Tensor Voting

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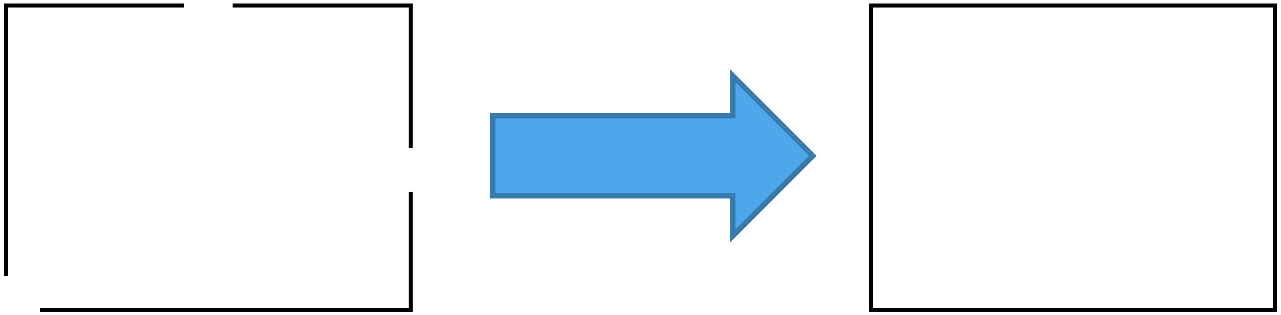


- Use features to strengthen each other and to expand features into possible nearby locations
  - Represent local feature by tensor
  - Every pixel (with a feature) cast a vote to each nearby pixel
  - “Nearby” is defined by a voting field aligned with the local pixel
  - The vote is a tensor generated according to the local tensor and the position in the voting field
  - All the votes are added to generate the new local tensor

1. Tensor Representation
2. Voting Field
3. Tensor Voting
4. Efficient Tensor Voting

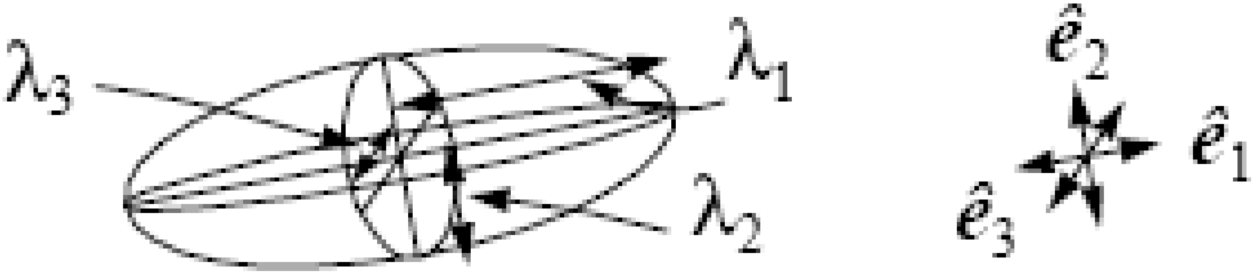
- **From Wikip dia**

  - **Tens** etric uced into mathem the notion of scalars, rices. Many physical qua ly regarded not as vectors th pondences betw e er. For examp ss tenso vector as input an duces another v as output and so expresses a relationship between the input and output vectors.

- Only second order tensor is used, which is an n by n matrix.
  - n is the dimension of the image.
- Can be expended into following format

$$T_{n\times n} = \sum_{i=1}^{n} \lambda_i \mathbf{e}_i \mathbf{e}_i^T$$

- Can be illustrated as a ellipsoid
- For example, the 3D case: [Medioni 2000]



- Parameterization (consider eigen values are in descending order)

$$\alpha_1 = \lambda_1 - \lambda_2$$

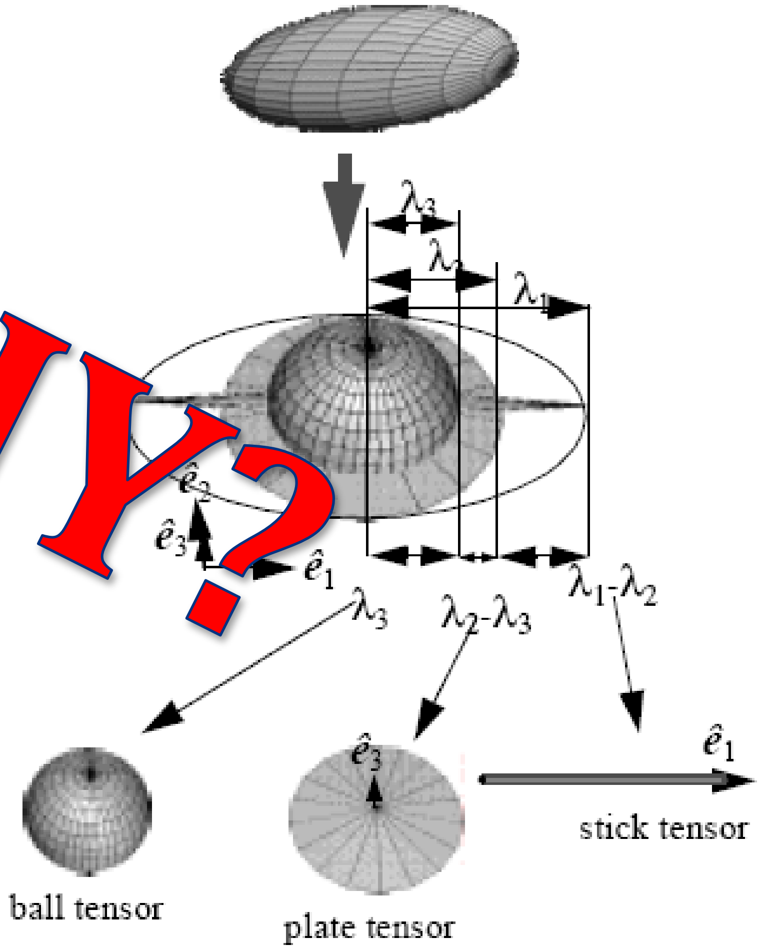
$$\alpha_2 = \lambda_2 - \lambda_3$$

$$\ldots$$

$$\alpha_{n-1} = \lambda_{n-1} - \lambda_n$$

$$\alpha_n = \lambda_n$$

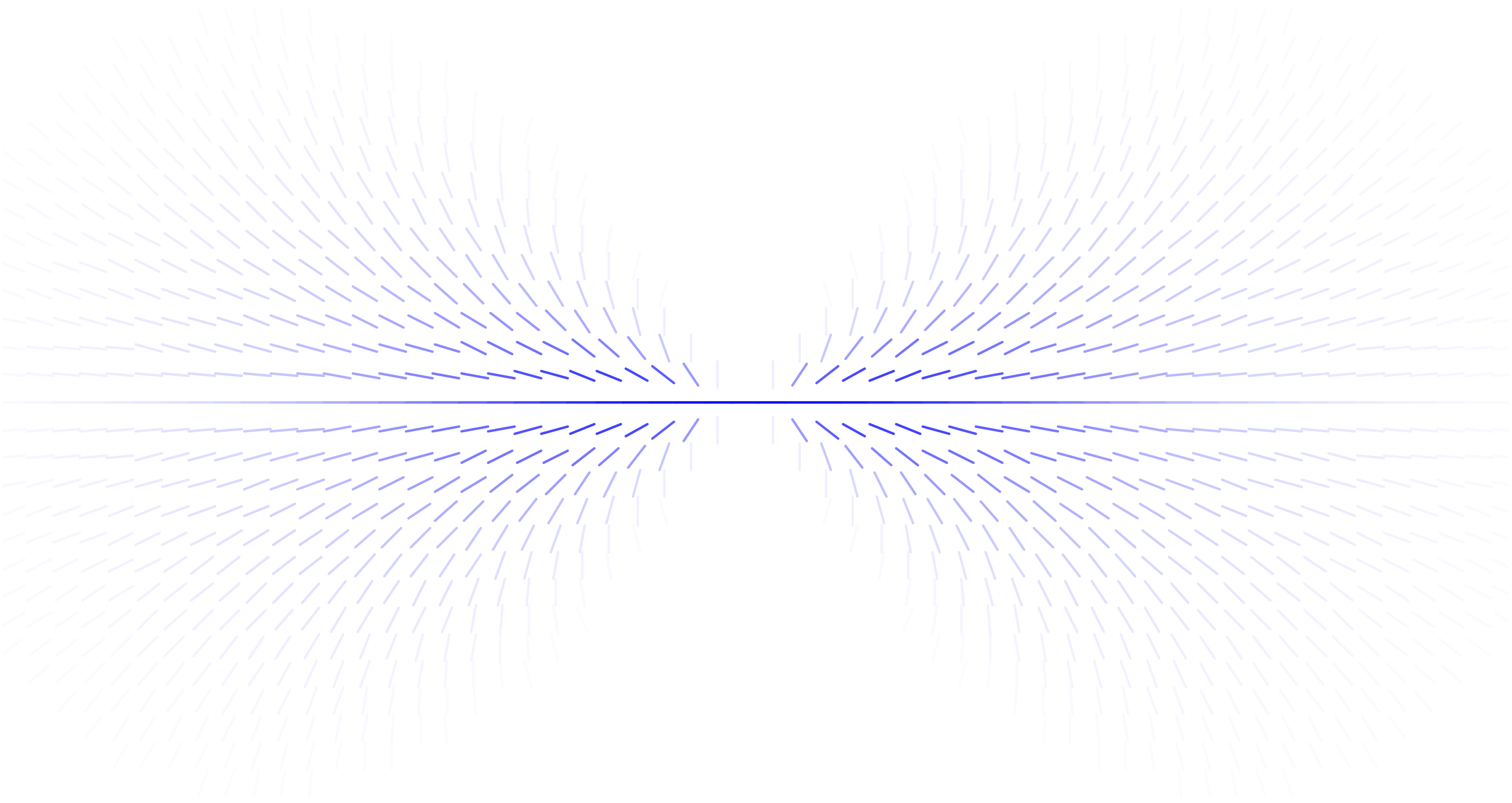
WHY?



3D illustration from [Medioni 2000]

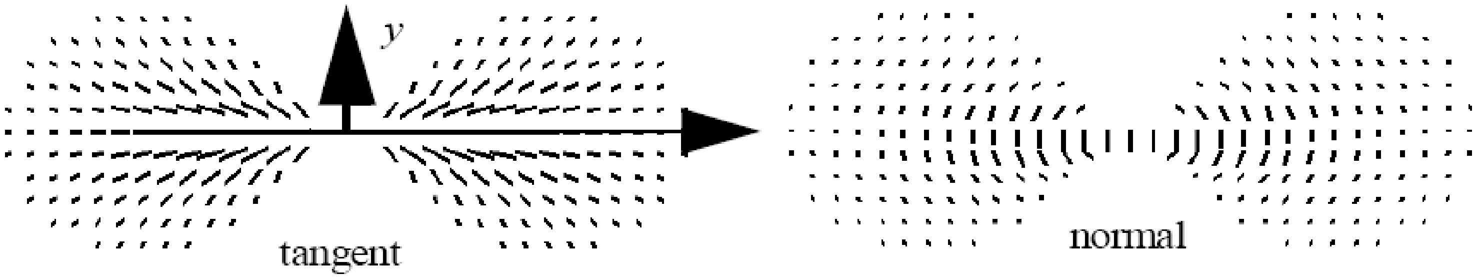
- Understand the parameterization, 3D case
  - α_1 shows the directional saliency (stickness), $\mathbf{e}_1$ shows the direction. In most case these shows the strength and the orientation of the local edge/flux.
  - The other α_i shows the i-dimensional “ballness”, which shows intersection or possible local i-dimensional plane.
  - Because solving $\mathbf{e}_i$ involves arcsin or arccos, eigen vector orientations are limited to $[0, \pi]$ or $[-\pi / 2, \pi / 2]$

- A typical voting field:



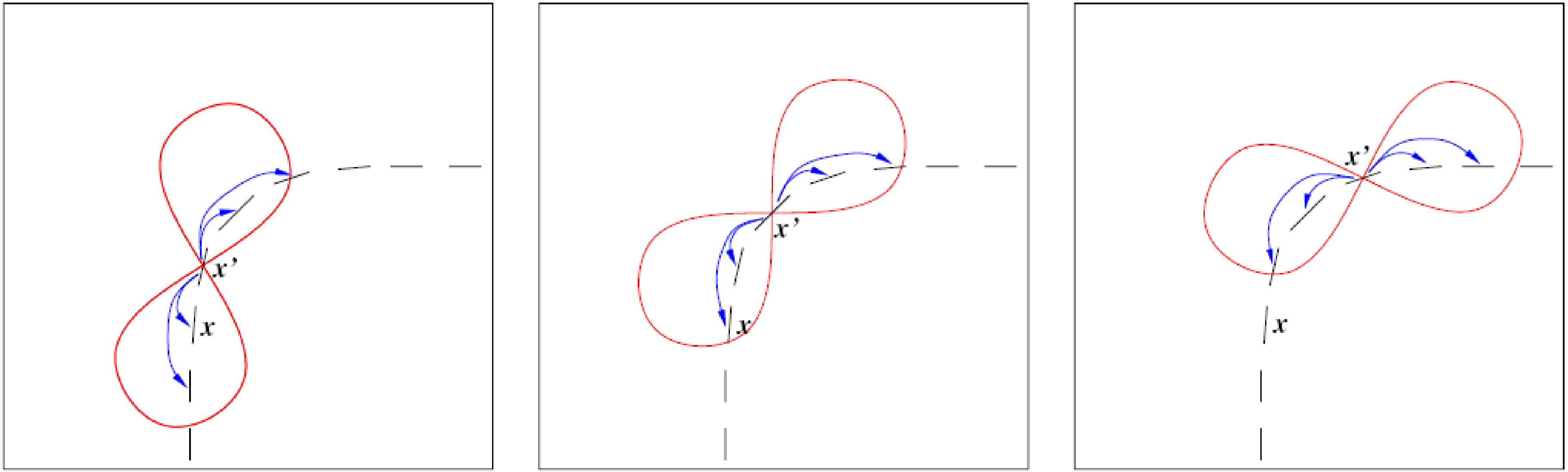
- The features should be smooth
  - The direction within the vote should be smooth relative to the local tensor
- Less impact to the pixels far away
  - The saliency within the vote should decrease with distance
- Affected by tensor representation
  - For example

Image is from [Franken 2006]

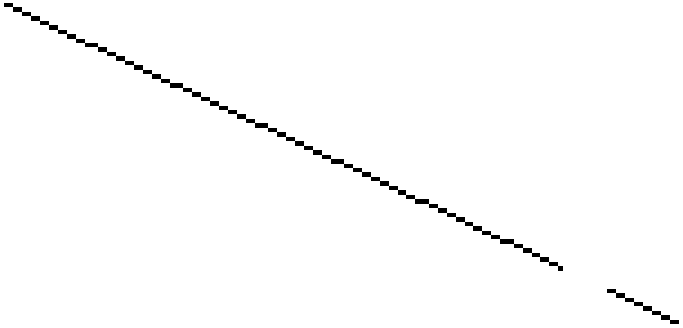


- General Idea: use local feature to strengthen each other and to expand into possible locations

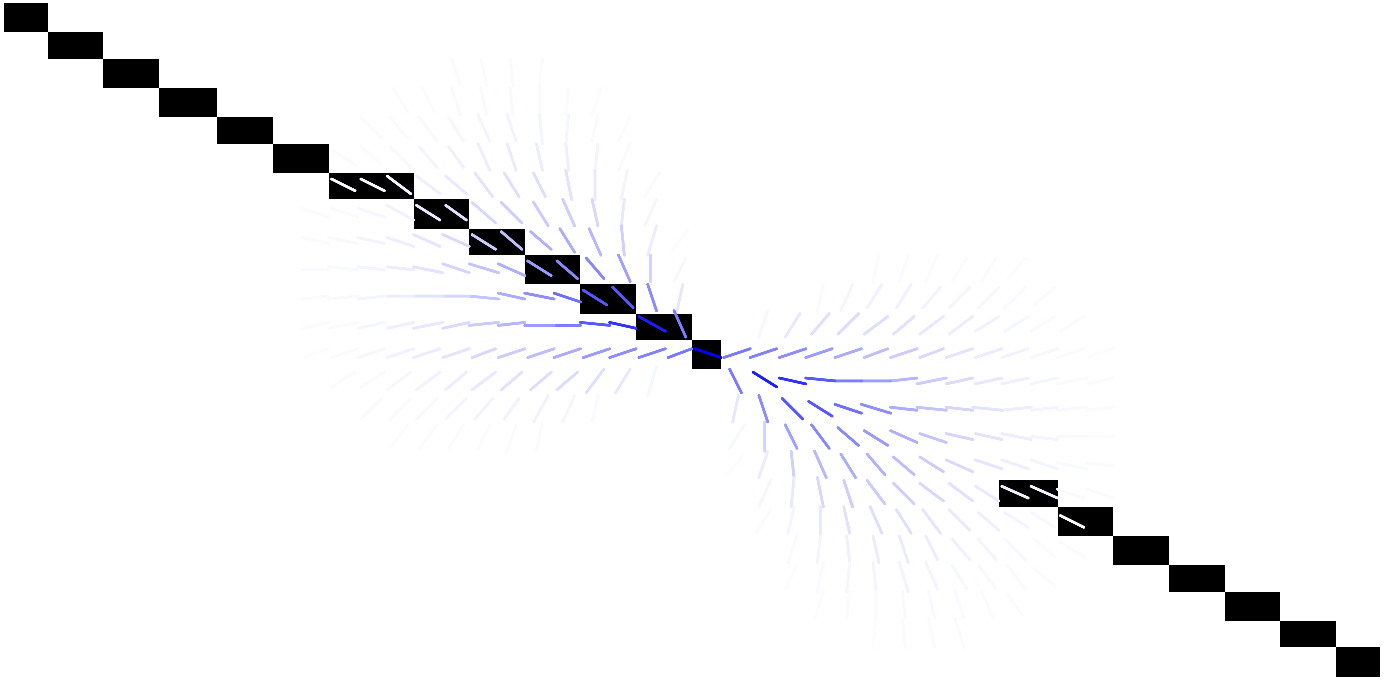
Image is from [Franken 2006]



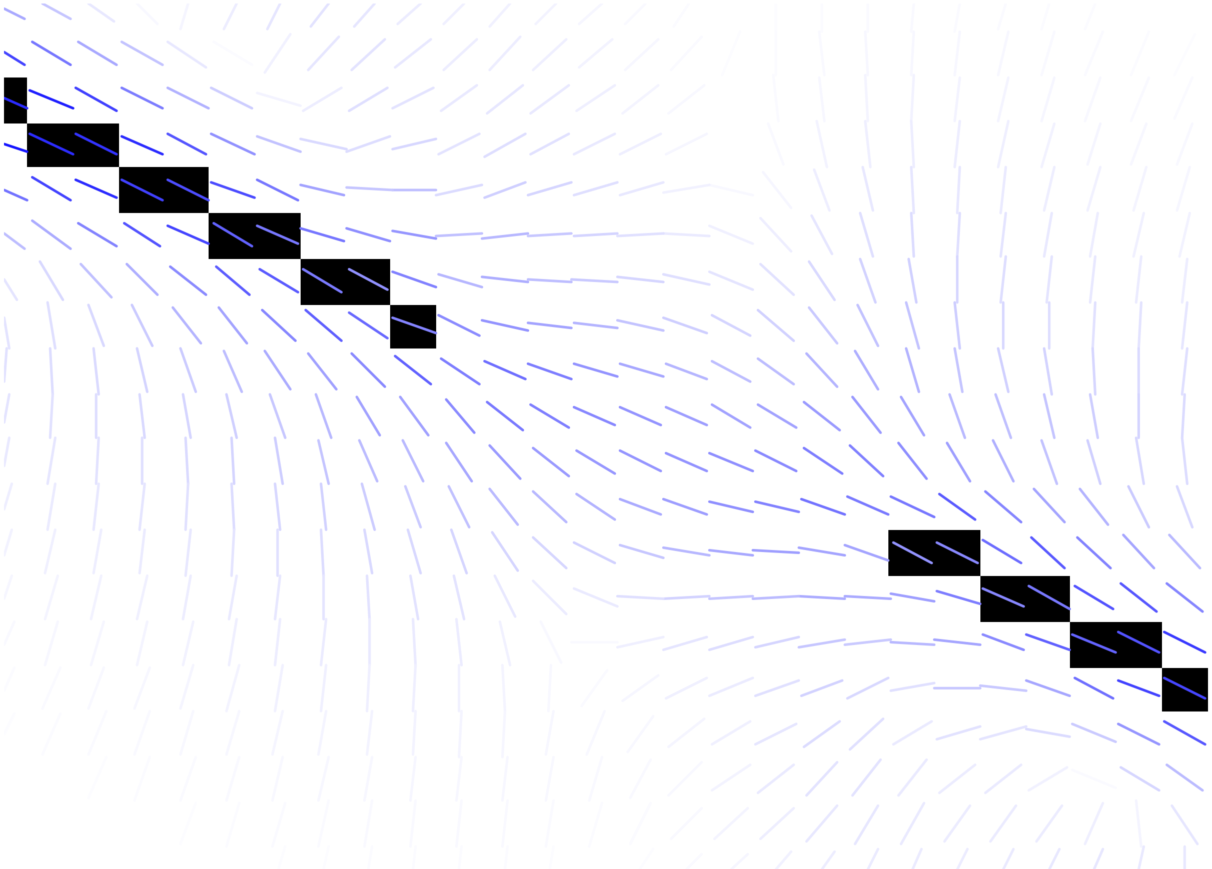
- Original Image



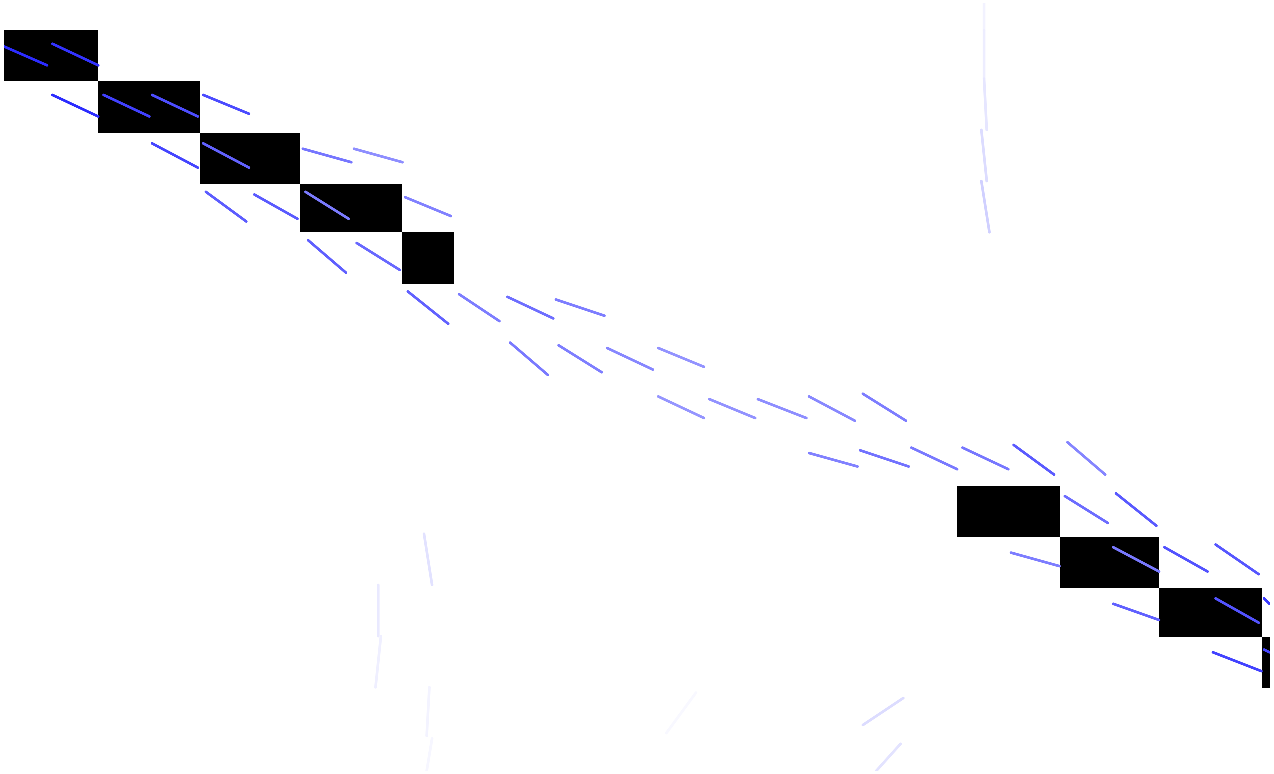
- Cast the vote



- Sum all the votes

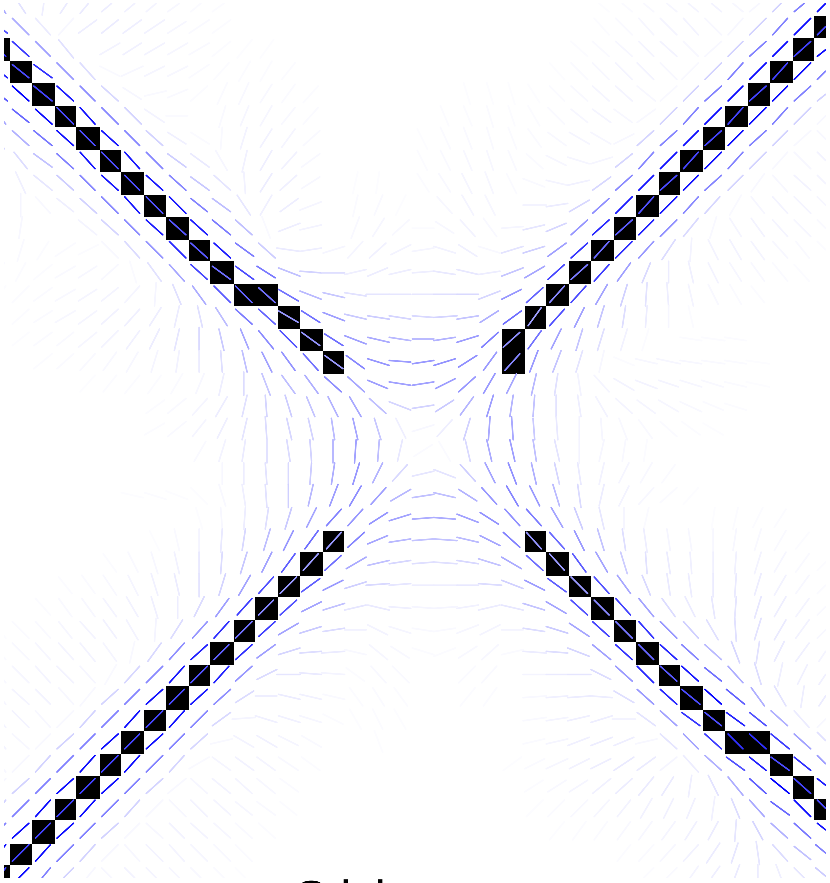


- After non-maxima suppression

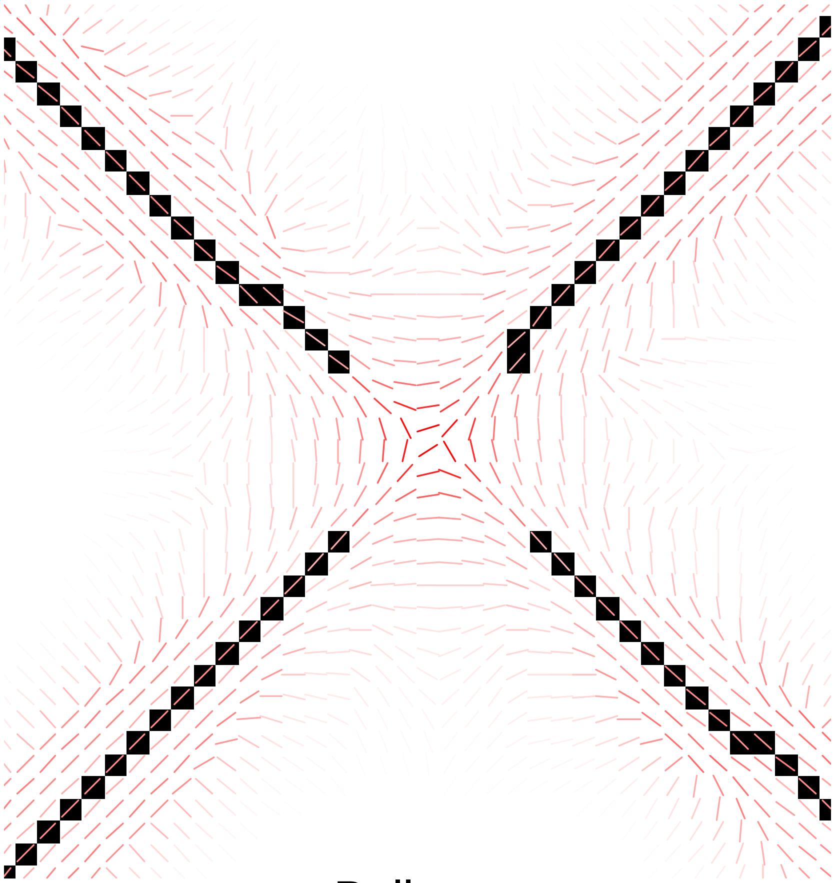


- Special Case: Intersection
  - The votes with similar saliencies and near perpendicular orientations will cancel each other, so the saliency of intersection will be very low.
  - But the ballness will be very significant
  - Better identification of intersection involves farther comparison between stickness and ballness.

- Special Case: Intersection

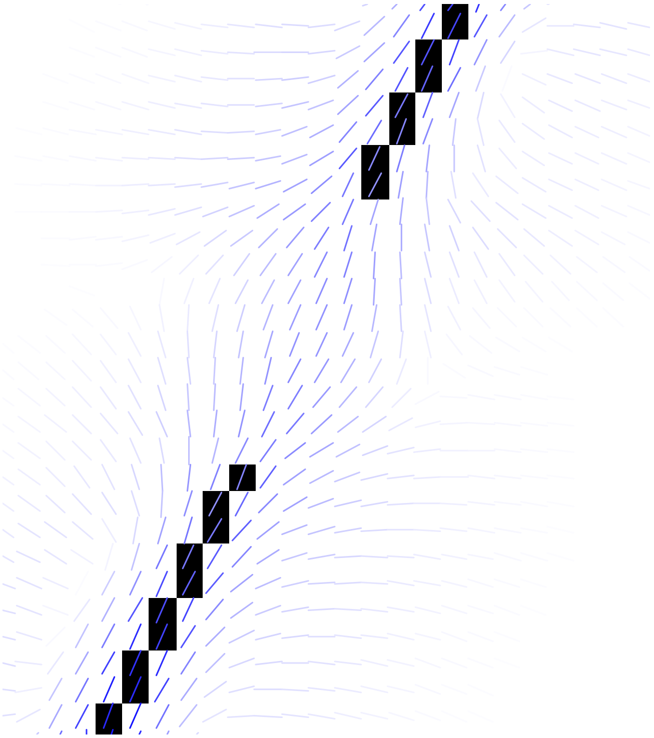


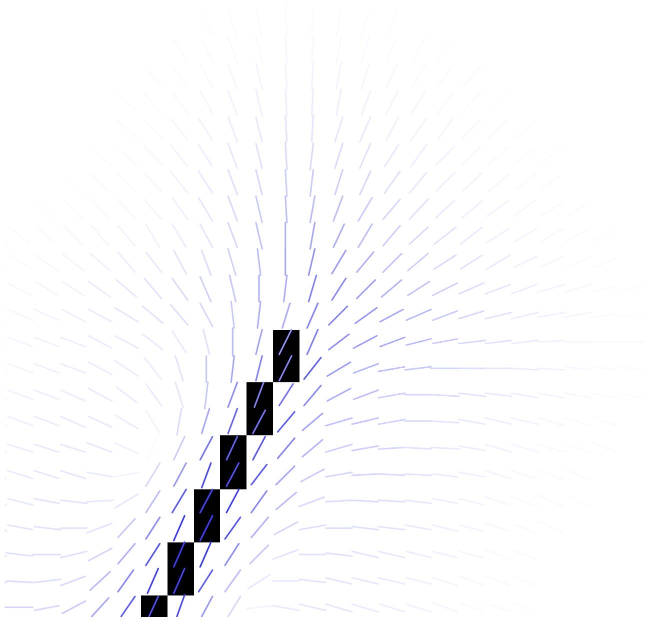
Stickness



Ballness

- Special Case: End Point
  - Tensor voting tends to expand open ends
  - Cannot Tell whether it is an opening or open end only by local saliency.





- Find out whether this is an end point
  - End point voting:
    - a vector voting (not tensor voting)
    - each vote is the vector directing from the casting pixel to the receiving pixel
    - the sum of all vote should be near zero for an end point and non-zeros for non-end point
    - weight of a vote is based on the tensor voting field
- Remove new end point
  - Compare the end points found in original tensor image and the one after tensor voting;
  - Eliminate the end points that only appears in new image.

- So far, tensor voting is time consuming, because all the operations are in space domain

How about do it in frequency domain?

- Steerable Filter (good for 2D)
  - By decompose a filter into several “major component”, the filter can be rotated easily with the weighted sum of the major components.
- Spherical Harmonics (good for higher order)
  - Tensors can be decomposed into spherical harmonics, which can be easily rotated.
- Alignment is then not necessary, tensor voting can be done with convolution
- Complexity can be reduced from $O(N^2M^2)$ to $O(N^2)$

- G Medioni, CK Tang, MS Lee. “Tensor Voting: Theory and Applications.” *Proceedings of RFIA*. Paris, France. 2000
- E Franken, M Almsick, P Rongen, L Florack, B Romany. “An Efficient Method for Tensor Voting Using Steerable Filters ”, *Medical Image Computing and Computer-Assisted Intervention*. 2006

Questions?

Thank You !