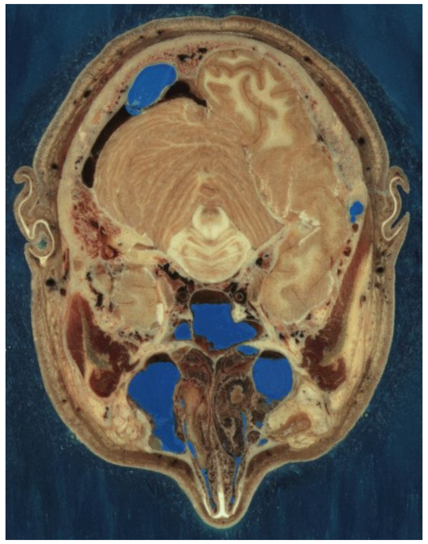
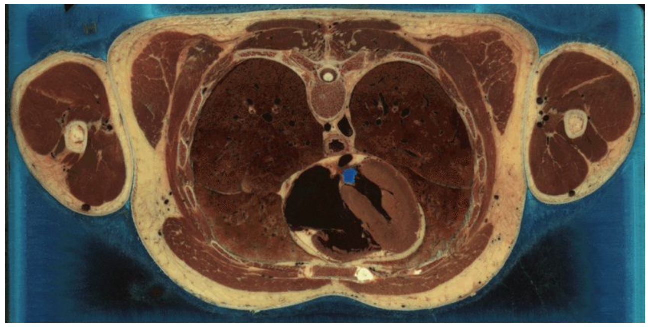
- Visualization of 1D, 2D, or 3D scalar fields
  - 1D scalar field: $\Omega \in R \rightarrow R$
  - 2D scalar field: $\Omega \in R^2 \rightarrow R$
  - 3D scalar field: $\Omega \in R^3 \rightarrow R$

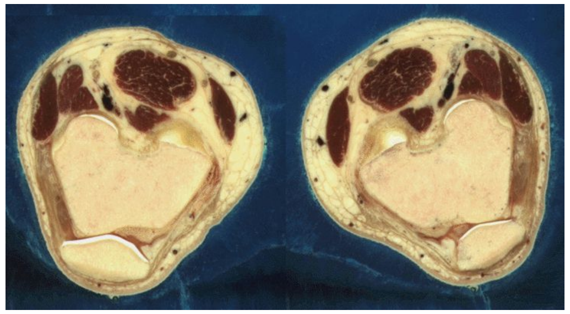
    $\rightarrow$ **Volume visualization!**

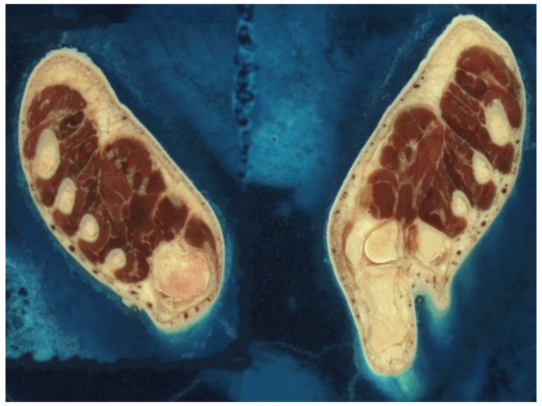
- Mapping to geometry
  - Function plots
  - Height fields
  - Isolines and isosurfaces
- Color coding
- Specific techniques for 3D data
  - Indirect volume visualization
  - Direct volume visualization
  - Slicing
- Visualization method depend heavily on dimensionality of domain

- NIH project established in 1989
- male: 1994
  - 1,871 4mm slices
  - 15GB
- female: 1995
  - 5,189 0.33mm slices
  - 40GB
- MRI, CT, and color









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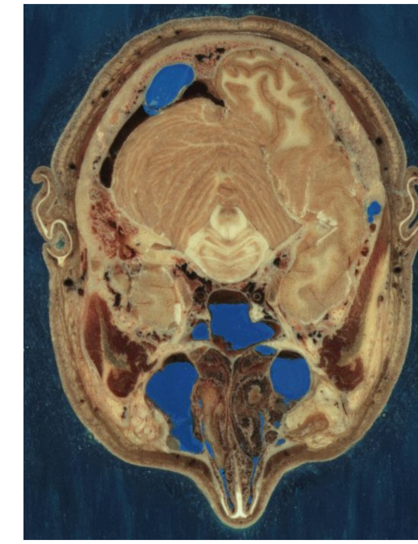
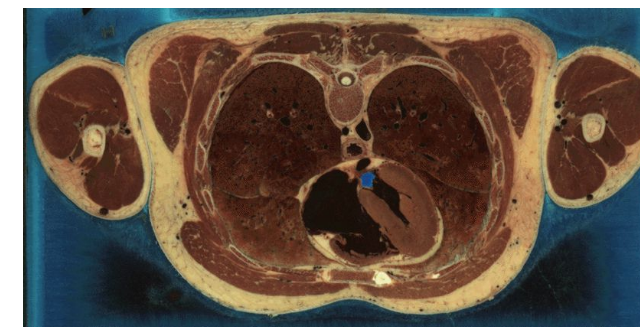
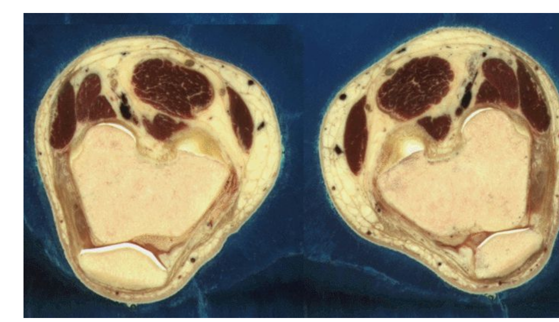
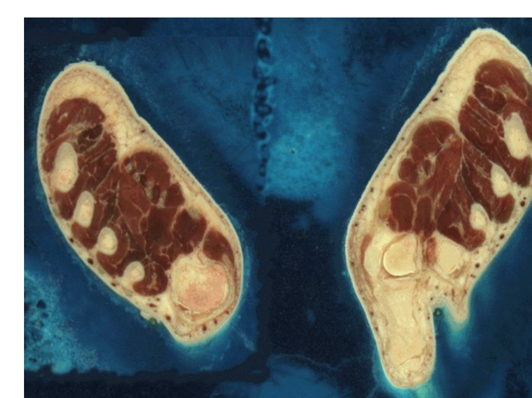


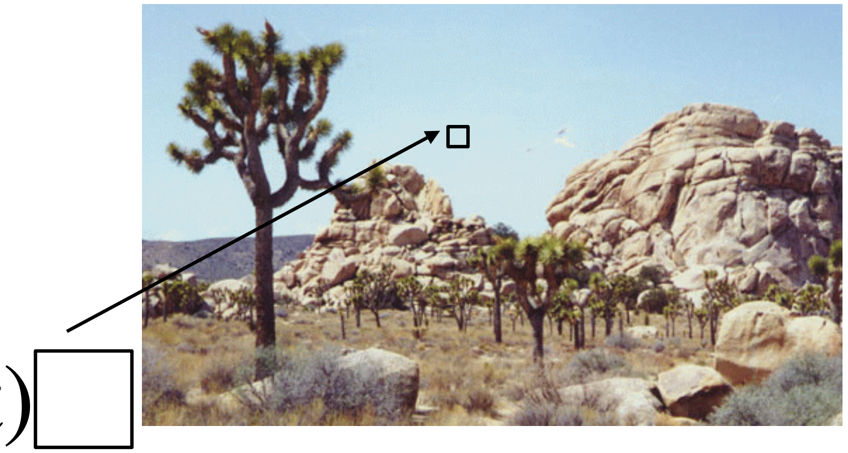




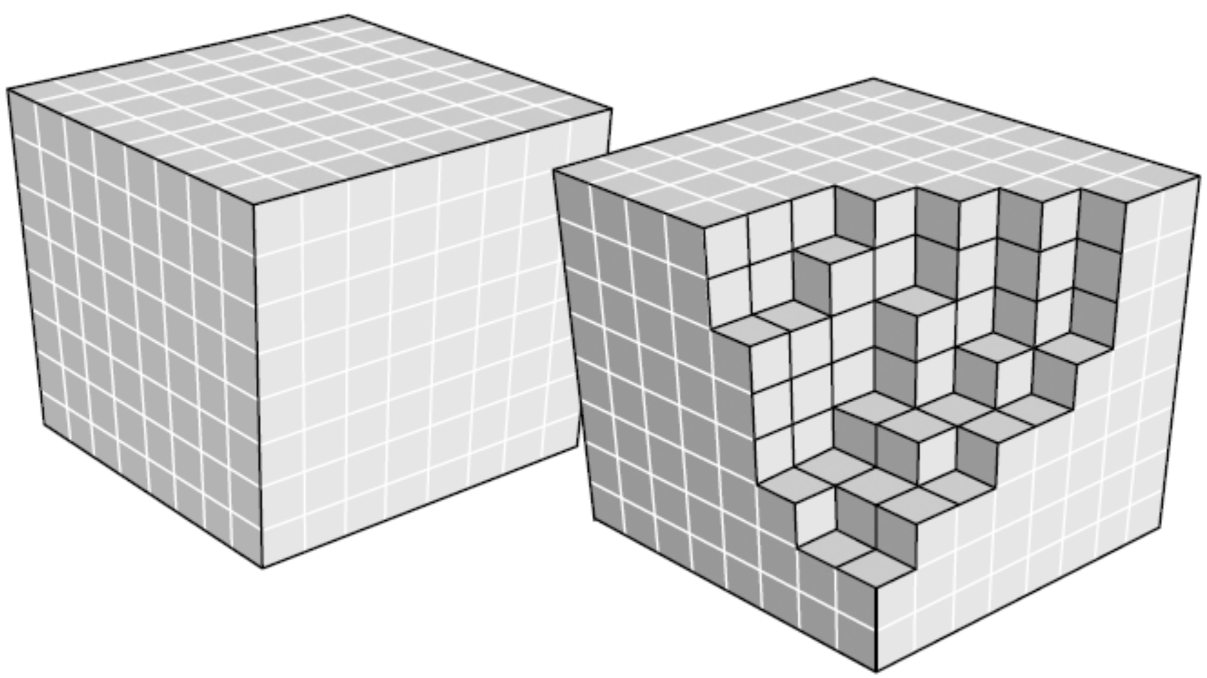


T. Fogal, J. Krueger. Size Matters - Revealing Small Scale Structures in Large Datasets, In IFMBE Proceedings, Vol. 25/13, Springer Berlin Heidelberg, pp. 41--44. 2009.

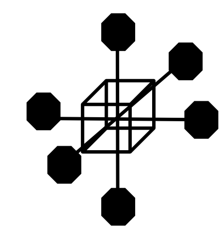
- Representation of scalar 3D data set $\Omega \in R^3 \rightarrow R$
- Analogy: pixel (picture element)

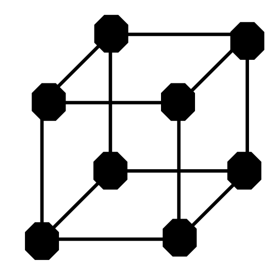


- Voxel (volume element), with two interpretations:
  - Values between grid points are resampled by interpolation



- Collection of voxels
- Uniform grid






voxel

Input Data

- Discrete positions (vertices)
- N dimensions, $N = 1, 2, 3, \ldots$
- With or without connectivity information
    - Structured
    - Unstructured
    - Scattered

grid topology

Grid Structure

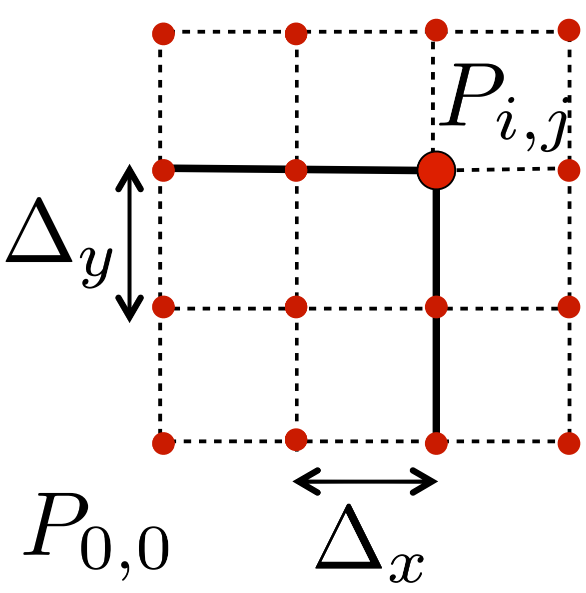
Classification

- Geometry
  - Position of vertices in Euclidean space
  - Structured / unstructured
- Topology
  - Cells
  - Connectivity information
  - Neighborhood definition
  - Structured / unstructured

Grid Geometry

Uniform

- implicit relationship between points
- positions can be computed (procedural)

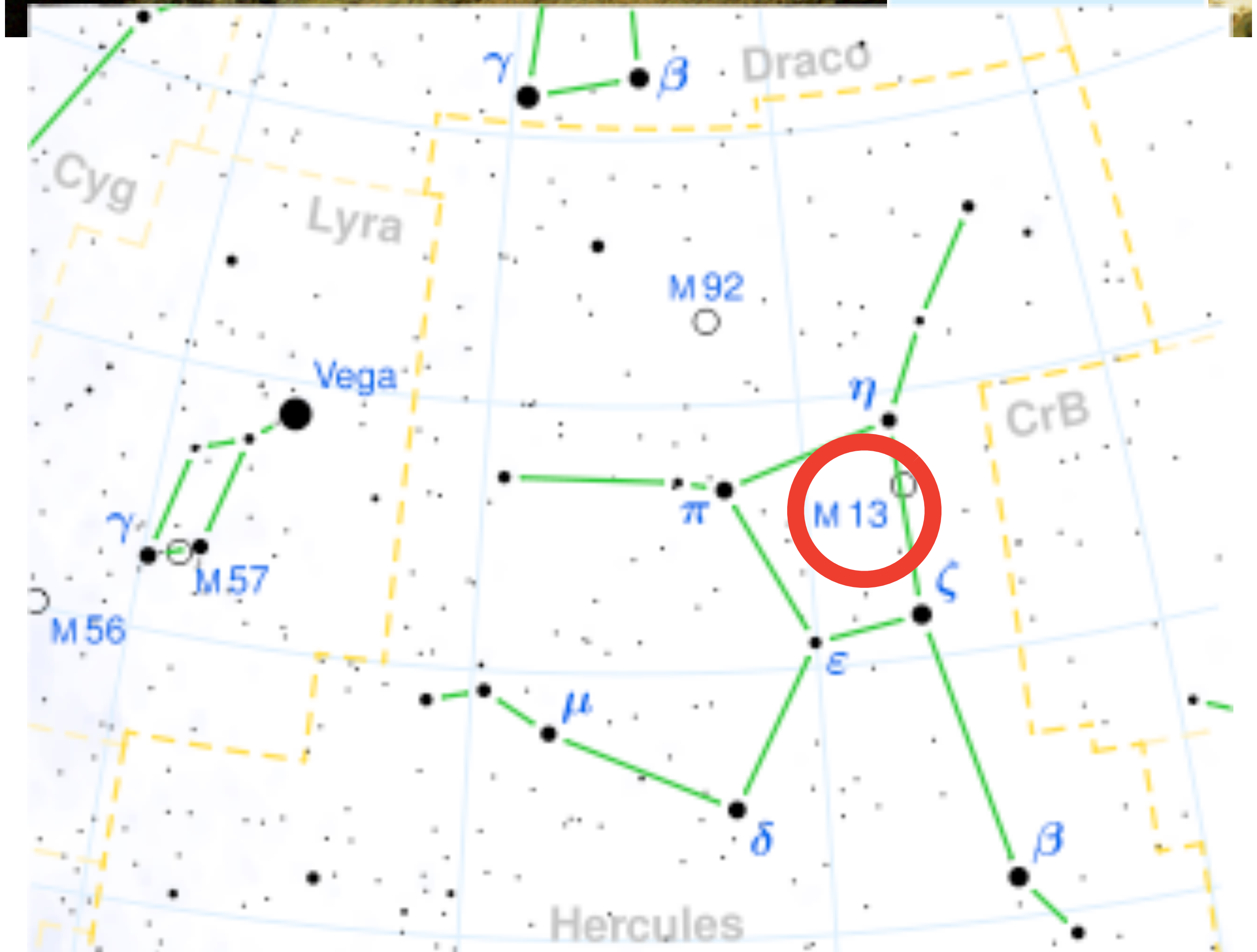


$$P_{i,j,k} = P_{0,0} + i\Delta_x\vec{e}_x + j\Delta_y\vec{e}_y + k\Delta_z\vec{e}_z$$

Arecibo Message

http://en.wikipedia.org/wiki/Arecibo_message

- Way of understanding mechanics of raster image representation
- Radio telecope in Puerto Rico
- built in 1964, renovated in 1974
- To celebrate: Frank Drake and Carl Sagan (Cornell University) sent message to M13 in Hercules (25,000 light years away)
- 1679 bits, frequency modulate 2380 MHz



The Message

http://www.physics.utah.edu/~cassiday/p1080/lec06.html

1679 bits were encoded as 2380MHz plus and minus some frequency

0 0 0 0 0 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 1 0 1 1 0 0 1 0 1 0 1
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0 1 1 1 1 0 0 1 1 1 1 1 0 1 0 0 1 1 1 1 0 0 0

This is a **1-D** sequence of bits in time

How will an alien understand this list of bits?

(will have different symbols than “0” “1”)

No meta-information!

Understanding the message

- Perhaps some “visual” representation of bits

  - (what is black vs white?)

- Aliens notice 1679 = 23 x 73 (product of two primes)

- Perhaps its not a linear sequence: 2-D array

- Two ways of sequencing values in 2D array

- Various ways of laying them out in 2D space

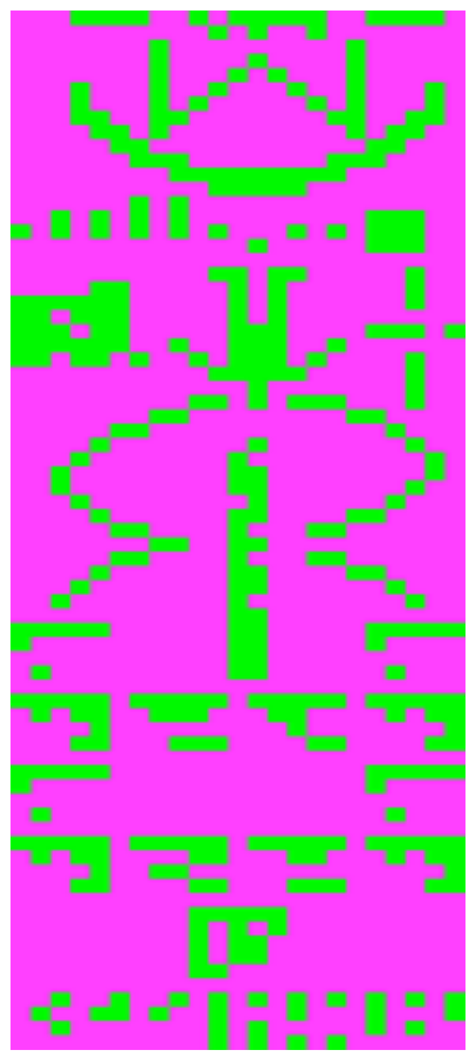
- Then: have to decipher it!

73 x 23

73 x 23

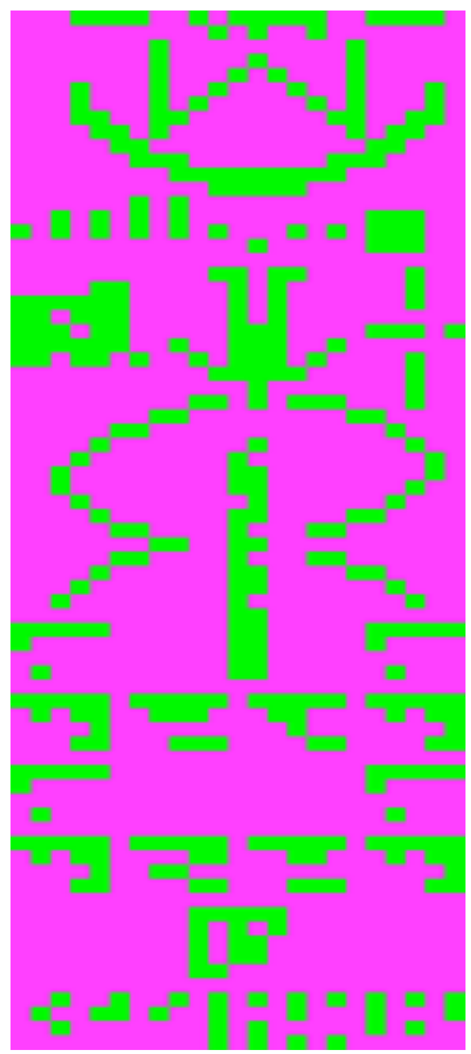
23 x 73: what was different?

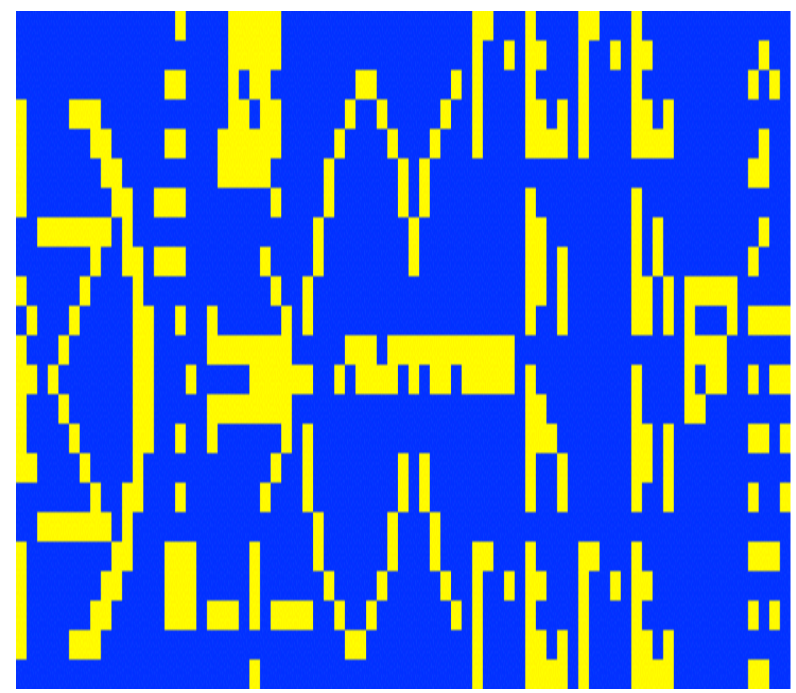
73 x 23



23 x 73: what was different?

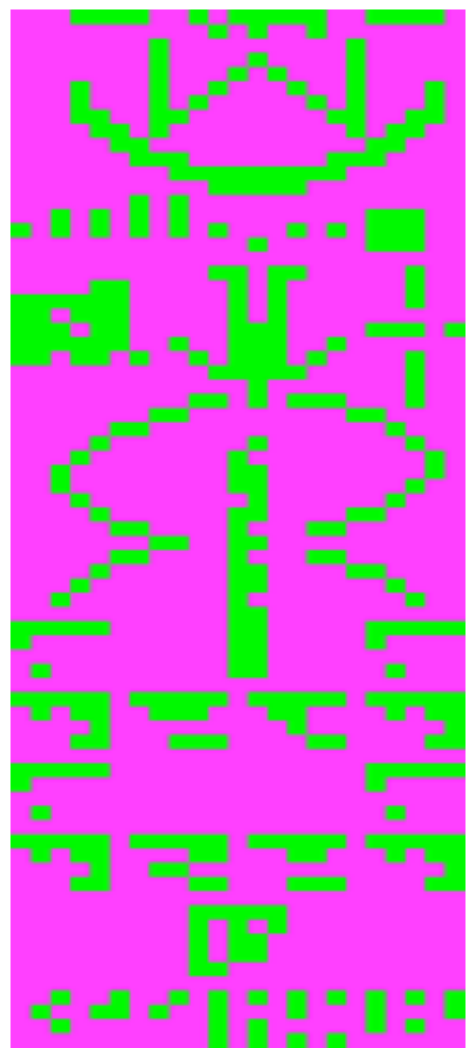
73 x 23

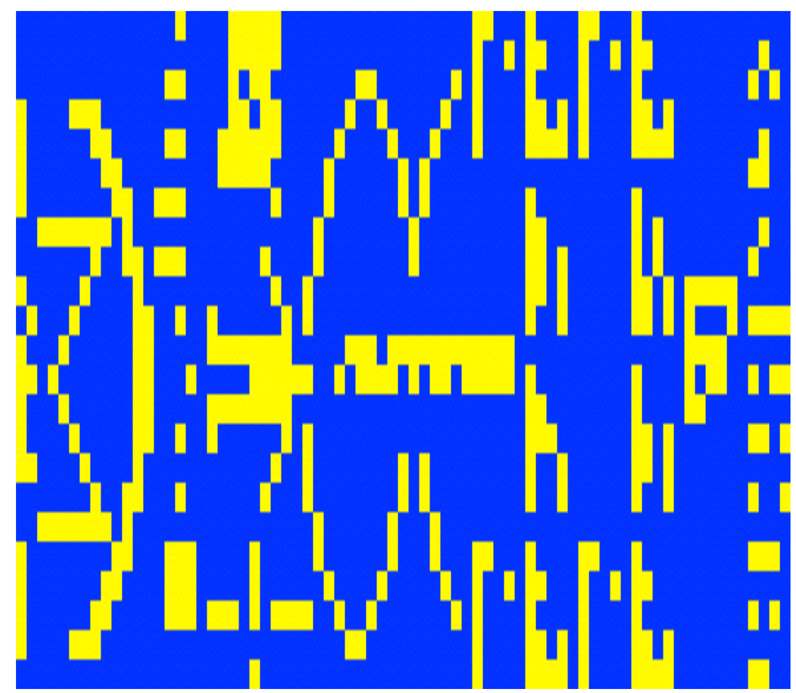




23 x 73: what was different?

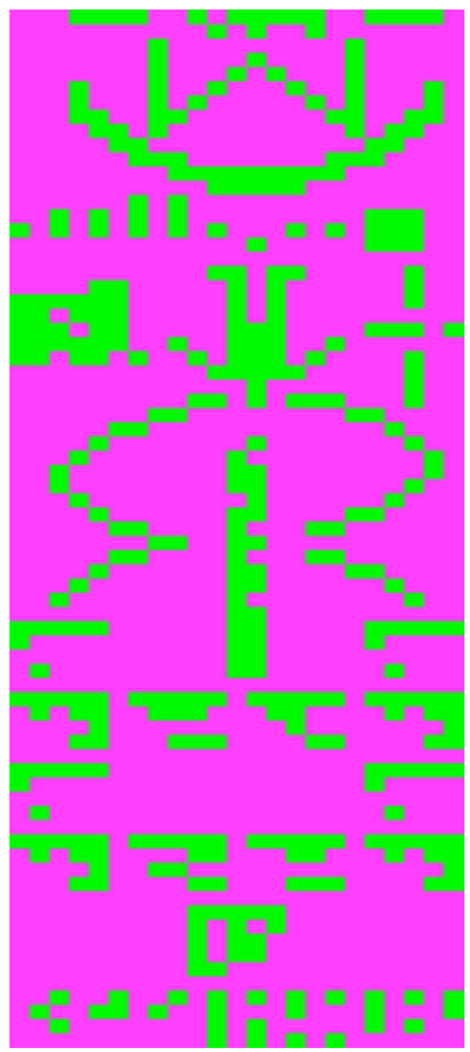
73 x 23

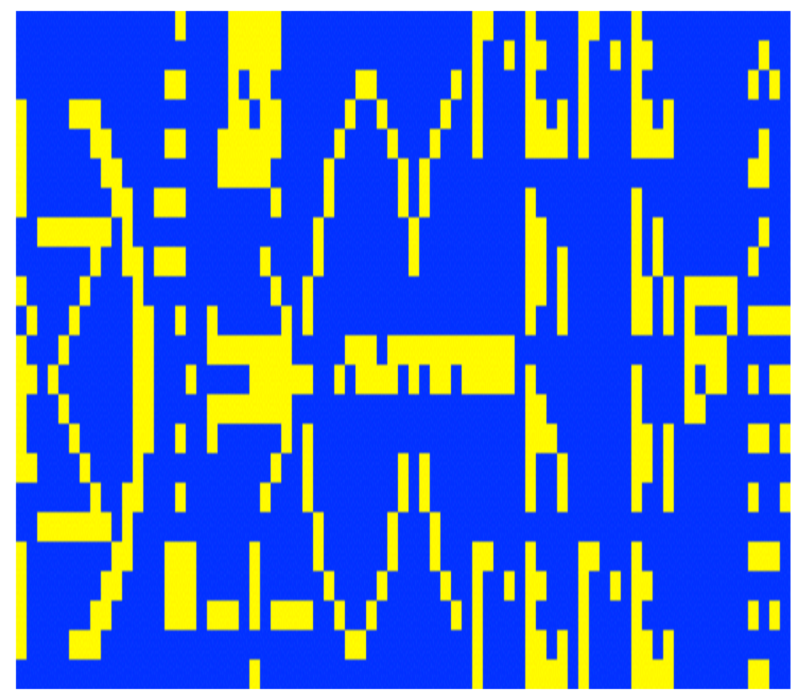




23 x 73: what was different?

73 x 23





compare to:
http://en.wikipedia.org/wiki/Arecibo_message

23 x 73: what was different?

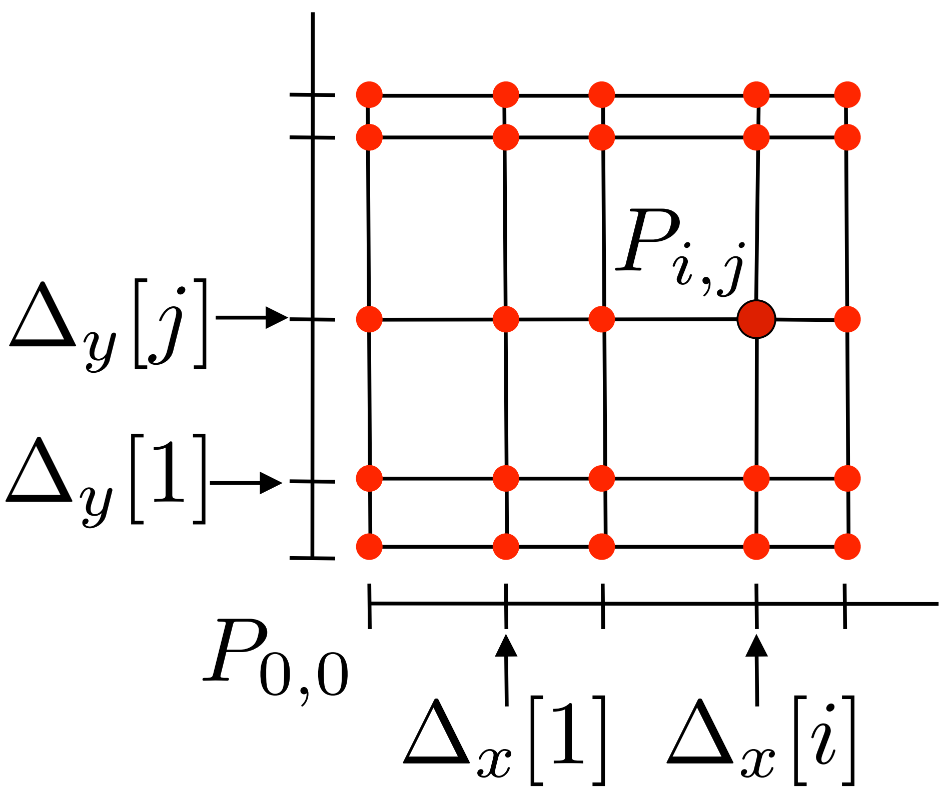
4 basic pieces of image metadata

- Interpretation of individual values
  - units, scalars, vectors, tensors, measurement frame
- Dimension of array
  - dimension of domain sampled
  - # axes, or # indices for getting a single sample
- Choice of axis ordering (fast-to-slow, or slow-to-fast)
  - Culturally specific
- # samples along each axis
  - “640-by-480 image” or “N-by-M matrix”

Grid Geometry

Structured

- implicit relationship between points
- positions can be computed (procedural)

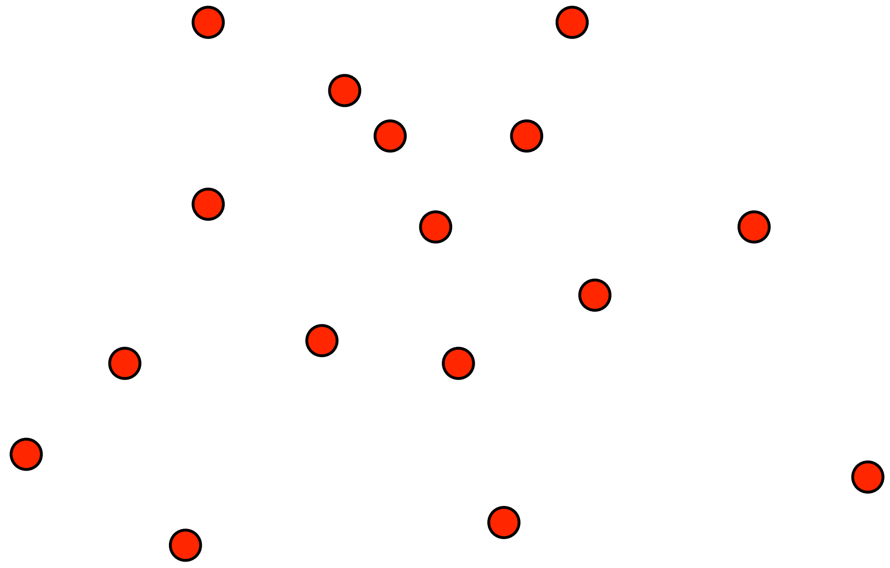


$$P_{i,j,k} = P_{0,0} + \Delta_x[i]\vec{e}_x + \Delta_y[j]\vec{e}_y + \Delta_z[k]\vec{e}_z$$

Grid Geometry

Unstructured

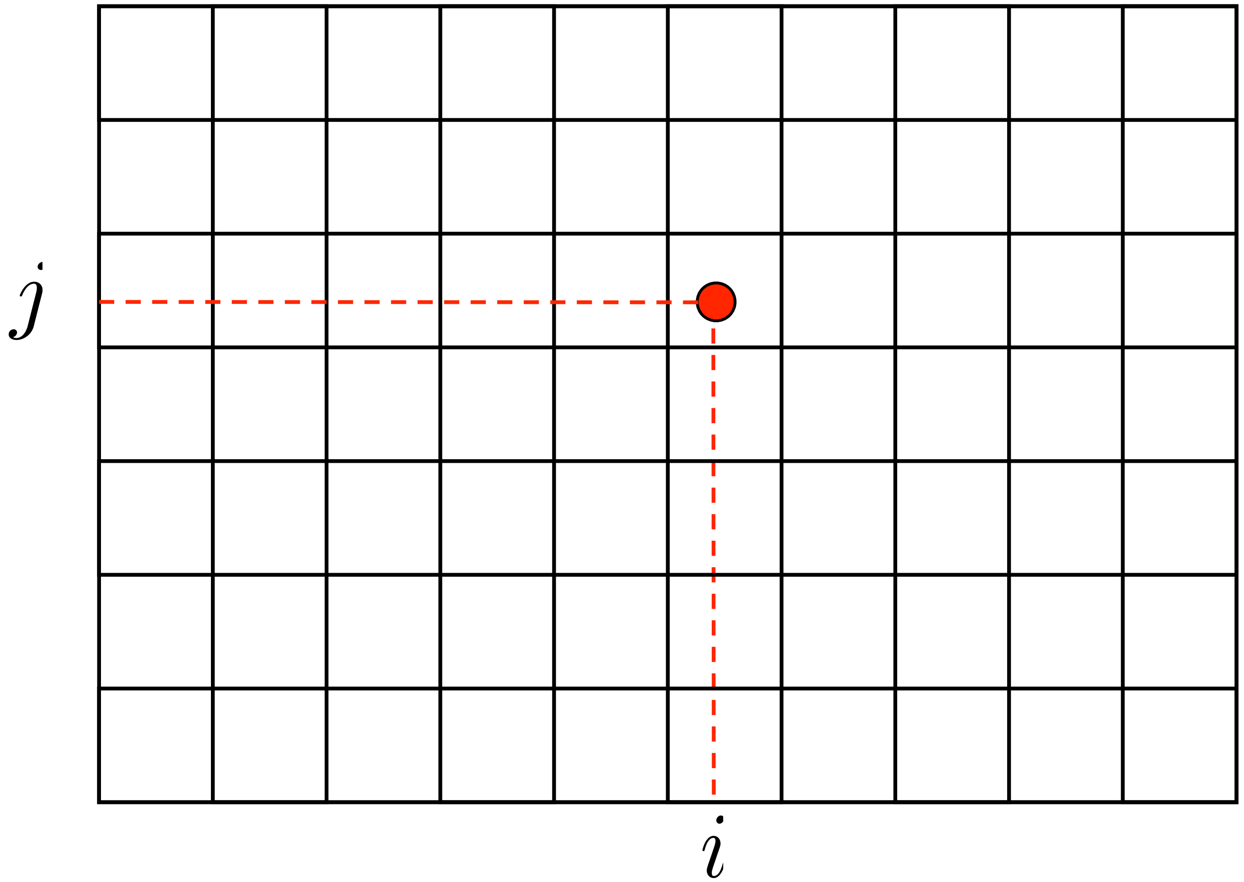
- No underlying structure
- Requires explicit knowledge of every vertex's position: (x_0, y_0, z_0), (x_1, y_1, z_1), ...,



Grid Topology

Structured *(quadrilateral / hexahedron)*

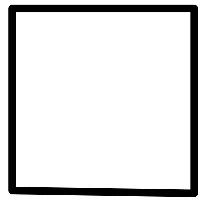
- Implicit connectivity between vertices
- Implicit cell definition



Associated with

structured geometry:

very efficient position location

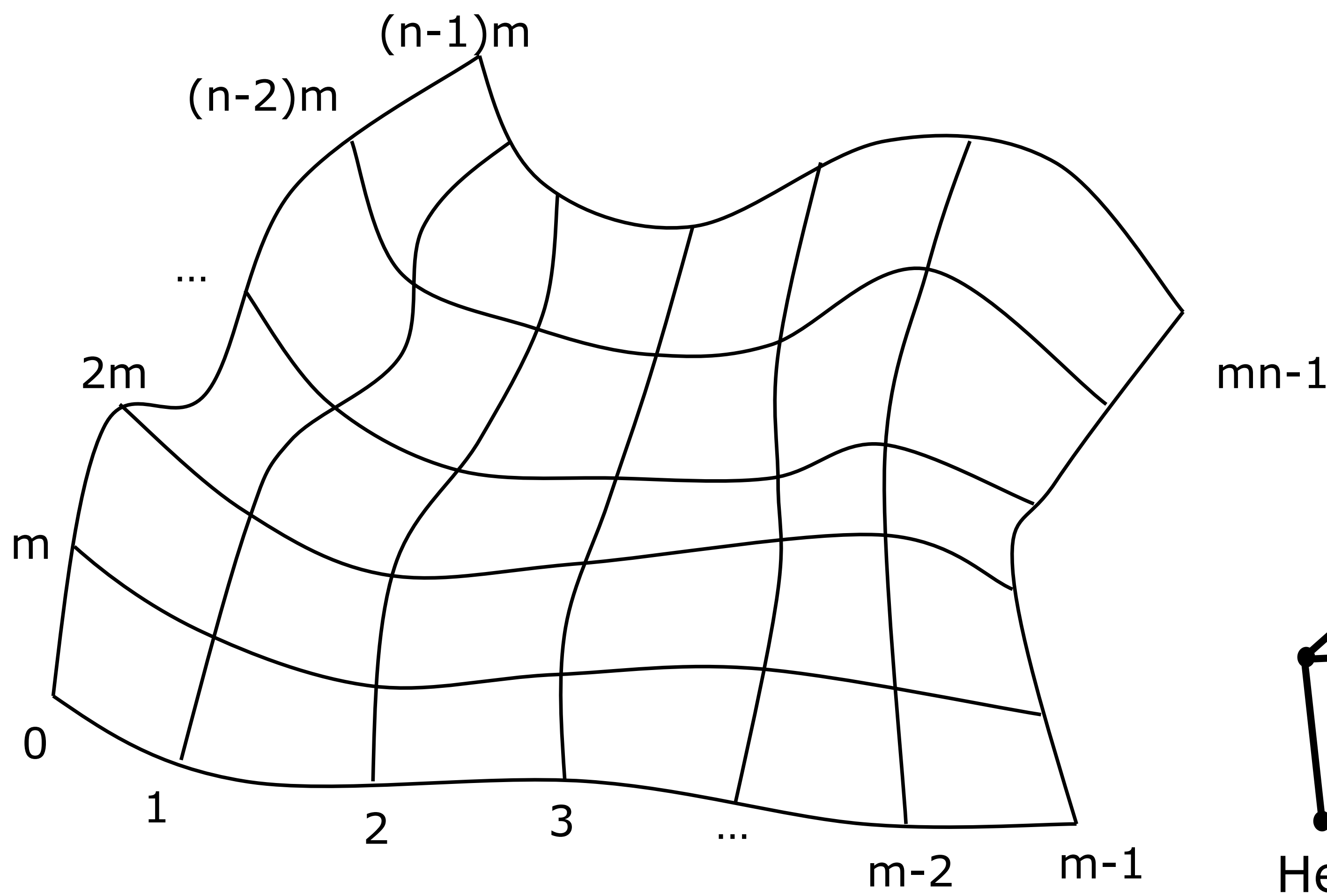


Pixel

Grid Topology

Structured *(quadrilateral / hexahedron)*

- Implicit connectivity between vertices
- Implicit cell definition

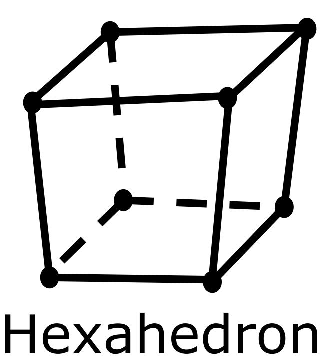


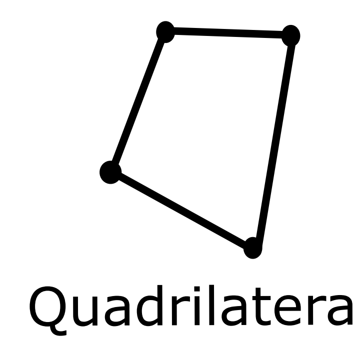
Associated with

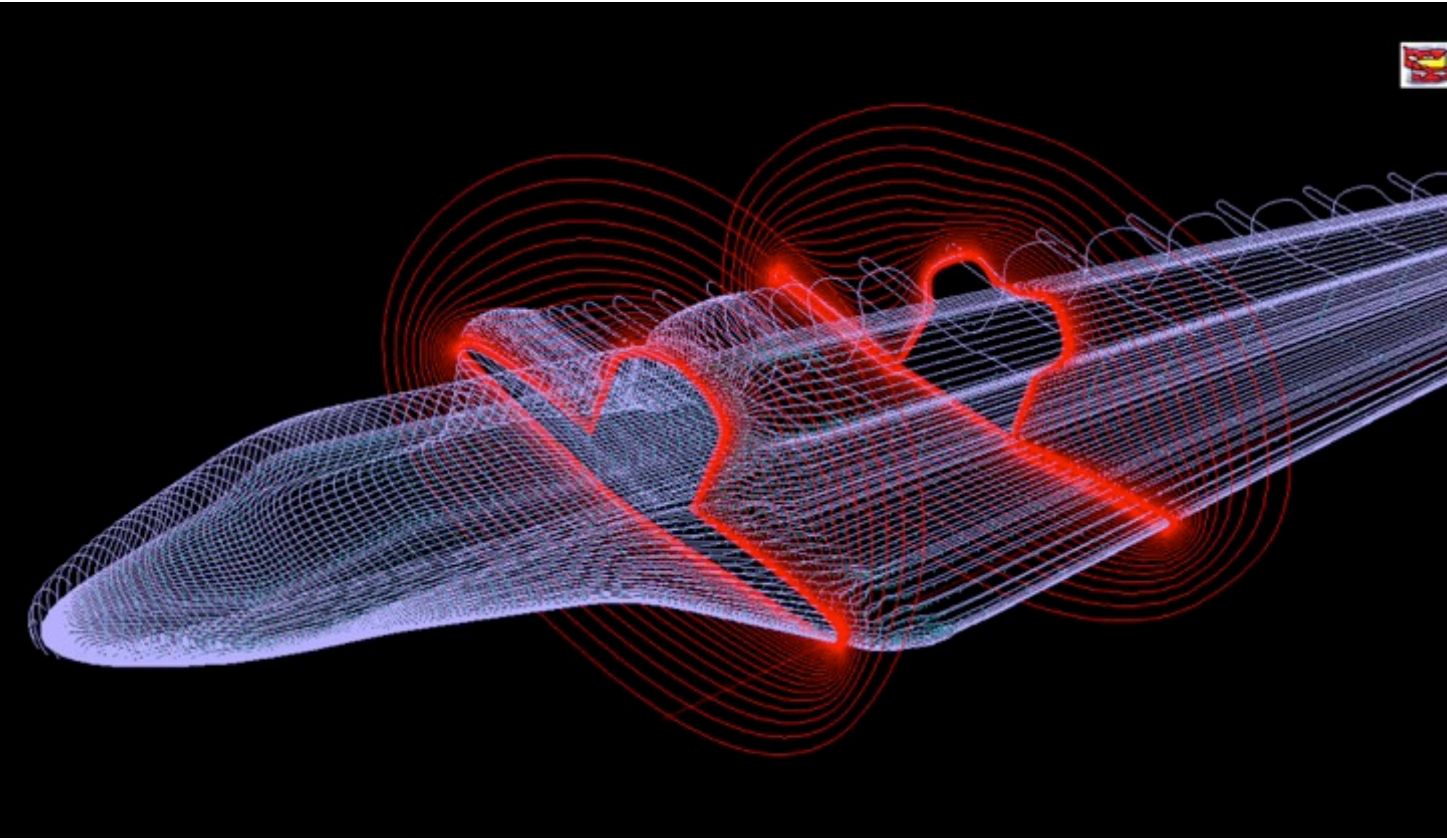
unstructured

geometry:

curvilinear grid



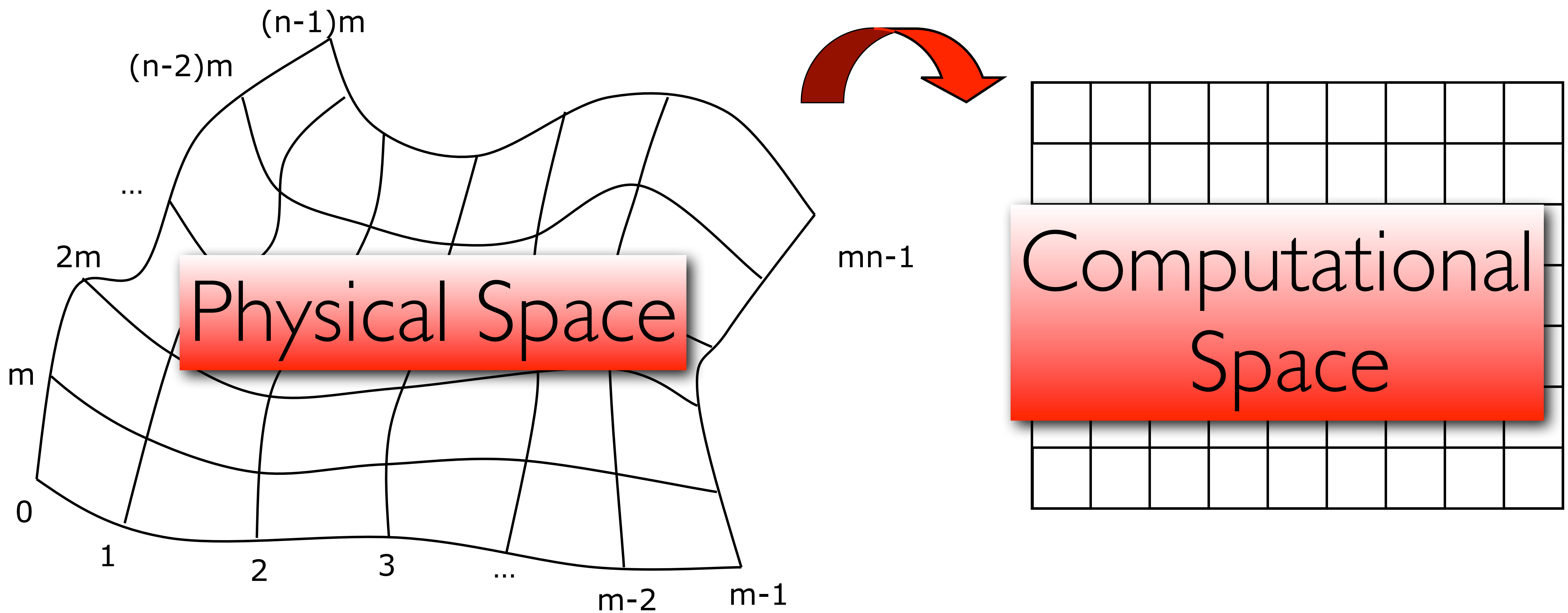




Grid Topology

Structured *(quadrilateral / hexahedron)*

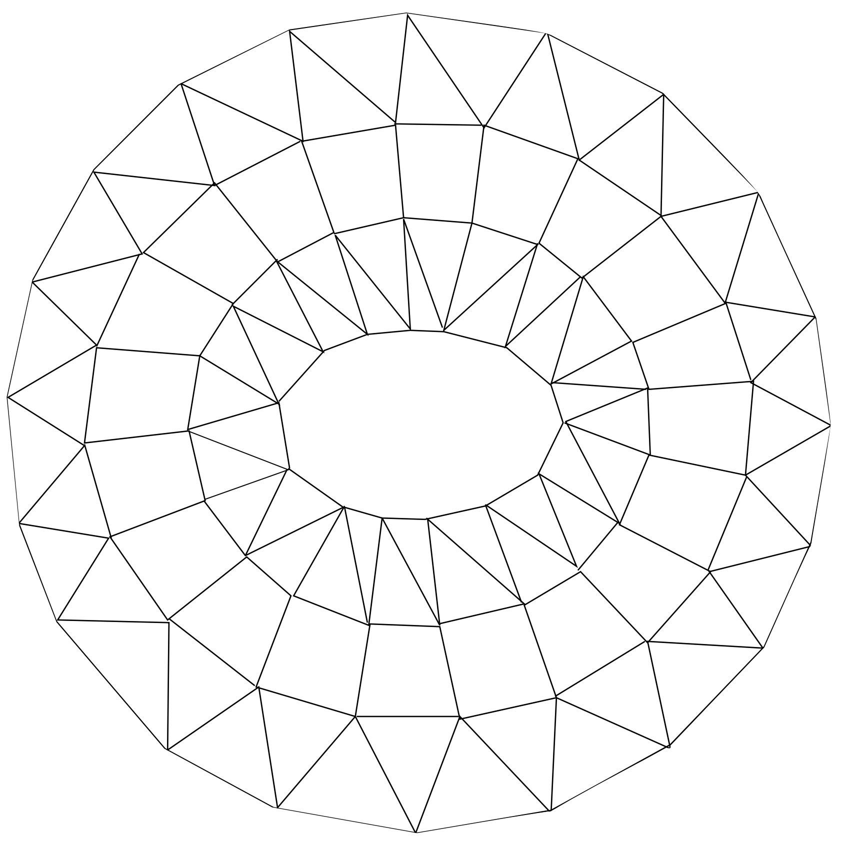
- Implicit connectivity between vertices
- Implicit cell definition

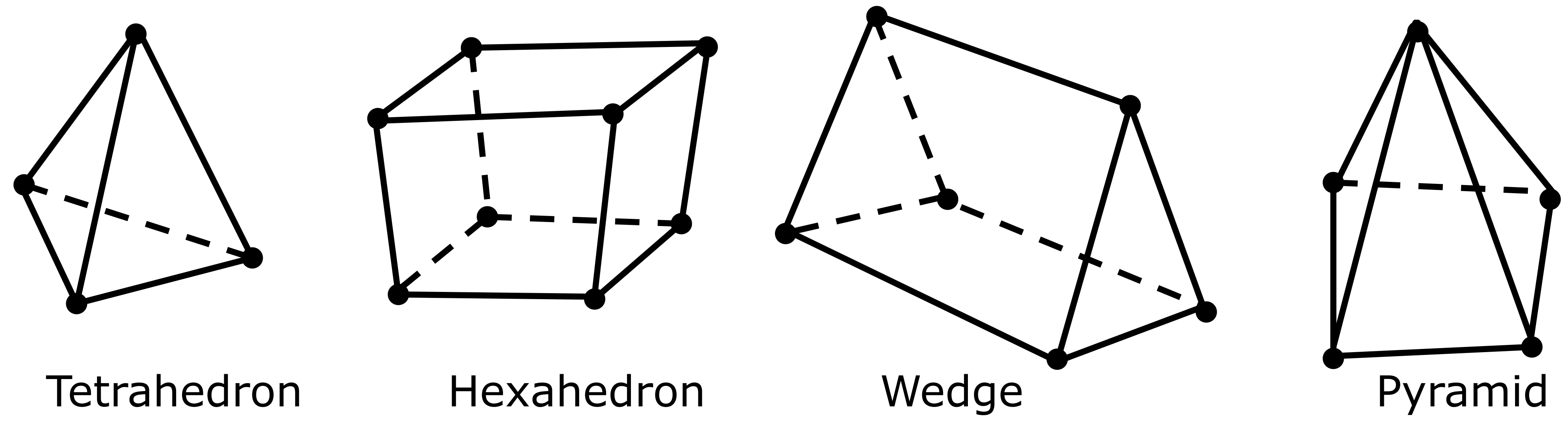


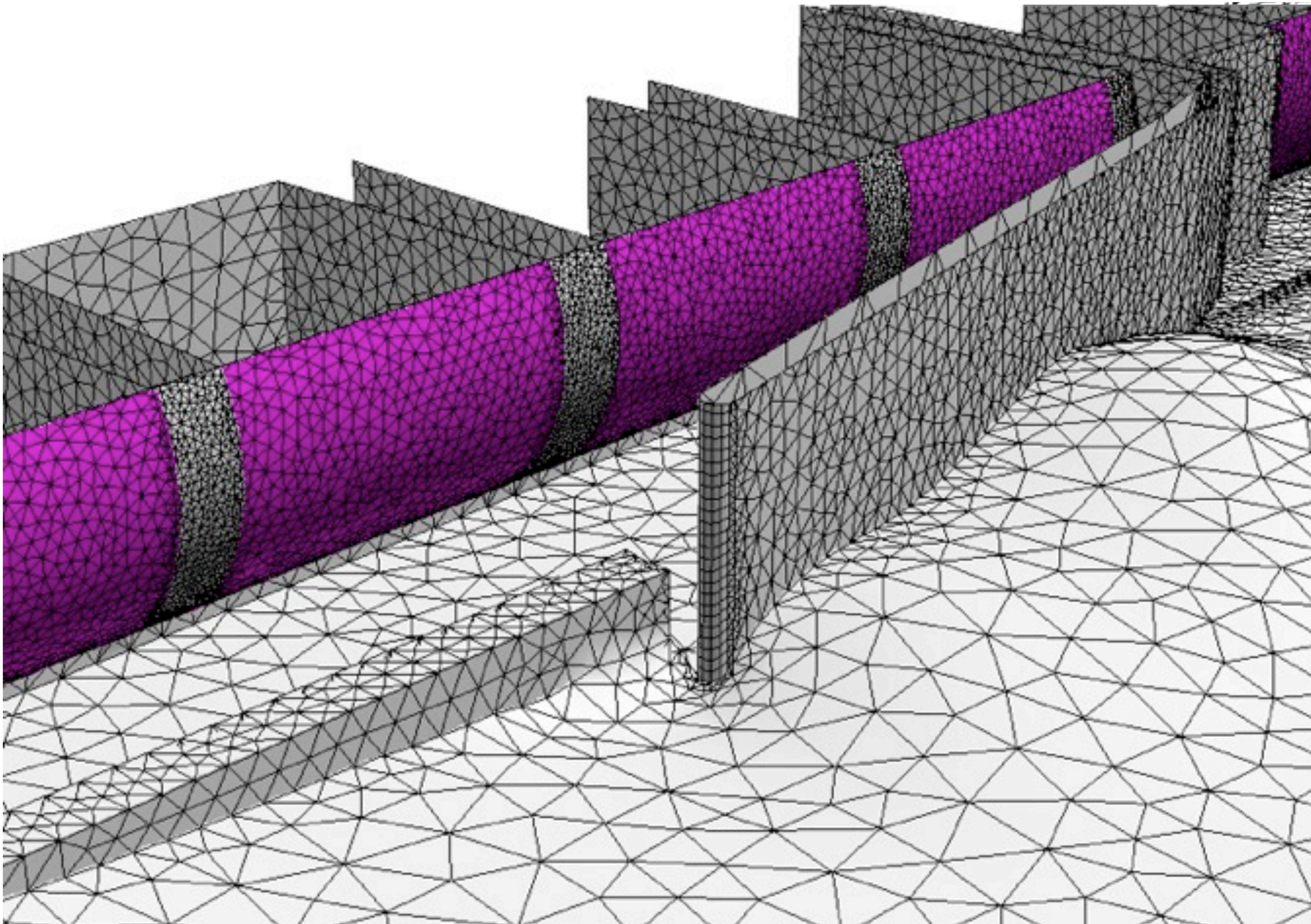
Grid Topology

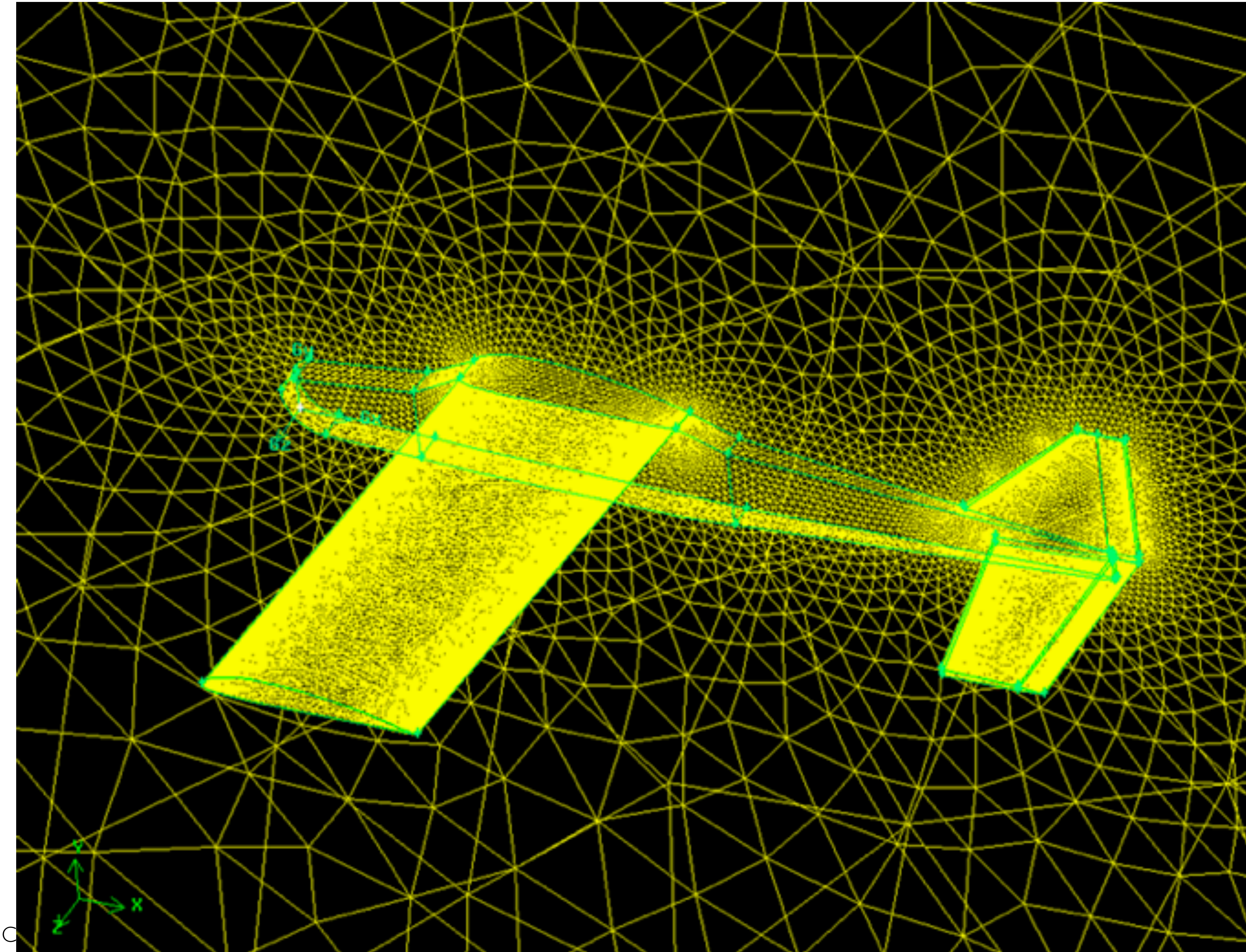
Unstructured *(any cell type)*

- Explicit cell definition
  - Types
  - Vertices







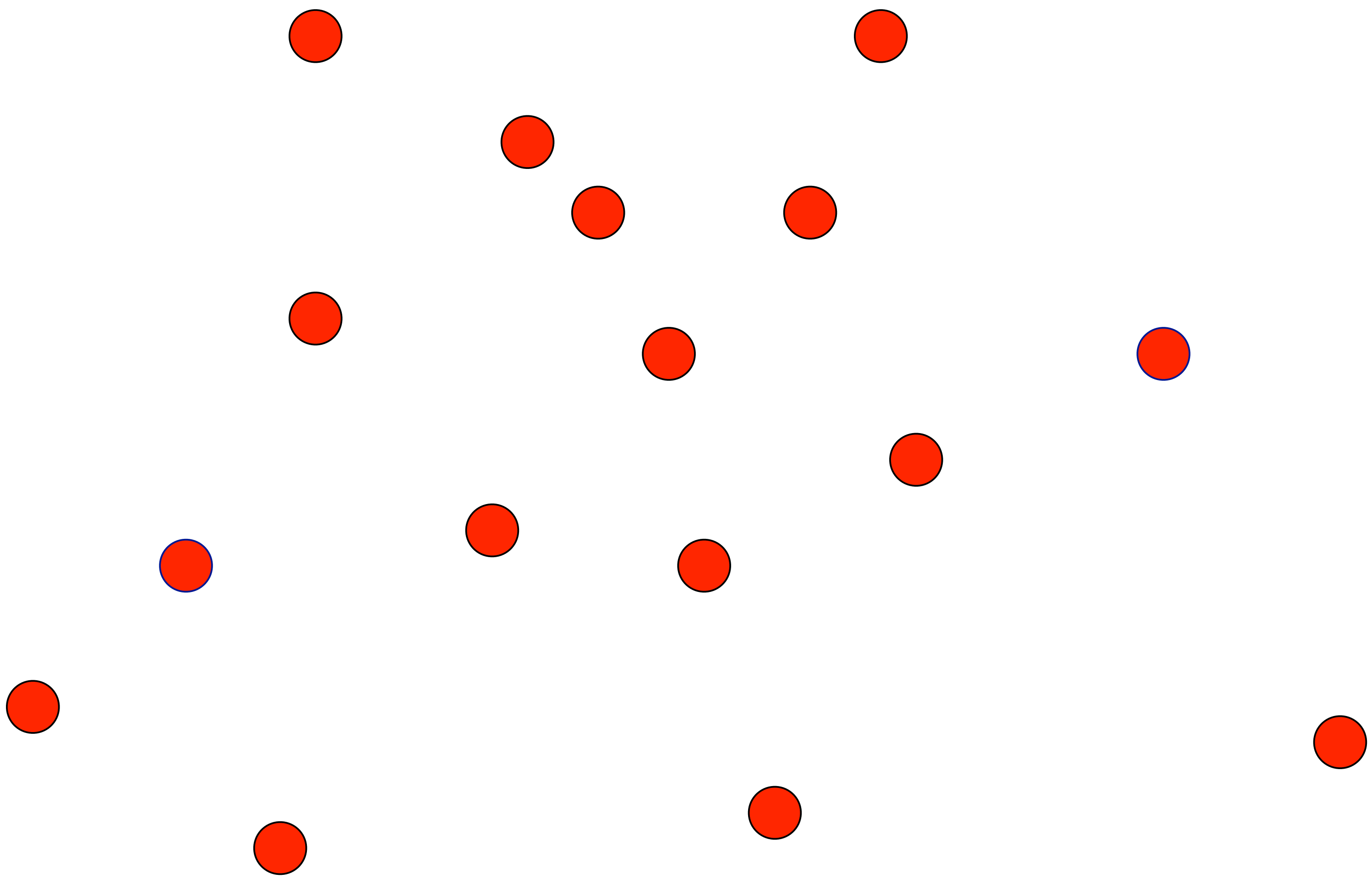

Y
X
Z

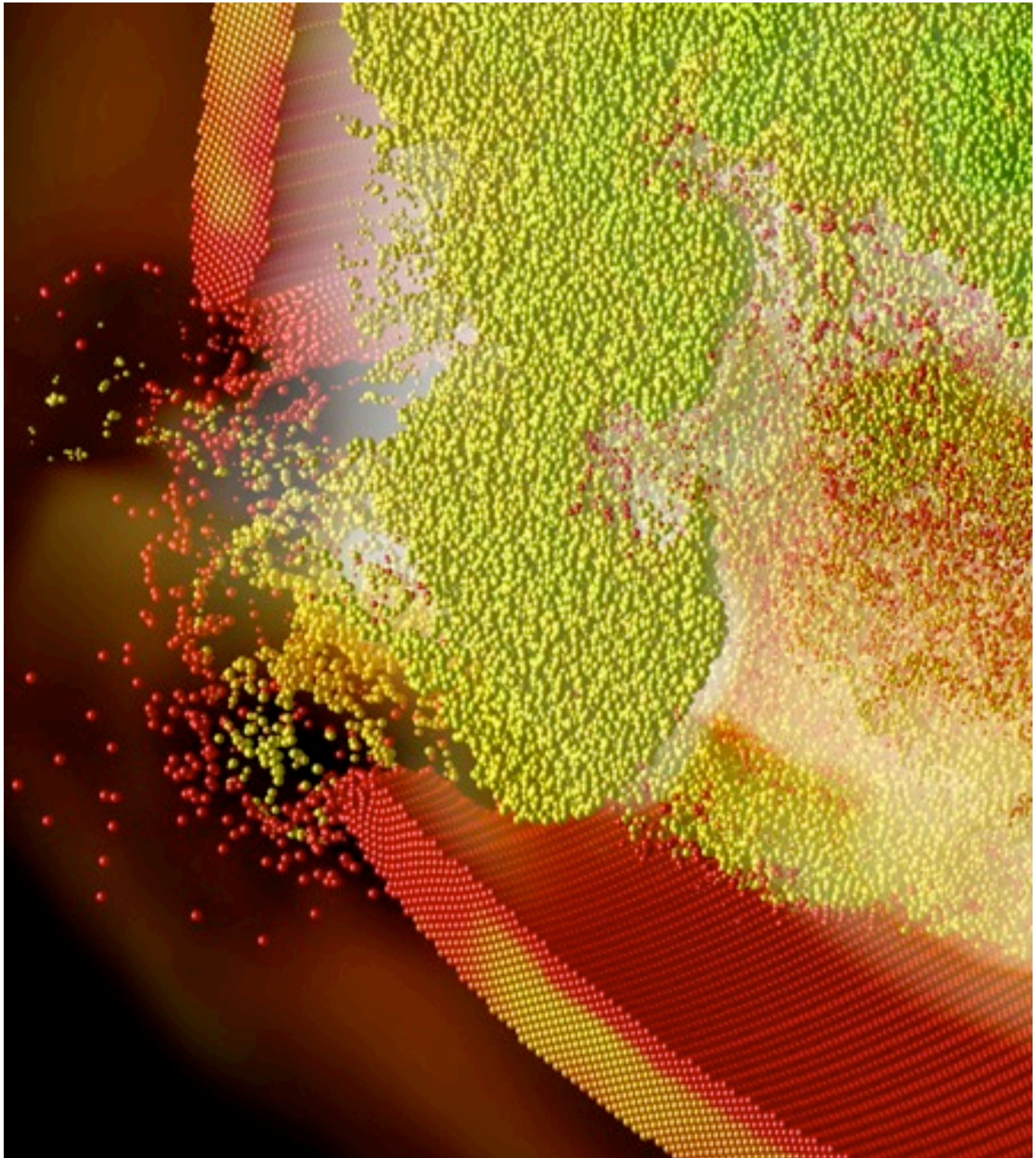
Grid Types

Geometry \ Topology	Structured	Unstructured
Uniform	**Image**	**Unstructured**
Structured	**Rectilinear**	**Unstructured**
Unstructured	**Curvilinear**	**Unstructured**

Grid Topology

- Mesh-free (no grid, no connectivity)

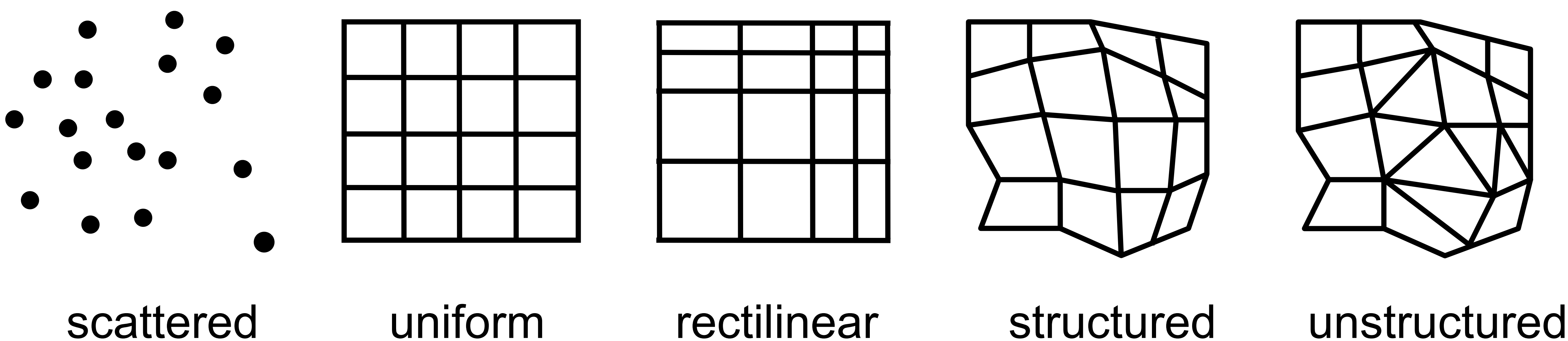




Grids (Meshes)

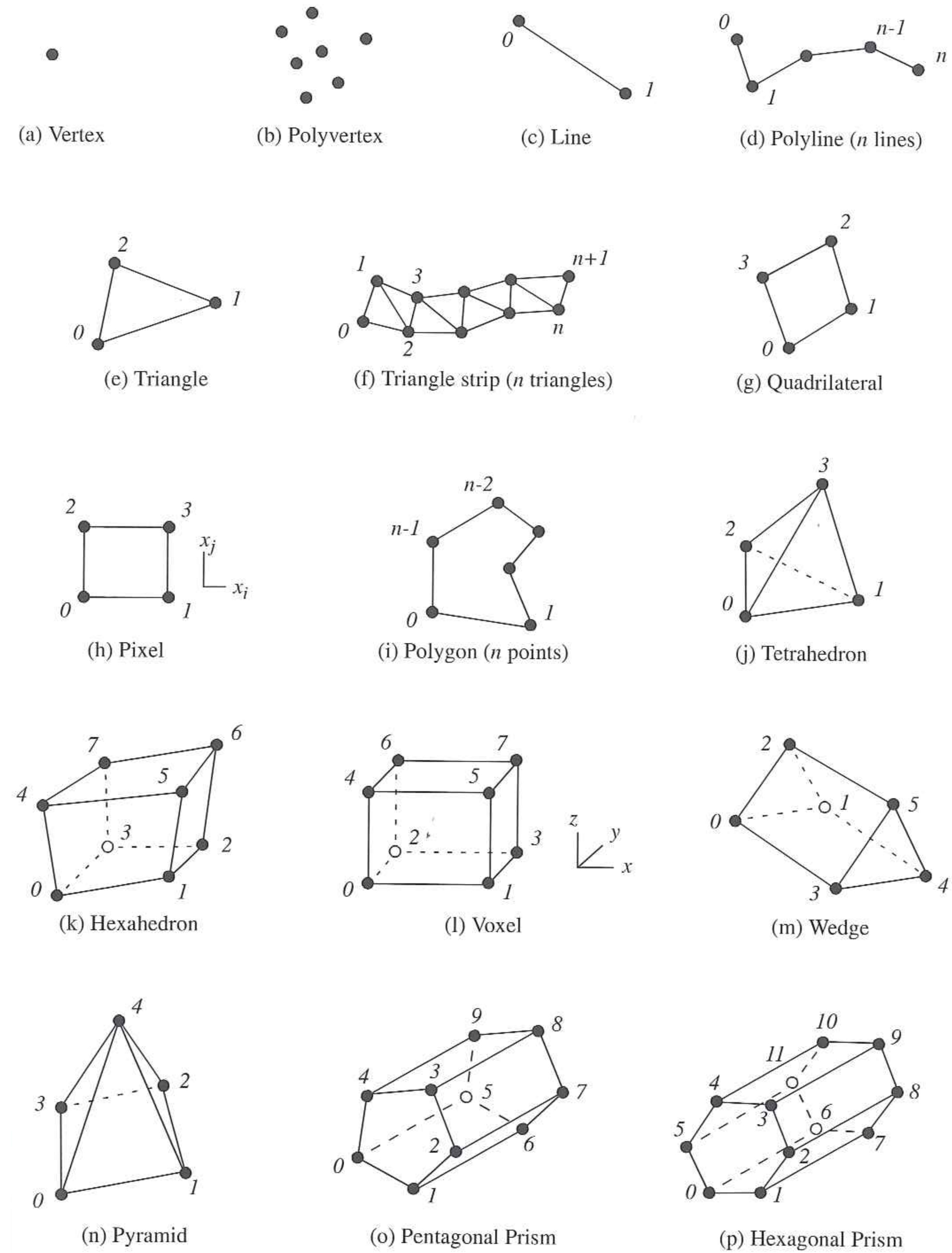
- Meshes combine positional information (geometry) with topological information (connectivity).

- Mesh type can differ substantial depending in the way mesh cells are formed.

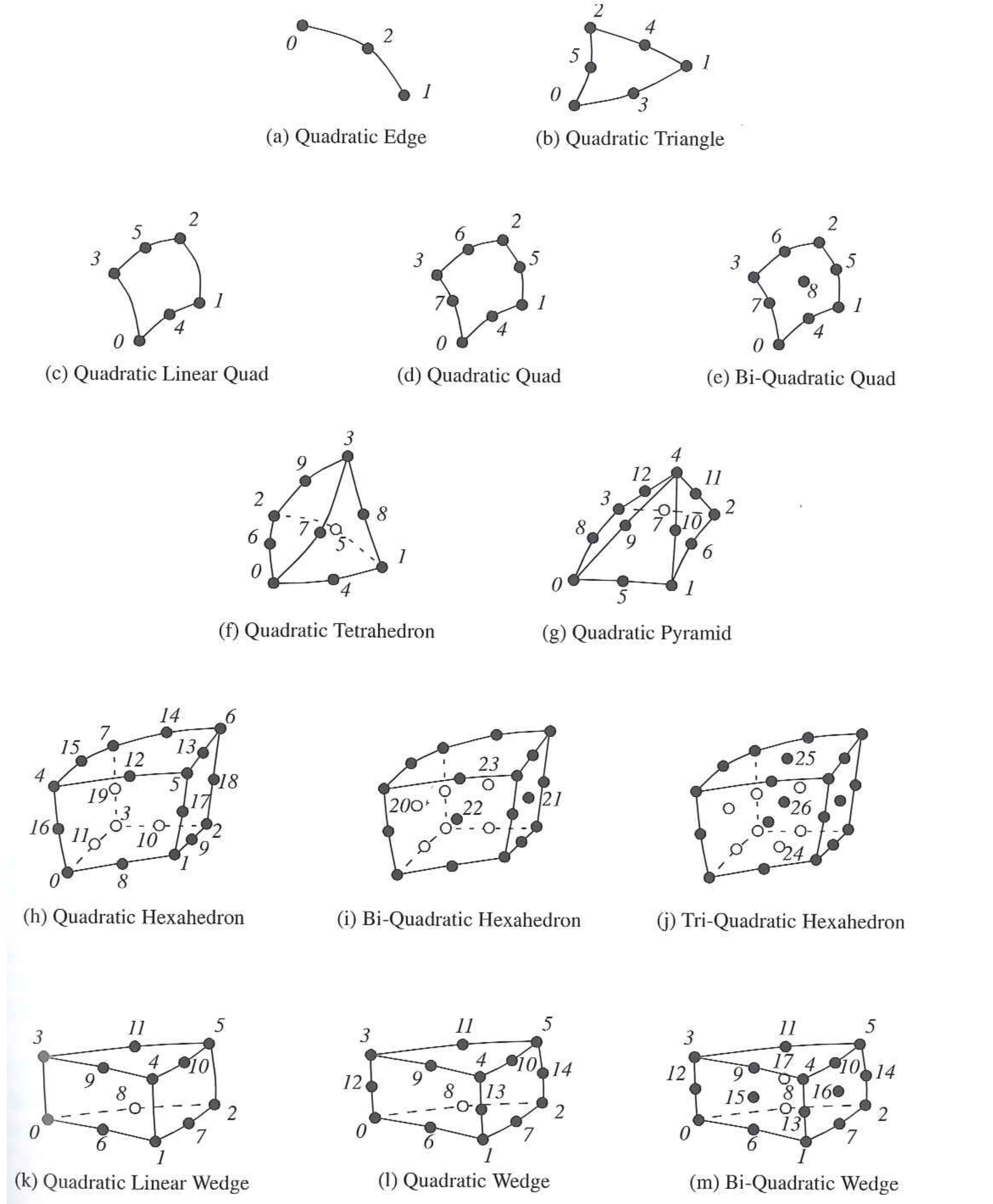


From Weiskopf, Machiraju, Möller

LINEAR CELLS



NONLINEAR CELLS

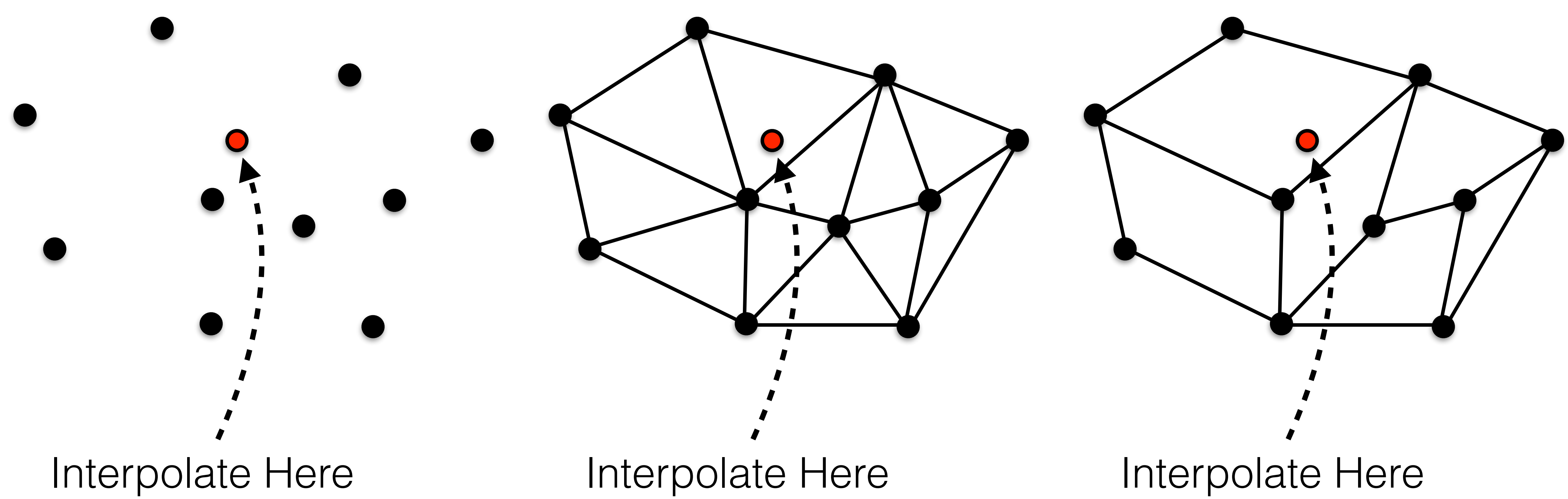


(a) Quadratic Edge
(b) Quadratic Triangle
(c) Quadratic Linear Quad
(d) Quadratic Quad
(e) Bi-Quadratic Quad
(f) Quadratic Tetrahedron
(g) Quadratic Pyramid
(h) Quadratic Hexahedron
(i) Bi-Quadratic Hexahedron
(j) Tri-Quadratic Hexahedron
(k) Quadratic Linear Wedge
(l) Quadratic Wedge
(m) Bi-Quadratic Wedge

INTERPOLATION

Mesh Choice Impacts How the Continuous Data is Interpreted

- Two key questions:
  - Sampling, or the choice of where attributes are measured
  - Interpolation, or how to model the attributes in the rest of space



Interpolation

- **Continuous** reconstruction of **discrete** input data

position

$$F : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$$(\mathbf{x}_i, f_i)$$

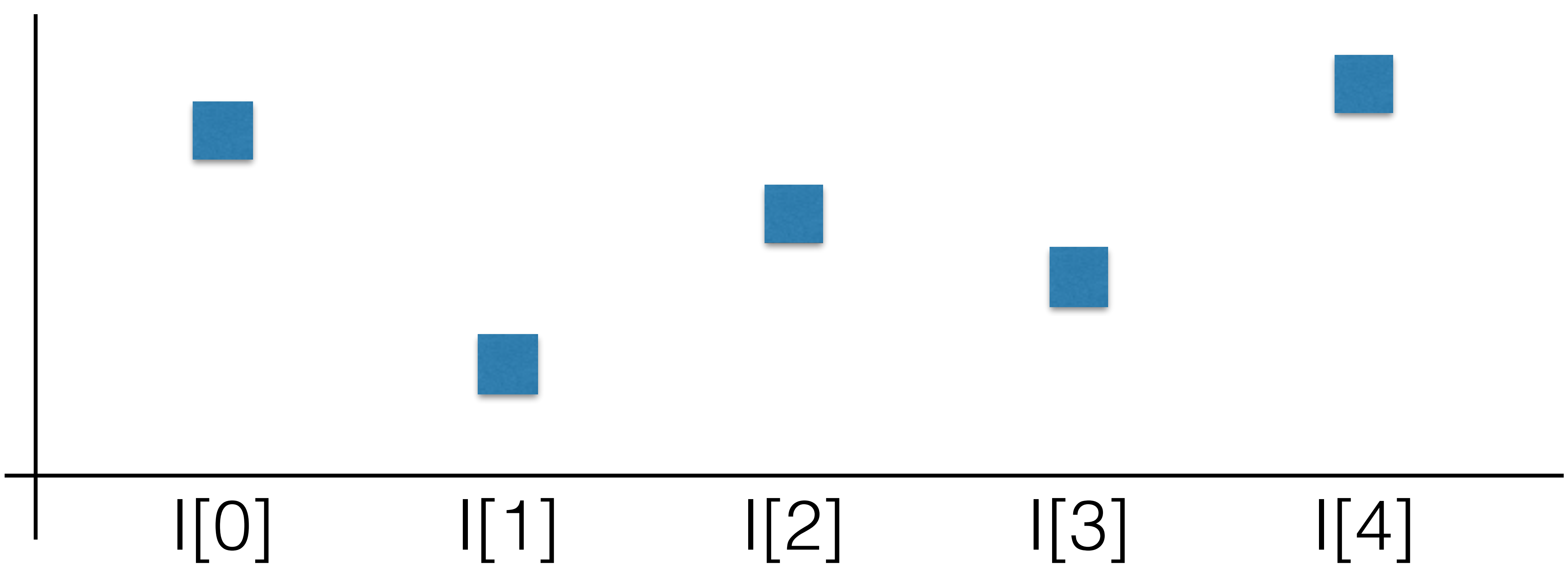
value

$$\forall i \in \{1, .., n\}, F(\mathbf{x}_i) = f_i$$

- Depends on grid structure (when available)
- Interpolation vs. approximation

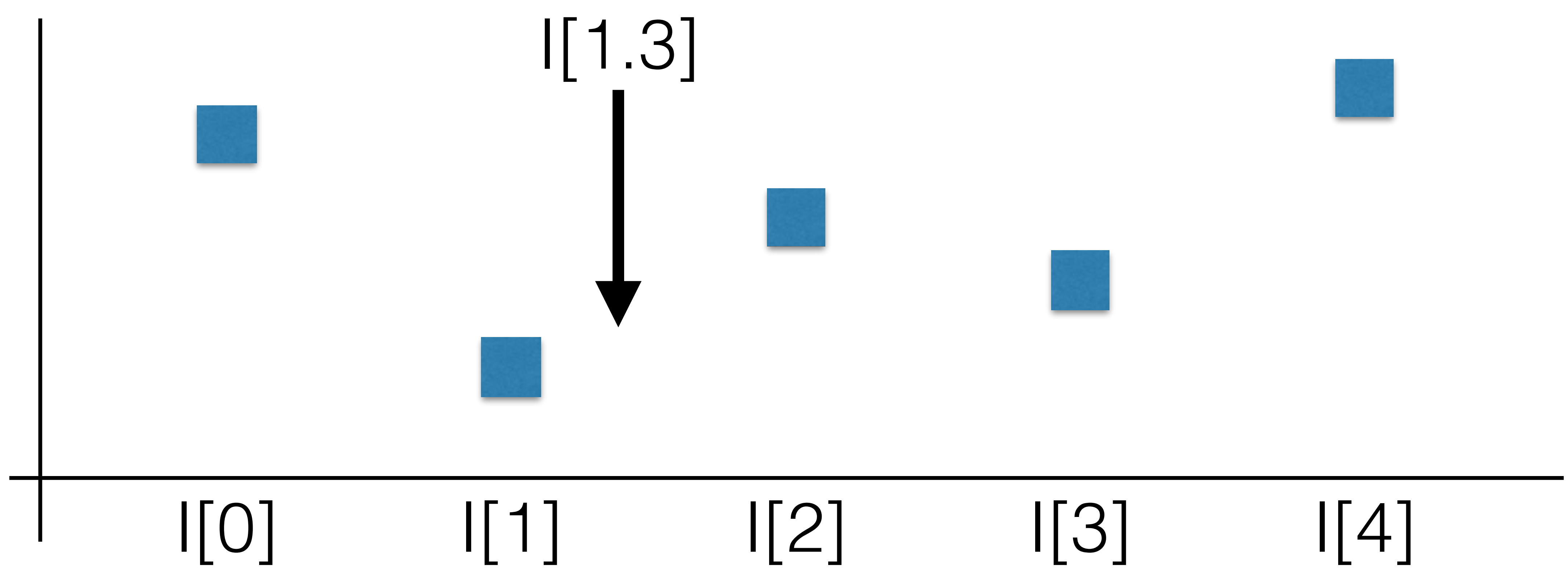
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?



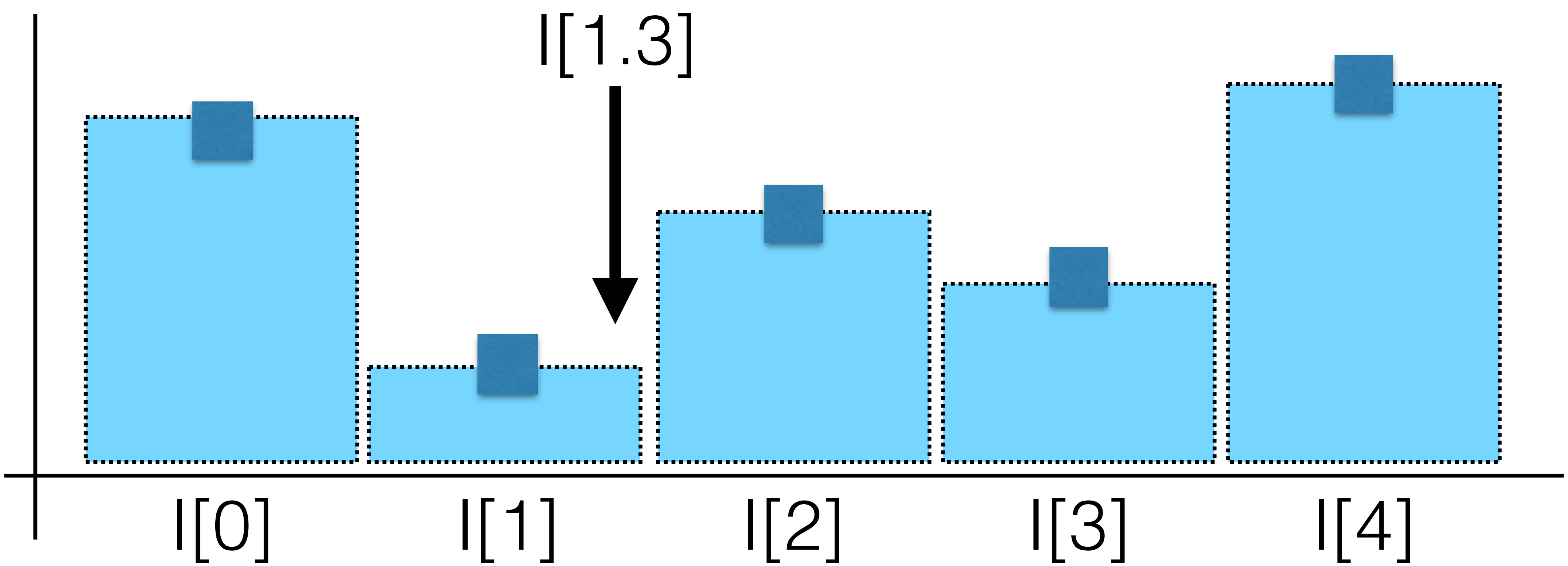
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?



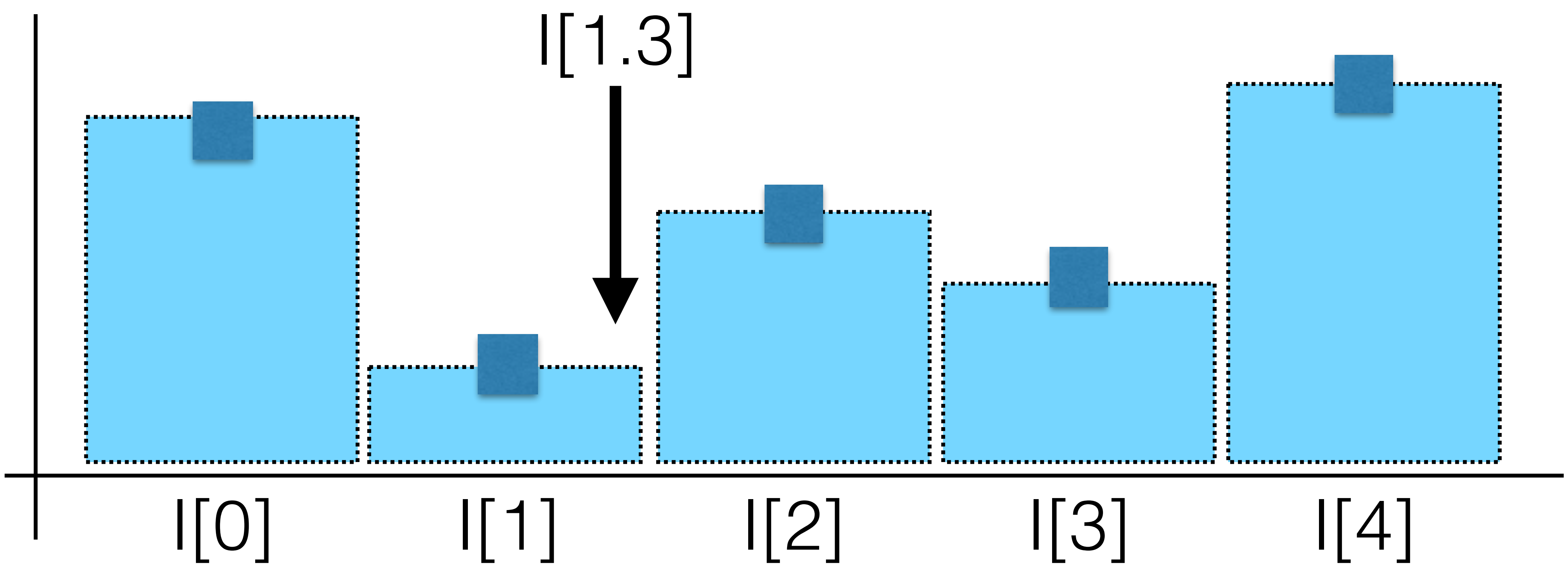
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?



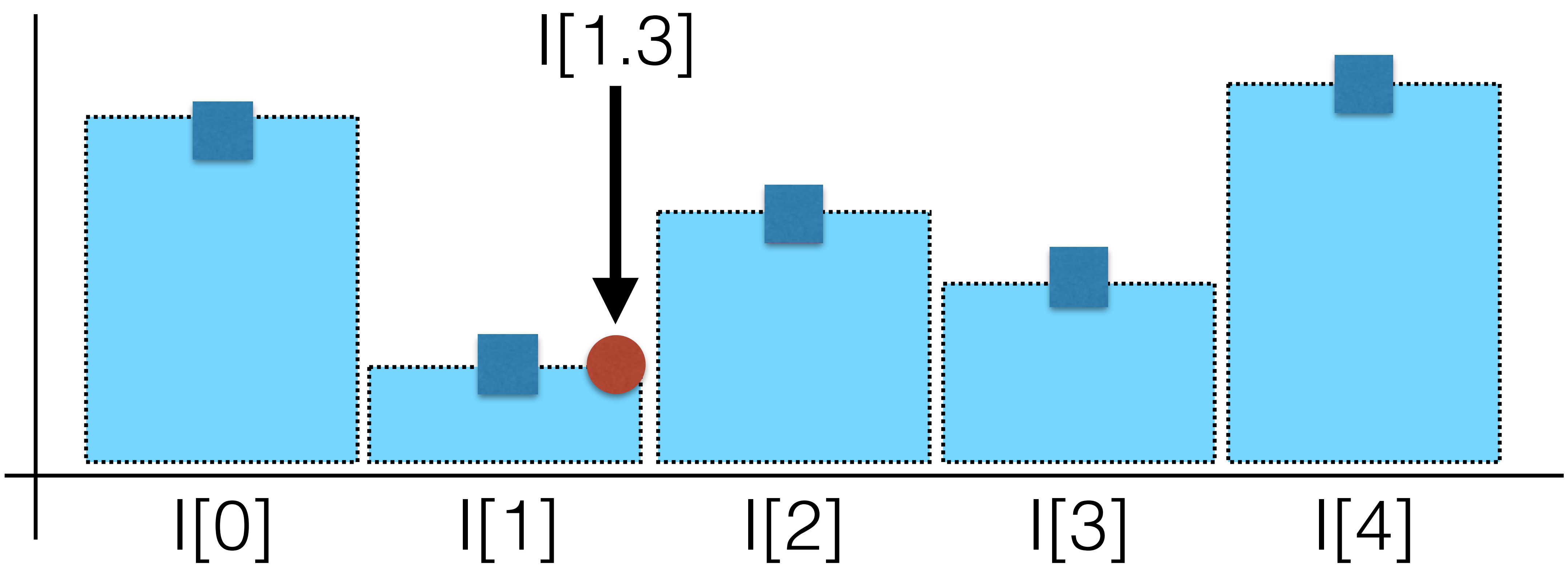
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?
  - I[1.3] = I[round(1.3)] = I[1]



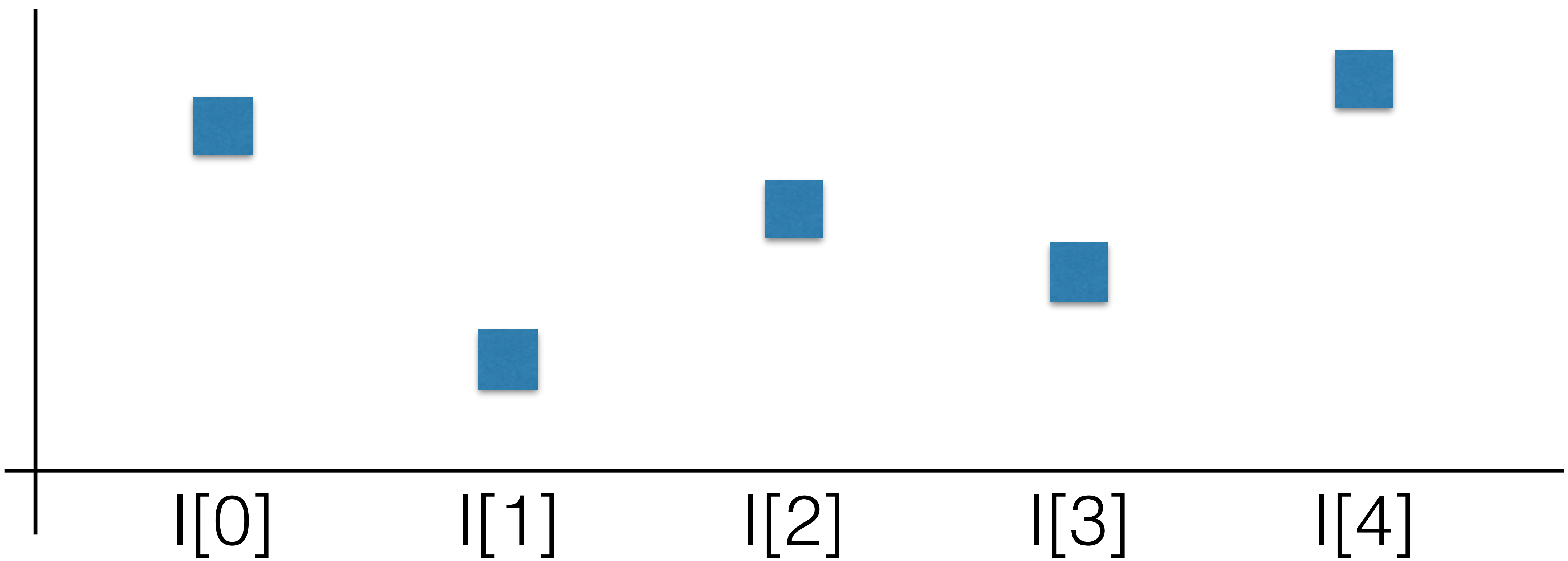
Nearest Neighbor Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?
  - I[1.3] = I[round(1.3)] = I[1]



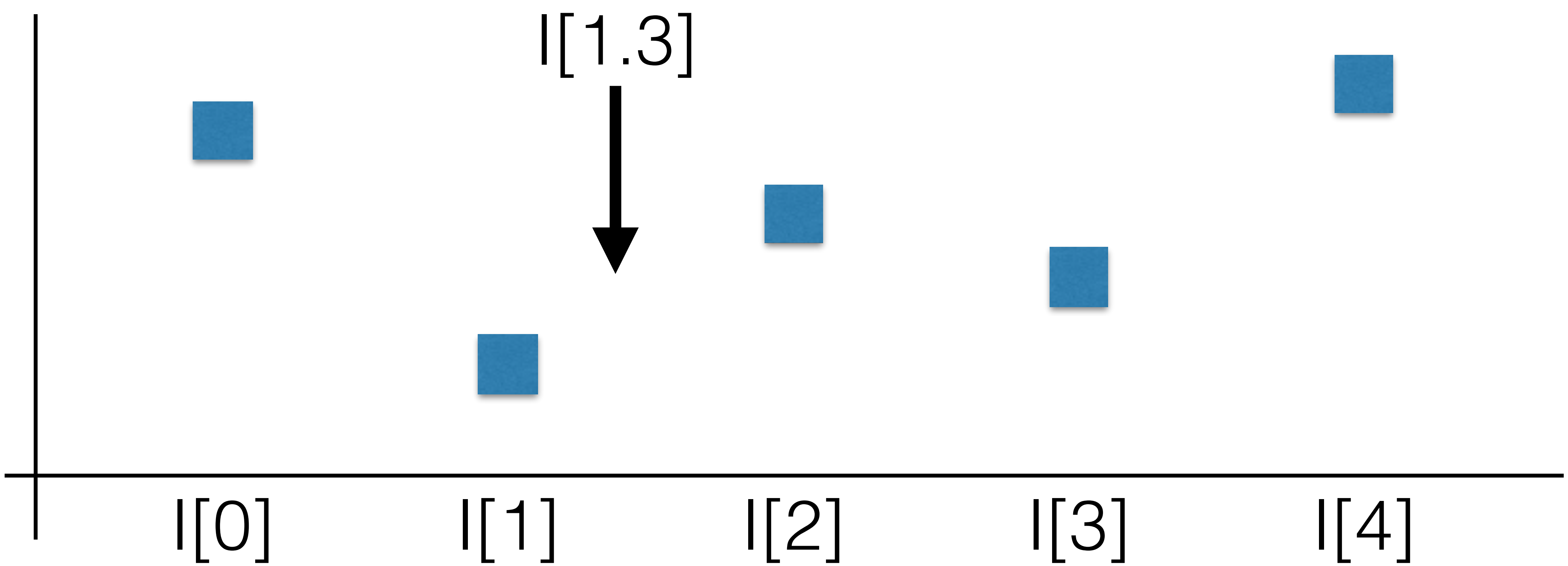
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?



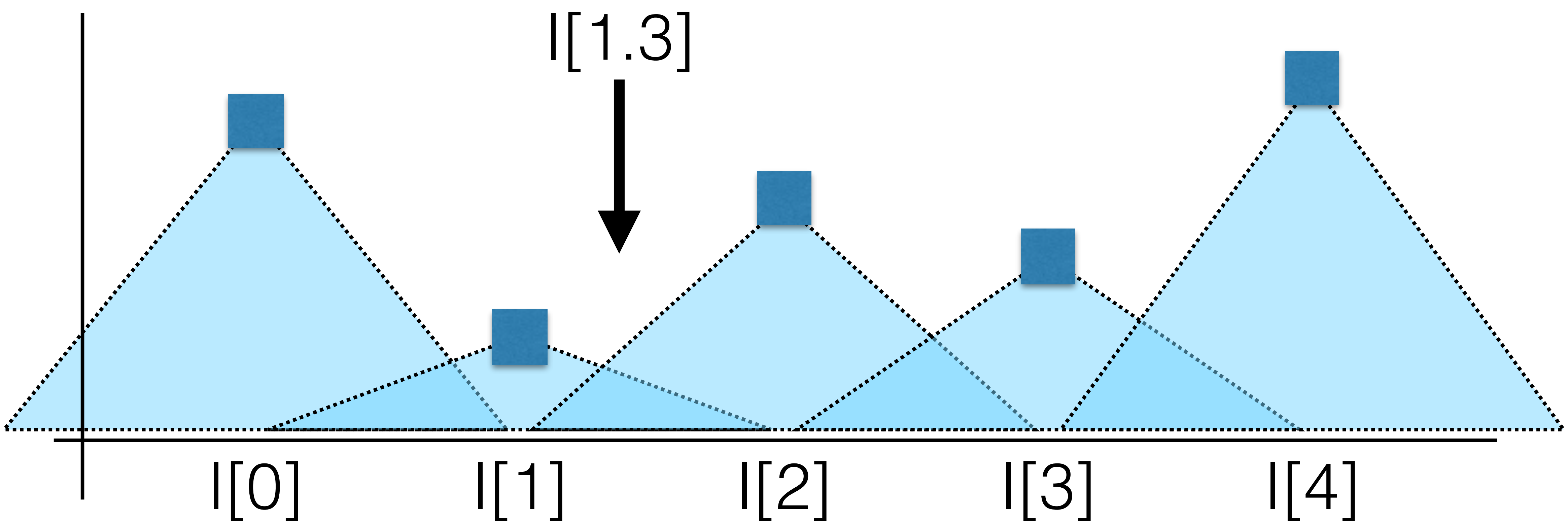
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?



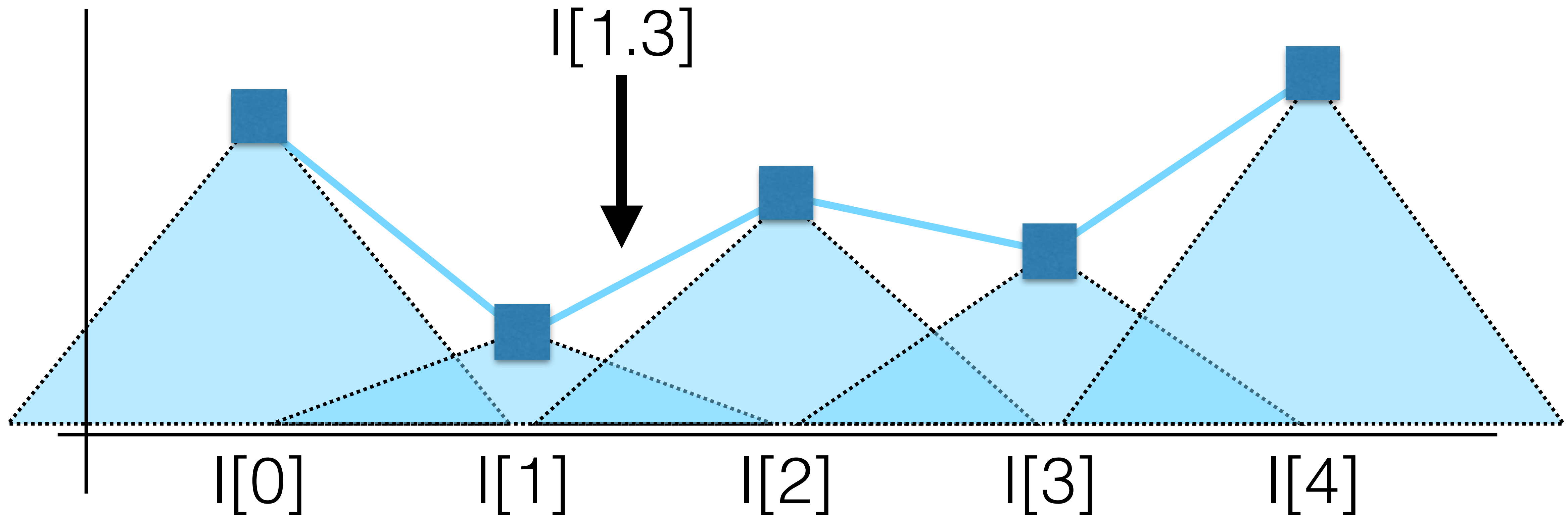
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?



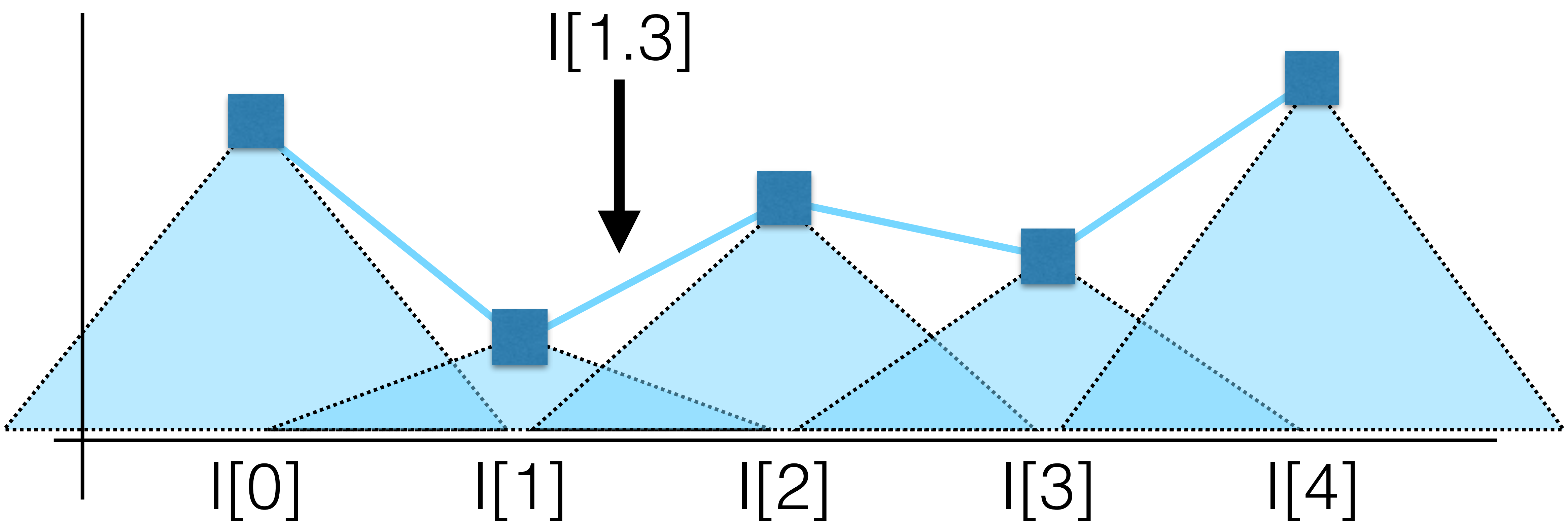
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?



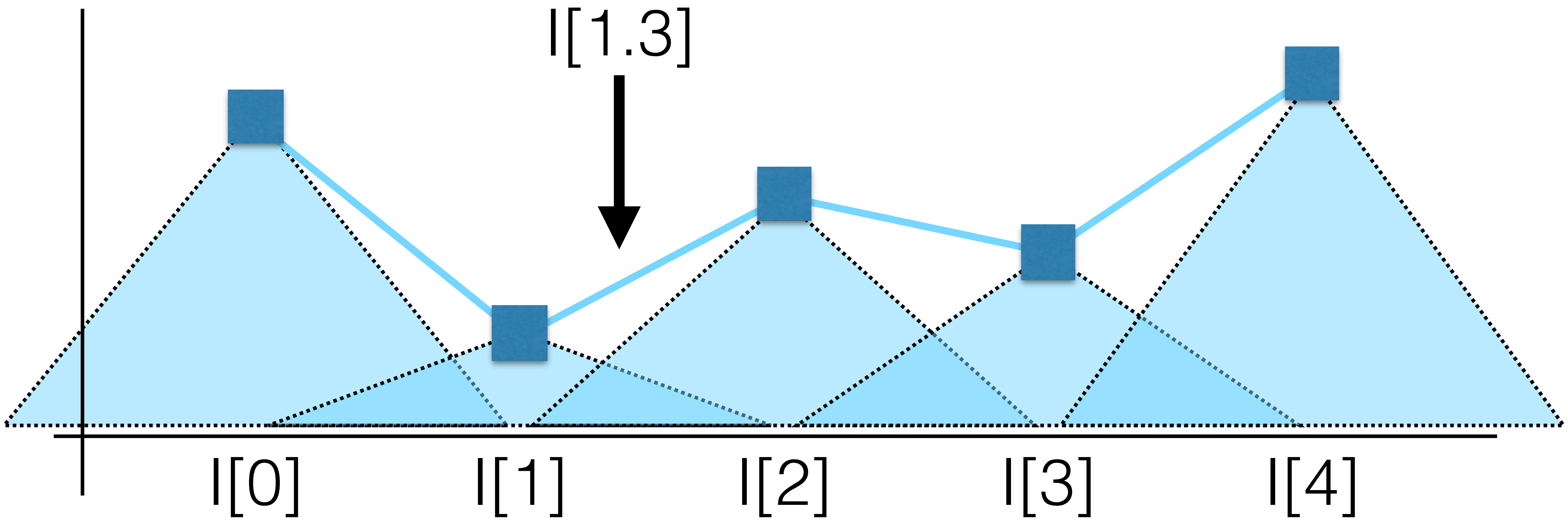
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?
    - Let s = 1.3 - round(1.3)



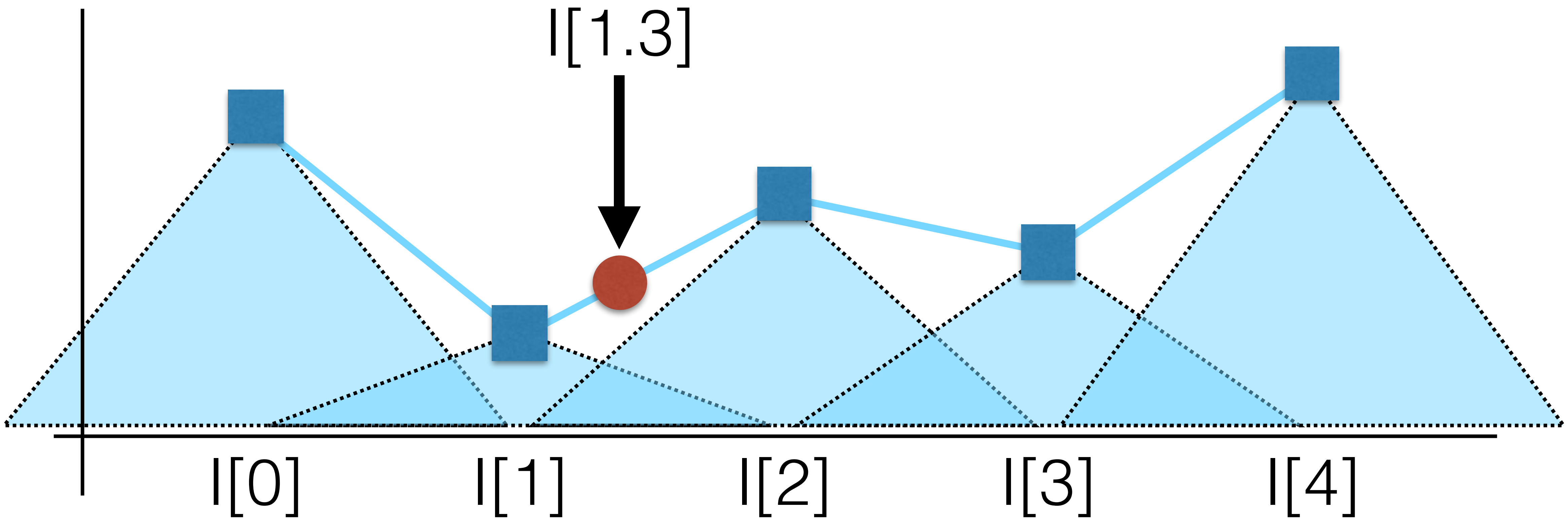
Linear Interpolation

- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?
  - Let s = 1.3 - round(1.3)
  - I[1.3] = 0.7*I[1] + 0.3*I[2] = (1-s)*I[1] + s*I[2]



Linear Interpolation

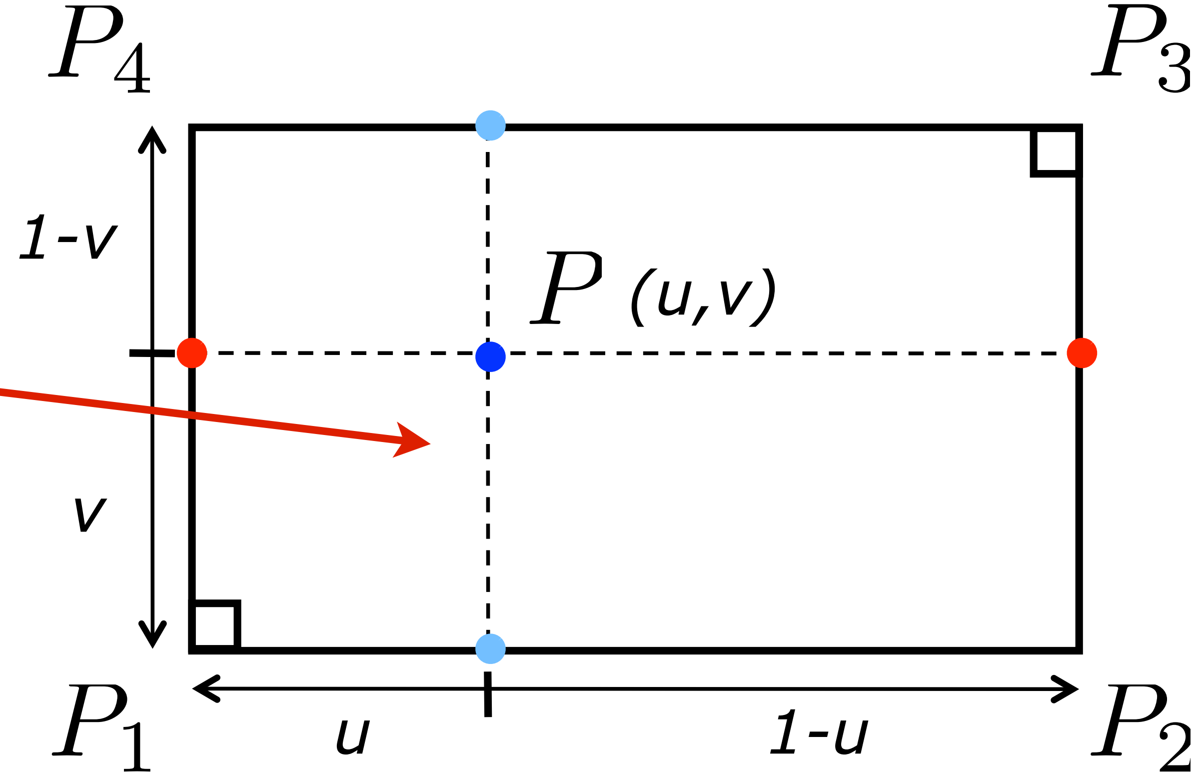
- Consider a 1-dimensional, grayscale image I spread horizontally
- What value is I[1.3] ?
  - Let s = 1.3 - round(1.3)
  - I[1.3] = 0.7*I[1] + 0.3*I[2] = (1-s)*I[1] + s*I[2]



Bilinear Interpolation

- In rectangle

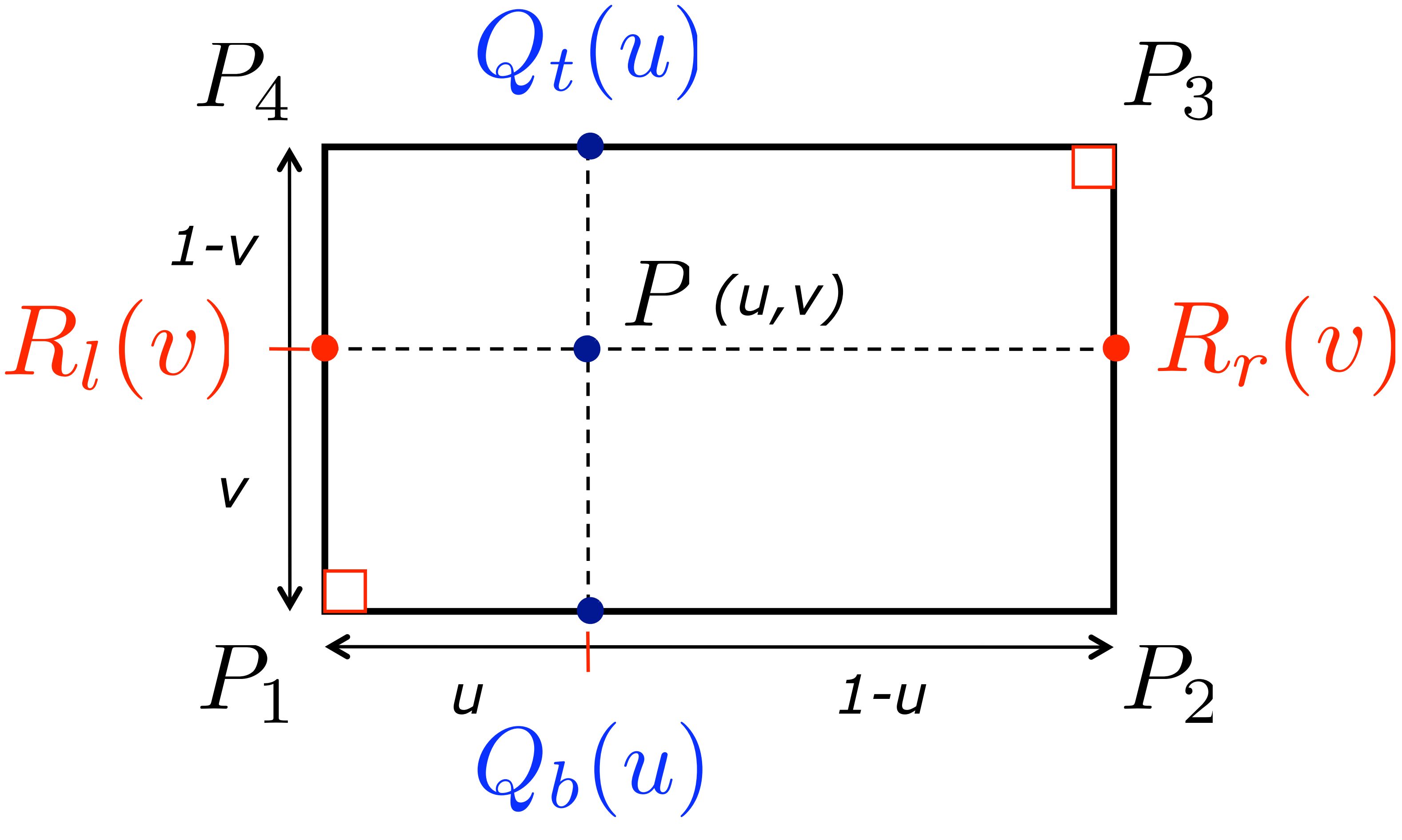
Combination of two consecutive linear interpolation



Bilinear Interpolation

- In rectangle

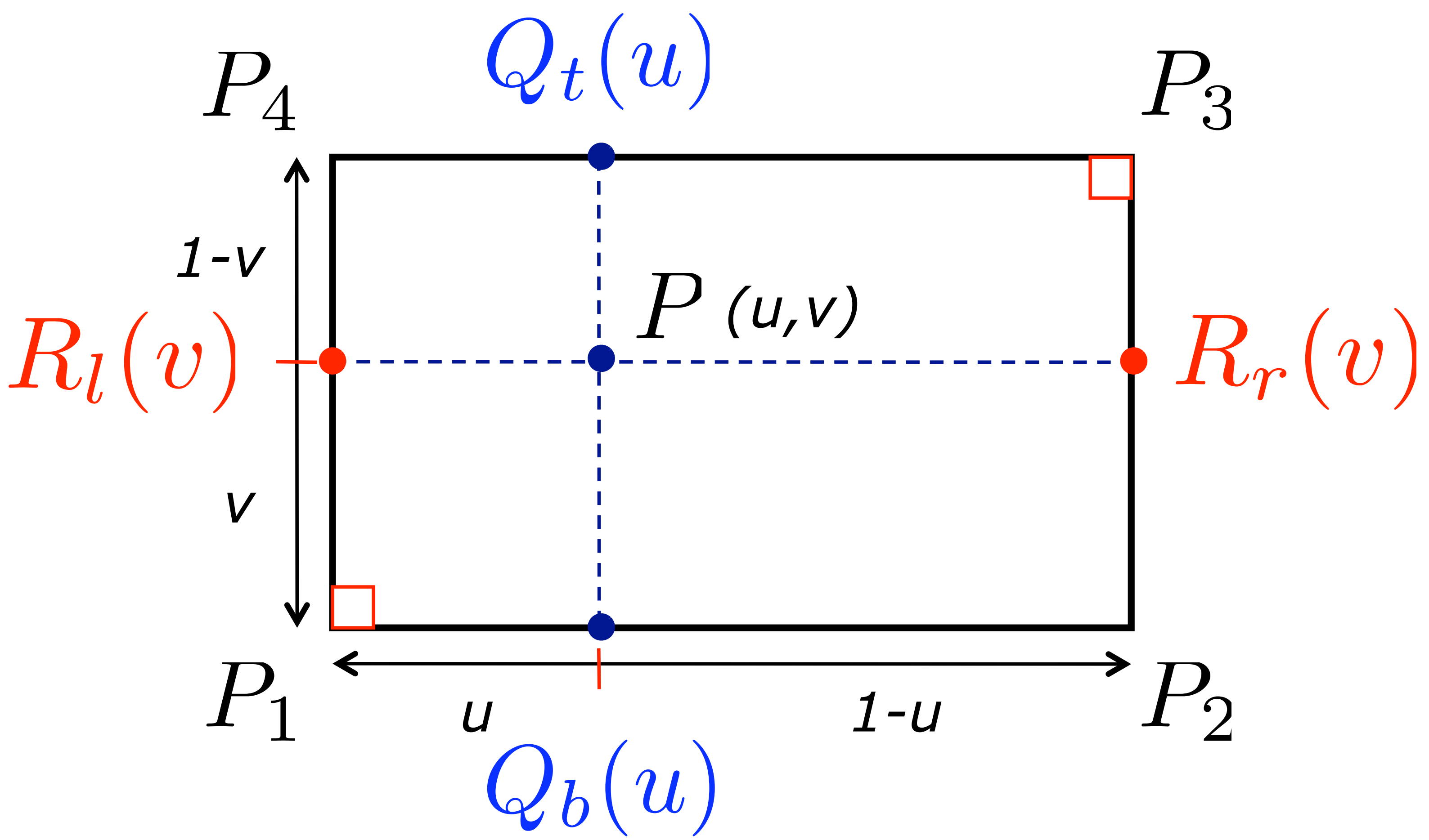
$$\boxed{\begin{aligned} P &= (1-v)Q_b(u) + vQ_t(u) \\ &= (1-u)R_l(v) + uR_r(v) \end{aligned}}$$



Bilinear Interpolation

- In rectangle

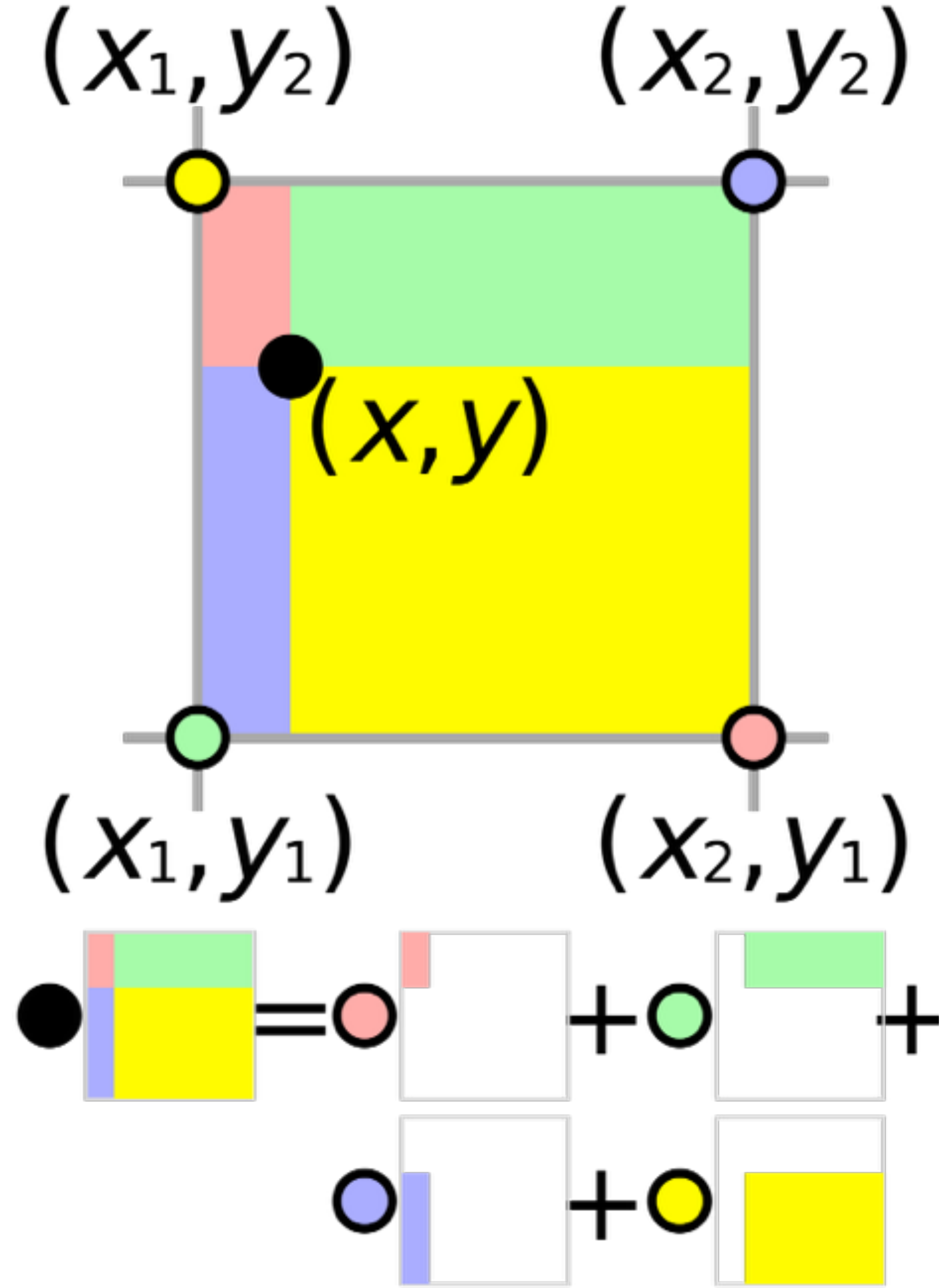
$$
\boxed{\begin{aligned} P = P_1 + u(P_2 - P_1) + v(P_4 - P_1) \\ +uv(P_1 - P_2 + P_3 - P_4) \end{aligned}}
$$



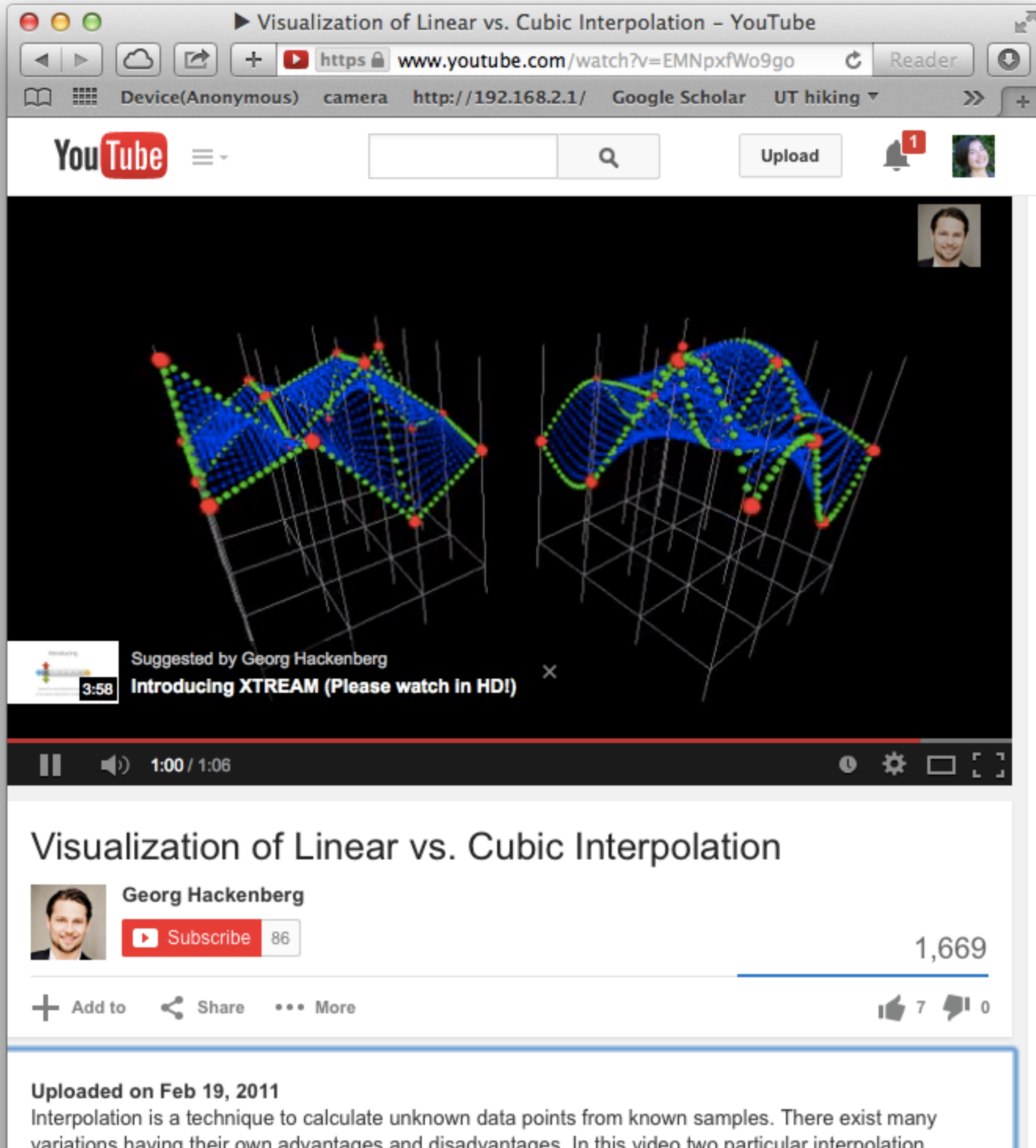
Bilinear Interpolation

- Alternate interpretation is a weighted sum of the four pixel values

- Weights defined by the area opposite each corner



http://en.wikipedia.org/wiki/Bilinear_interpolation


Visualization of Linear vs. Cubic Interpolation – YouTube
https
www.youtube.com/watch?v=EMNpxfWo9go
Reader
Device(Anonymous)
camera
http://192.168.2.1/
Google Scholar
UT hiking
Upload
Suggested by Georg Hackenberg
Introducing XTREAM (Please watch in HD!)
3:58
1:00 / 1:06
Visualization of Linear vs. Cubic Interpolation
Georg Hackenberg
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Uploaded on Feb 19, 2011
Interpolation is a technique to calculate unknown data points from known samples. There exist many

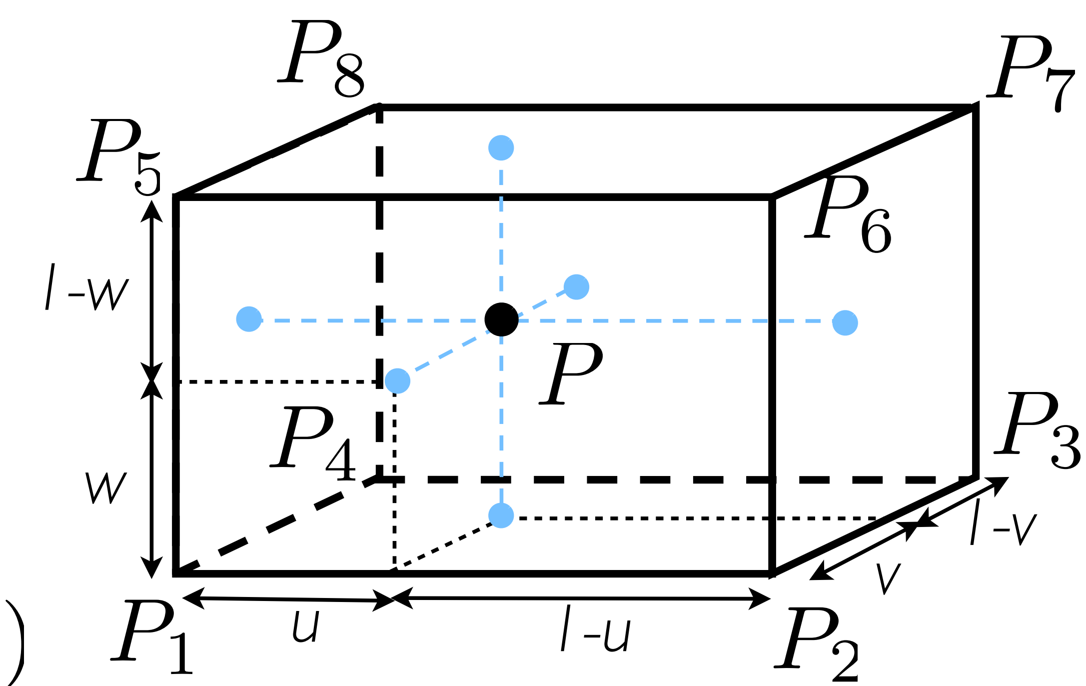
Trilinear Interpolation

- In a cuboid (axis parallel)
  - general formula

$$\phi(x,y,z) = axyz + bxy + cxz + dyz + ex + fy + gz + h$$

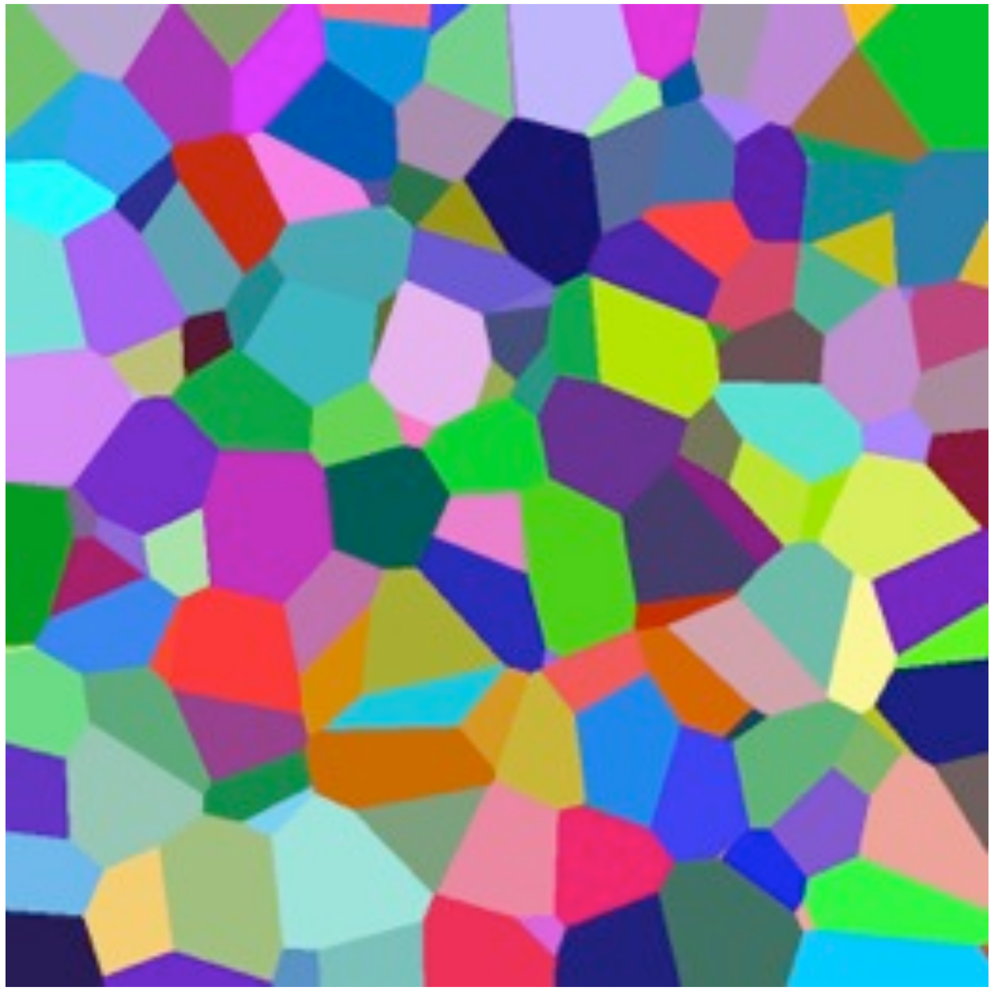
  - with local coordinates

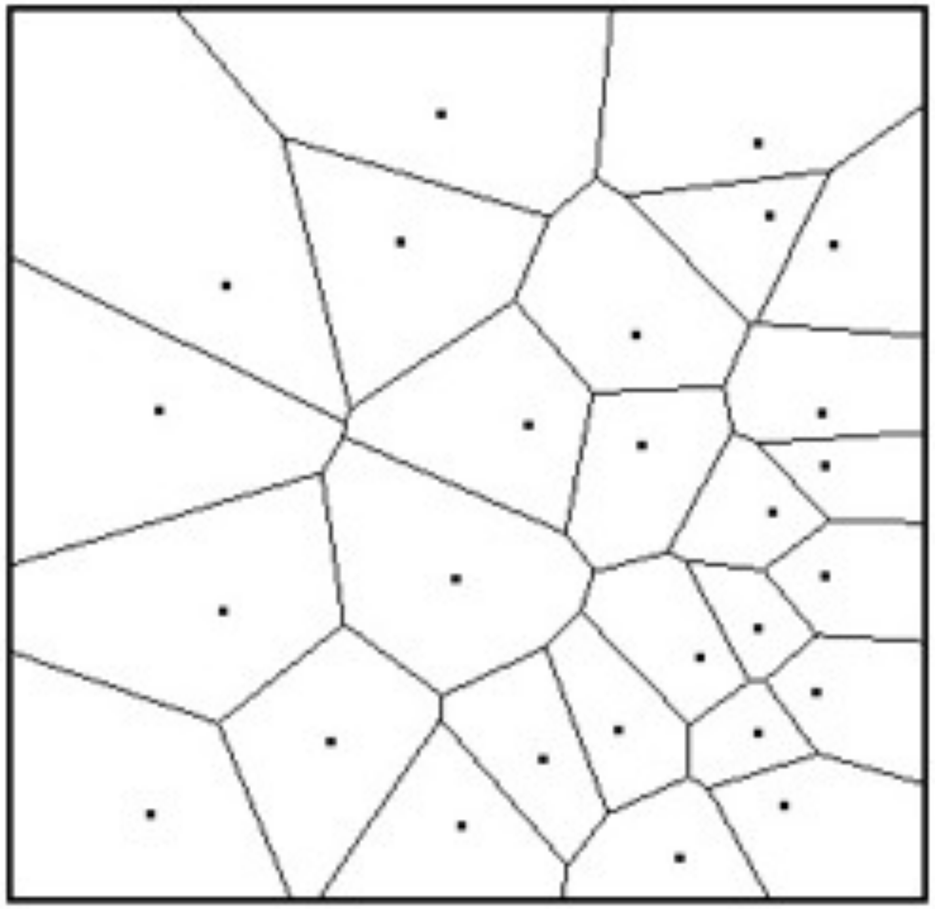
$$\begin{aligned} P = P_1 & \\ & +u(P_2 - P_1) \\ & +v(P_4 - P_1) \\ & +w(P_5 - P_1) \\ & +uv(P_1 - P_2 + P_3 - P_4) \\ & +uw(P_1 - P_2 + P_6 - P_5) \\ & +vw(P_1 - P_4 + P_8 - P_5) \\ & +uvw(P_1 - P_2 + P_3 - P_4 + P_5 - P_6 + P_7 - P_8) \end{aligned}$$



But Also...

- Nearest Neighbor interpolation

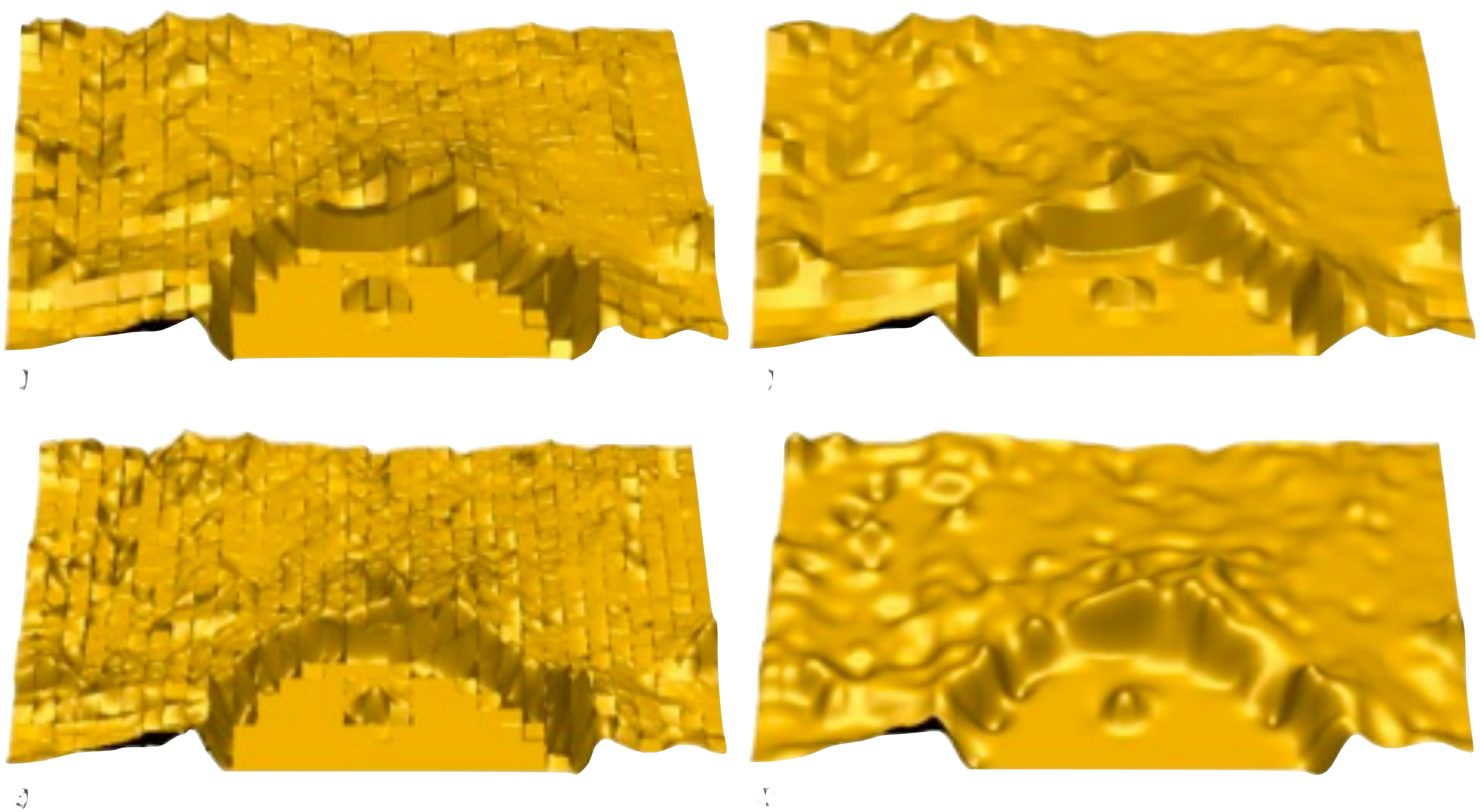




Voronoi diagram

But Also...

- Higher-order interpolation schemes
  - splines, local polynomial fit (interpolation, least sq., ...)
  - smooth reconstruction kernels (on uniform grids)



L17: Isosurfaces

REQUIRED READING

Chapter 8

Arrange Spatial Data

8.1 The Big Picture

For datasets with spatial semantics, the usual choice for *arrange* is to *use* the given spatial information to guide the layout. In this case, the choices of *express*, *separate*, *order*, and *align* do not apply because the position channel is not available for directly encoding attributes. The two main spatial data types are geometry, where shape information is directly conveyed by spatial elements that do not necessarily have associated attributes, and spatial fields, where attributes are associated with each cell in the field. (See Figure 8.1.) For scalar fields with one attribute at each field cell, the two main visual encoding idiom families are isocontours and direct volume rendering. For both vector and tensor fields, with multiple attributes at each cell, there are four families of encoding idioms: flow glyphs that show local information, geometric approaches that compute derived geometry from a sparse set of seed points, texture approaches that use a dense set of seeds, and feature approaches where data is derived with global computations using information from the entire spatial field.

MARCHING CUBES: A HIGH RESOLUTION 3D SURFACE CONSTRUCTION ALGORITHM

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Abstract

We present a new algorithm, called *marching cubes*, that creates triangle models of constant density surfaces from 3D medical data. Using a divide-and-conquer approach to generate inter-slice connectivity, we create a case table that defines triangle topology. The algorithm processes the 3D medical data in scan-line order and calculates triangle vertices using linear interpolation. We find the gradient of the original data, normalize it, and use it as a basis for shading the models. The detail in images produced from the generated surface models is the result of maintaining the inter-slice connectivity, surface data, and gradient information present in the original 3D data. Results from computed tomography (CT), magnetic resonance (MR), and single-photon emission computed tomography (SPECT) illustrate the quality and functionality of *marching cubes*. We also discuss improvements that decrease processing time and add solid modeling capabilities.

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acetabular fractures [6], craniofacial abnormalities [17,18], and intracranial structure [13] illustrate 3D's potential for the study of complex bone structures. Applications in radiation therapy [27,11] and surgical planning [4,5,31] show interactive 3D techniques combined with 3D surface images. Cardiac applications include artery visualization [2,16] and nongraphic modeling applications to calculate surface area and volume [21].

Existing 3D algorithms lack detail and sometimes introduce artifacts. We present a new, high-resolution 3D surface construction algorithm that produces models with unprecedented detail. This new algorithm, called *marching cubes*, creates a polygonal representation of constant density surfaces from a 3D array of data. The resulting model can be displayed with conventional graphics-rendering algorithms implemented in software or hardware.

After describing the information flow for 3D medical applications, we describe related work and discuss the drawbacks of that work. Then we describe the algorithm as well as efficiency and functional enhancements, followed by case studies using three different medical imaging techniques to il-