Digital Image Enhancement and Noise Filtering by Use of Local Statistics

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Abstract—Computational techniques involving contrast enhancement and noise filtering on two-dimensional image arrays are developed based on their local mean and variance. These algorithms are nonrecursive and do not require the use of any kind of transform. They share the same characteristics in that each pixel is processed independently. Consequently, this approach has an obvious advantage when used in real-time digital image processing applications and where a parallel processor can be used. For both the additive and multiplicative cases, the *a priori* mean and variance of each pixel is derived from its local mean and variance. Then, the minimum mean-square error estimator in its simplest form is applied to obtain the noise filtering algorithms. For multiplicative noise a statistical optimal linear approximation is made. Experimental results show that such an assumption yields a very effective filtering algorithm. Examples on images containing 256 × 256 pixels are given. Results show that in most cases the techniques developed in this paper are readily adaptable to real-time image processing.

Index Terms—Digital image enhancement, local statistics, noise filtering, real-time processing.

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INTRODUCTION

Image processing on digital computers has been gaining in acceptance in recent years [1]–[3]. Early techniques in image processing concentrated mostly on procedures that were carried out computationally in the frequency domain (Fourier or Walsh), which was a natural extention of one-dimensional linear signal processing theory. In due course it became well-known that computing a two-dimensional transform for a large data array is a very time-consuming activity even with fast transform techniques on large computers. Hence, implementation of frequency domain procedures for real-time processing of images appears less than promising. More recent works based on an application of Kalman filtering algorithm [4] or Bayesian estimation extended to two-dimensional arrays led to the concept of a recursive filtering algorithm [5], [6]. The power of recursive algorithms for real-time one-dimensional signal processing are well established. However, when applied to a two-dimensional array, the algorithm operates in the spatial domain in which pixels have to be processed sequentially. As a consequence, the procedure is no longer computationally efficient and loses its attraction for real-time processing.

Algorithms developed in this paper share a particular characteristic in that each pixel can be processed separately without waiting for its neighboring pixels to be processed. This characteristic permits a direct implementation of these algorithms for real-time image processing. Applying local statistics to image processing is not a new idea. Ketcham *et al.* [7] used the entire local histogram for real-time image enhancement, and Wallis [8] applied local mean and variance to filter out scan line noise with striking results. This paper extends this family of algorithms to contrast enhancement and noise filtering. Both additive white noise and multiplicative white noise cases are considered. Most additive noise filtering approaches utilize the fast Fourier transform, convolution, or recursive algorithms. In the transform and convolution methods, the autocorrelation between pixels is either assumed or approximated, and in the recursive algorithm, a linear causal or semi-causal autoregressive image model is generally assumed. The techniques based on the use of local mean and variance described in this paper deviate from these approaches. The basic assumption is that the sample mean and variance of a pixel is equal to the local mean and variance of all pixels within a fixed range surrounding it. The validity of this assumption is debatable but so are most other statistical image representations encountered in the current practice. In the additive noise filtering case, the *a priori* mean (variance) of the estimated image is calculated as the difference between the mean (variance) of the noise corrupted image and the mean (variance) of the noise by itself. This technique is extended to include multiplicative noise filtering and also the case involving both multiplicative and additive noise. Although this simple approach may not have the mathematical elegance and sophistication of a few other techniques, our experimental results and those reported by Wallis [8] indicate that the local mean and variance technique is a very effective tool in both contrast stretching and noise filtering applications.

Let x_{ij} be the brightness of the pixel (i, j) in a two-dimensional $N \times N$ image. The local mean and variance are calculated over a $(2n + 1) \times (2m + 1)$ window. The local mean is defined as

$$m_{i,j} = \frac{1}{(2n+1)(2m+1)} \sum_{k=i-n}^{n+i} \sum_{l=j-m}^{m+j} x_{k,l}. \tag{1}$$

Similarly, the local variance is

$$v_{i,j} = \frac{1}{(2n+1)(2m+1)} \sum_{k=i-n}^{i+n} \sum_{l=j-m}^{j+m} (x_{k,l} - m_{i,j})^2. \tag{2}$$

In this paper, Section II is devoted to spatial contrast enhancement for which only the local mean is required. Section III deals with additive noise filtering. Section IV treats images corrupted by multiplicative noise, and Section V extends the results of Sections III and IV to images corrupted by both additive and multiplicative noise. In Section V, a simplified algorithm is discussed to approximate the local mean and variance and future research using local statistics is briefly mentioned.

II. Spatial Contrast Enhancement

Gray level rescaling, high-pass filtering, and histogram redistribution [9] are the most popular techniques in image contrast enhancement. Wallis [8] suggested an algorithm based on local mean and variance in which each pixel is required to have a "desirable" local mean m_d and a "desirable" local variance v_d such that

$$x'_{i,j} = m_d + \sqrt{\frac{v_d}{v_{i,j}}}\,(x_{i,j} - m_{i,j}) \tag{3}$$

where, in (3), $m_{i,j}$ and $v_{i,j}$ are local mean and variance as given by (1) and (2). It is easy to verify that the $x'_{i,j}$ has a mean m_d and variance v_d if we consider $m_{i,j}$ and $v_{i,j}$ as the true mean and variance of $x_{i,j}$. The main drawback of this technique is that it tends to enhance subtle details at the expense of the principal features which are lost in the process. Fig. 1 shows the original image and the image processed by (3). The river in the original image and other large objects are difficult to recognize in the processed image. To compensate for this, an algorithm is designed such that a pixel $x_{i,j}$ will maintain its local mean, and yet permits its variance to be modified by a factor of its original local variance. The modified algorithm is

$$x'_{i,j} = m_{i,j} + k(x_{i,j} - m_{i,j}) \tag{4}$$

where k, the gain, is the ratio of new local standard deviation to the original standard deviation. This scheme has a distinct computational advantage over the scheme rooted in (3). The computation of local variance $v_{i,j}$ is not required and only $m_{i,j}$ need be computed. If $k > 1$, the image will be sharpened as if acted upon by a high-pass filter. If $0 \leq k < 1$, the image will be smoothed as if passed through a low-pass filter. In the extreme case, $k = 0$ and $x'_{i,j}$ is equal to its local mean $m_{i,j}$.

This algorithm can be easily extended to enhance images with an ill-distributed histogram. Let $g(x)$ be the gray level rescaling function [9], then (4) is modified to

$$x'_{i,j} = g(m_{i,j}) + k(x_{i,j} - m_{i,j}). \tag{5}$$

In the case of a linear gray level stretch, $g(x)$ is a linear relation. Several images are processed and shown in Fig. 1 using $g(x) = ax + b$, where $a = 0.9$ and $b = 13$ to allow contrast enhancement at both ends of gray scale (between 0 and 255). The linear function $g(x)$ used in this picture yields an effective contrast stretch in both the highlights and the dark areas of the image. The window size of 3 X 3 or 5 X 5 is large enough to carry out contrast enhancement. For noise filtering (to be discussed in later sections), however, a 7 X 7 or higher dimensional window is more desirable but at the expense of image resolution. All images of Fig. 1 are processed by the use of a 5 X 5 window. Fig. 1(c) shows that for $k = 2$, both the gross features and subtle details are enhanced in the same proportion. The case $k = 1$ has no effect on the image. For $k = 0.5$, the image is blurred as if passed through a low-pass filter. For $k = 0$, the image is averaged over the 5 X 5 window.

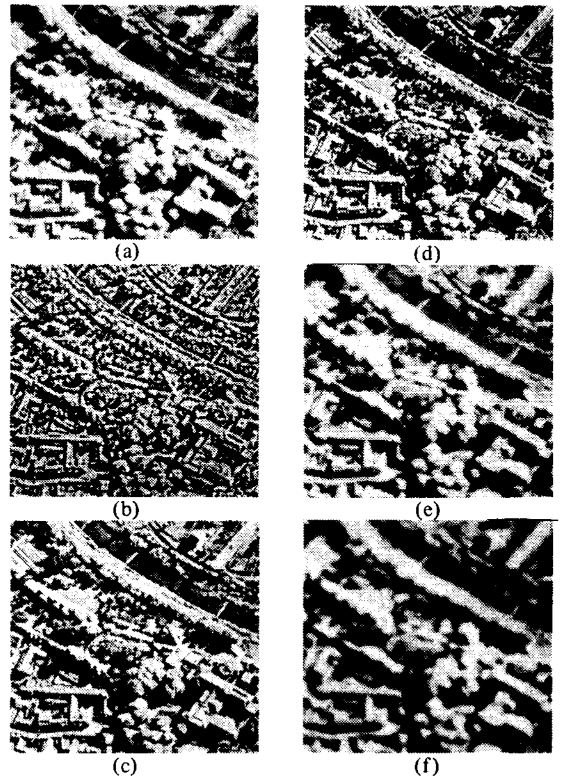

Fig. 1. (a) Original (or $k = 1$). (b) Wallis' algorithm. (c) $k = 2$. (d) $k = 3$. (e) $k = 0.5$. (f) $k = 0$.

III. Additive Noise Filtering

In this section, attention is focused on filtering of images degraded by white additive noise. Most current approaches to this problem employ frequency domain techniques, direct inversion, or recursive Kalman filtering. In the present paper, we derive a very simple algorithm based on the use of local mean and variance of an image. Let $z_{i,j}$ be the degraded pixel $x_{i,j}$; then

$$z_{i,j} = x_{i,j} + w_{i,j} \tag{6}$$

where $w_{i,j}$ is a white random sequence with

$$E[w_{i,j}] = 0$$

and $E[w_{i,j}w_{k,l}] = \sigma_1^2 \delta_{i,k}\delta_{j,l}$, where $\delta_{i,k}$ is the Kronecker delta function and E is the expectation operator. (The distribution of $w_{i,j}$ is irrelevant in this derivation.) The problem is to estimate $x_{i,j}$, given $z_{i,j}$ and the noise statistics.

From (6), we have

$$\bar{x}_{i,j} \triangleq E[x_{i,j}] = E[z_{i,j}] = \bar{z}_{i,j} \tag{7}$$

and

$$Q_{i,j} \triangleq E[(x_{i,j} - \bar{x}_{i,j})^2] = E[(z_{i,j} - \bar{z}_{i,j})^2] - \sigma_1^2. \tag{8}$$

In most filtering techniques, the *a priori* mean and variance of $x_{i,j}$ are derived from an assumed autocorrelation model or, recursively, from an autoregressive Markov sequence. In either method, it is an approximation. Assume that $\bar{x}_{i,j}$ and $Q_{i,j}$ are the *a priori* mean and variance of $x_{i,j}$, which in turn are approximated by the local mean and variance of (7) and (8). Under this assumption, it is very easy to obtain a filtering algorithm either by minimizing the mean-square error or by weighted least-square estimation [10]. Both methods will produce the same algorithm. The estimated $x_{i,j}$, $\hat{x}_{i,j}$ is computed by

$$\hat{x}_{i,j} = \bar{x}_{i,j} + k_{i,j}(z_{i,j} - \bar{x}_{i,j}) \tag{9}$$

where the gain is given by

$$k_{i,j} = \frac{Q_{i,j}}{Q_{i,j} + \sigma_1^2}. \tag{10}$$

Equation (9) can be written as

$$\hat{x}_{i,j} = (1 - k_{i,j})\bar{x}_{i,j} + k_{i,j}(z_{i,j}). \tag{11}$$

Since $Q_{i,j}$ and σ_1^2 are both positive, $k_{i,j}$ will lie between 0 and 1. A simple intuitive interpretation is that for a low signal-to-noise ratio region $Q_{i,j}$ is small compared with σ_1^2, $k_{i,j} \simeq 0$, and the estimated $\hat{x}_{i,j}$ is $\bar{x}_{i,j}$. Conversely, for a high signal-to-noise ratio region, $Q_{i,j}$ is much larger than σ_1^2, $k_{i,j} \simeq 1$, and $\hat{x}_{i,j} \simeq z_{i,j}$, the corrupted pixel. The use of different window sizes will greatly affect the quality of processed images. If the window is too small, the noise filtering algorithm is not effective. If the window is too large, subtle details of the image will be lost in the filtering process. Our experiments indicate that a 7 X 7 window is a fairly good choice. All images presented in this and later sections are processed by the 7 X 7 window.

Fig. 2 shows the original image, the image contaminated with additive noise and the estimated image produced by the local mean and variance algorithm. Clearly, in a smooth area, the pixel is averaged over the window and in a high contrast or edge area, the noise corrupted pixels are weighted higher than their local mean value. Fig. 2(d), (e), and (f) are the plots of intensity along a specific scan line for the original, the noise corrupted, and the processed images, respectively. This algorithm works equally well for an image corrupted by a Gaussian noise. Results for the latter case are shown in Fig. 3(a) and (b).

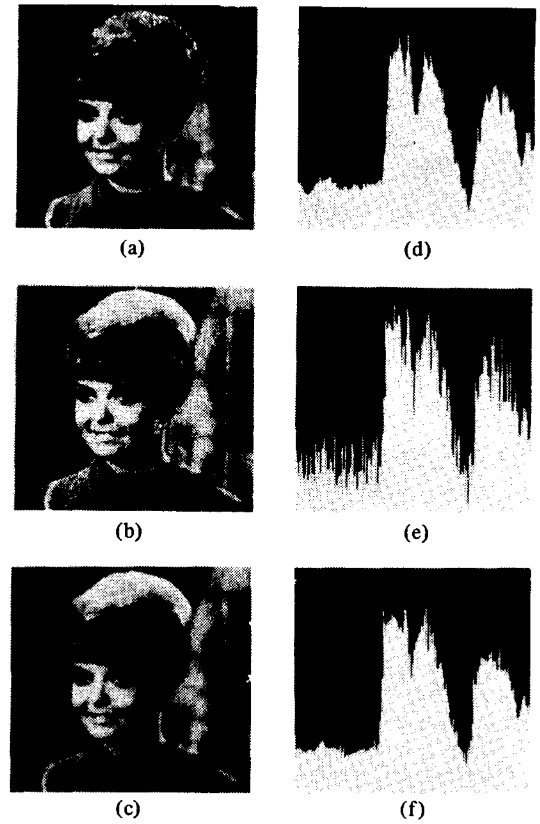

Fig. 2. (a) Original. (b) Original plus additive uniform noise (−30, 30). (c) Additive noise removed with (7 × 7) mesh, $\sigma_1^2 = 300$. (d) Original intensity profile along a scan line. (e) Profile at (d) contaminated by additive noise. (f) Profile at (e) filtered for noise.

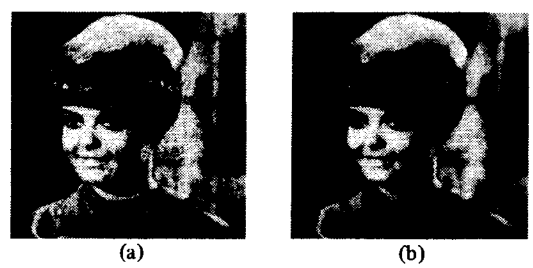

Fig. 3. (a) Original plus additive Gaussian noise, $\sigma_1^2 = 300$. (b) Noise removed.

IV. MULTIPLICATIVE NOISE FILTERING

Images containing multiplicative noise have the characteristic that the brighter the area the noisier it is. Mathematically, the degraded pixel can be represented by

$$z_{i,j} = x_{i,j}u_{i,j} \tag{12}$$

where $E[u_{i,j}] = \bar{u}_{i,j}$ and

$$E[(u_{i,j} - \bar{u}_{i,j})(u_{k,l} - \bar{u}_{k,l})] = \sigma_2^2 \delta_{i,k}\delta_{j,l}.$$

Nahi and Naraghi [11] treat this problem via the Kalman-Bucy approach which necessitates solving nonlinear estimation problem by numerical integration. In this paper, the nonlinearity in (12) is treated differently. An optimal linear approximation of (12) is used to produce a filtering algorithm similar to that for the additive noise case. Experimental results show that the derived algorithm is a very promising one. Let

$$z'_{i,j} = A\, x_{i,j} + B\, u_{i,j} + C \tag{13}$$

where A, B, and C are nonrandom variables. They are to be chosen to minimize the mean-square error between $z_{i,j}$ and $z'_{i,j}$ and also to make $z'_{i,j}$ an unbiased estimate of $z_{i,j}$. For $z'_{i,j}$ to be unbiased estimate of $z_{i,j}$, we must have

$$A\, \bar{x}_{i,j} + B\, \bar{u}_{i,j} + C = \bar{x}_{i,j}\, \bar{u}_{i,j}$$

or

$$C = \bar{x}_{i,j}\, \bar{u}_{i,j} - A\, \bar{x}_{i,j} - B\, \bar{u}_{i,j}. \tag{14}$$

Substituting (14) into (13) and forming the mean-square error, we arrive at the performance index to be minimized,

$$J = E[A(x_{i,j} - \bar{x}_{i,j}) + B(u_{i,j} - \bar{u}_{i,j}) - (x_{i,j}u_{i,j} - \bar{x}_{i,j}\,\bar{u}_{i,j})]^2.$$

Upon carrying out the necessary mathematical procedures, we obtain the following relation:

$$z_{i,j} = \bar{u}_{i,j}\, x_{i,j} + \bar{x}_{i,j}(u_{i,j} - \bar{u}_{i,j}). \tag{15}$$

It is not surprising to find that (15) actually is the first-order Taylor series expansion of $z_{i,j}$ about $(\bar{x}_{i,j}, \bar{u}_{i,j})$.

The *a priori* mean and variance of $x_{i,j}$ are computed from (12) and are given by

$$\bar{x}_{i,j} = \bar{z}_{i,j}/\bar{u}_{i,j} \tag{16}$$

and

$$Q_{i,j} = \frac{\mathrm{var}(z_{i,j}) + \bar{z}_{i,j}^2}{\sigma_2^2 + \bar{u}_{i,j}^2} - \bar{x}_{i,j}^2 \tag{17}$$

in which $\mathrm{var}(z_{i,j})$ is the variance of $z_{i,j}$. The quantities $\bar{z}_{i,j}$ and $\mathrm{var}(z_{i,j})$ are approximated by the local mean and local variance of the corrupted image. Using (16) and (17), and applying the Kalman filtering algorithm to (15), we have

$$\hat{x}_{i,j} = \bar{x}_{i,j} + k_{i,j}(z_{i,j} - \bar{u}_{i,j}\bar{x}_{i,j}), \tag{18}$$

in which

$$k_{i,j} = \frac{\bar{u}_{i,j} Q_{i,j}}{\bar{x}_{i,j}^2 \sigma_2^2 + \bar{u}_{i,j}^2 Q_{i,j}}. \tag{19}$$

An experimental example is shown in Fig. 4, in which the original image is corrupted by multiplicative noise uniformly distributed between 0.7 and 1.0, and the estimated image has been processed by the algorithm developed in this section. Considerable improvement is shown in the processed image, thus substantiating the effectiveness of the local mean and variance technique.

V. Filtering of Combined Additive and Multiplicative Noise

It is very easy to extend the algorithms of previous sections to deal with images corrupted by both additive and multiplicative noise. A noise-corrupted image is described by

$$z_{i,j} = x_{i,j} u_{i,j} + w_{i,j} \tag{20}$$

in which the statistical characteristics are the same as given in Sections III and IV. Assume that $u_{i,j}$ and $w_{i,j}$ are independent white noises. This independence assumption can be removed, but the result is a more complicated formulation. Following the idea of an optimal linear approximation of Section IV, we have

$$z'_{i,j} = \bar{u}_{i,j} x_{i,j} + \bar{x}_{i,j}(u_{i,j} - \bar{u}_{i,j}) + w_{i,j}.$$

The formulas for the *a priori* mean and variance of $x_{i,j}$ of Section IV are modified to read

$$\bar{x}_{i,j} = (\bar{z}_{i,j} - \bar{w}_{i,j})/\bar{u}_{i,j} \tag{21}$$

and

$$Q_{i,j} = \frac{\operatorname{var}(z_{i,j}) + \bar{z}_{i,j}^2}{\sigma_2^2 + \bar{u}_{i,j}^2} - \bar{x}_{i,j}^2 - \sigma_1^2.$$

The filtering algorithm is

$$\hat{x}_{i,j} = \bar{x}_{i,j} + k_{i,j}(z_{i,j} - \bar{u}_{i,j}\bar{x}_{i,j} - \bar{w}_{i,j}) \tag{22}$$

in which

$$k_{i,j} = \frac{\bar{u}_{i,j} Q_{i,j}}{\bar{x}_{i,j}^2 \sigma_2^2 + \bar{u}_{i,j}^2 Q_{i,j} + \sigma_1^2}. \tag{23}$$

Fig. 5(a) shows the image corrupted by an additive noise uniformly distributed between gray levels −20 and +20 and also a multiplicative noise uniformly distributed between multiplicative factors 0.7 and 1.0. The processed image, Fig. 5(b) shows a very significant improvement over the original image.

VII. Remarks and Conclusions

The principal computational load of the developed algorithms is in the calculation of the local means and variances, especially the latter. To lighten this burden Wallis [8] proposed a fast algorithm in which the image is partitioned into square subregions over which the local mean and variance are computed. Then the local mean and variance of a particular pixel are approximated by the use of two-dimensional interpolation formulas. Results, as reported by Wallis [8], are quite favorable. It is believed that Wallis' approach will yield an equally impressive improvement when applied to our contrast enhancement and noise filtering algorithms.

(a) (b)

Fig. 4. (a) Multiplicative noise, $U(0.7, 1.0)$. (b) Multiplicative noise removed.

(a) (b)

Fig. 5. (a) Image corrupted by additive and multiplicative noise. (b) Restored.

In conclusion, image processing algorithms presented in this paper, based on considerations of the local image statistics, have a structure which makes them naturally suitable for parallel processing. Since the latter approach offers great computational economy, real or near real-time processing can be achieved. Future research in this area is to extend the method to image restoration of motion blur and other degradations characterized by local correlations around pixels.

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