Scale-Space Representation Using Anisotropic Diffusion

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Gaussian Aperture Model

Image is viewed through an Gaussian aperture

- $L(x, y) = L_0(x, y) \otimes G(x, y; \sigma)$

By varying σ, we create images at different scales

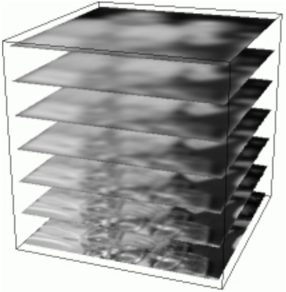

Image from http://cvr.yorku.ca/members/gradstudents/kosta/compvis/.

Pros/Cons of Gaussian Scale-Space

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- Gaussians have many nice properties
  - Linearly separable
  - Convolution of two Gaussians is a Gaussian
- Efficient to implement via FFT
- Meets causality criteria
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Cons

- Poor localization at coarse scales
  - Requires tracing through scale space
- Regions are blurred together before large scale features are recognized
  - Example: leaves of a tree blend with sky before leaves blend with each other

Modified Criteria

Causality

- No spurious features

Immediate Localization

- Region boundaries should be sharp at every scale

Piecewise Smoothing

- Intra-region smoothing favored over inter-region smoothing

Anisotropic Diffusion

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What is the ideal solution using this formulation?

- $c = 0$ at region boundaries
- $c = 1$ everywhere else

Designing a Solution

- We need an estimation of edge strength E
- We need some function $g(||E||)$
  - Must be non-negative monotonically decreasing
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Now we must make reasonable choices for E and g.

Choosing E and G

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- Image gradient gives a simple and effective edge estimation

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- Image gradient gives a simple and effective edge estimation

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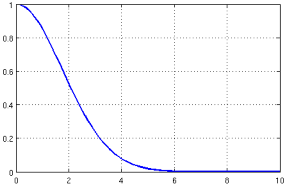
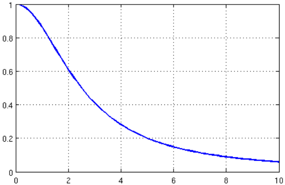
Choose g

- There are many choices for g

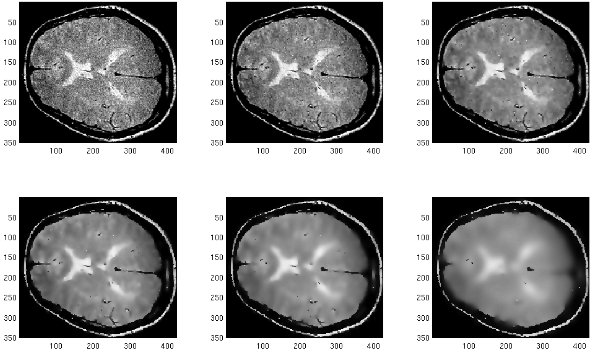


$g(\nabla I) = \exp -(||\nabla I||/K)^2$



$g(\nabla I) = \frac{1}{1+\left(\frac{||\nabla I||}{K}^2\right)}$

Example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