Geodesic Snakes

Level-Set Evolution

CS7960
Advanced Image Processing
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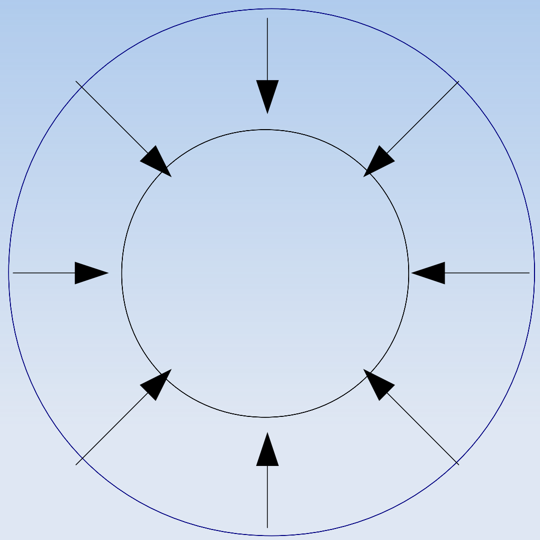
Motivation

- Drawbacks of previous Snake formulations:
  - Explicit Representation
    - Parameterization / Reparameterization issues
    - Approximating Discrete Derivatives
  - Fixed Topology
  - Extention to 3D very complex (active meshes)

Motivation

- New Approach:
  - Embed contour in higher order surface
  - Implicit Representation
  - Insensitive to Topology
  - Easily extends to 3D

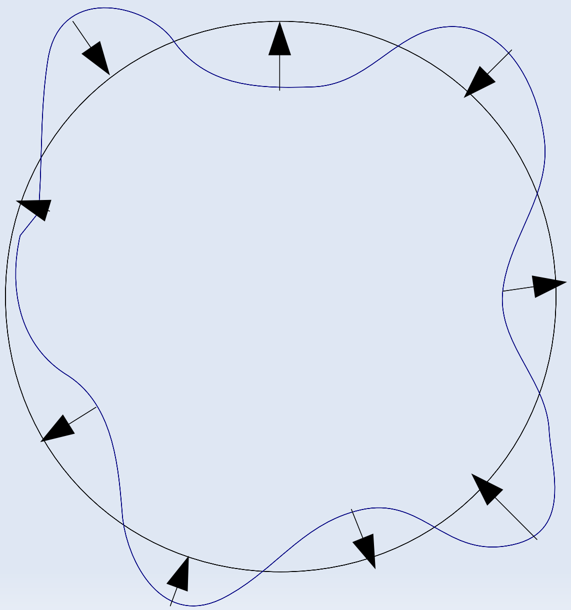
Mathematical Framework



$$\frac{dC}{dt} = \beta \vec{N}$$

β : Speed

$\vec{N}$: Normal



$$\frac{dC}{dt} = \kappa \vec{N}$$

κ : Curvature

Mathematical Framework

- Combining terms simple:
  - $$\frac{dC}{dt} = (\beta + \kappa)\vec{N}$$
- Still want:
  - Ability to slow/stop on edges/lines/etc
  - Image force term
  - $$\frac{dC}{dt} = g(I)(\beta + \kappa)\vec{N}$$
- Where have we seen this before?

Mathematical Framework

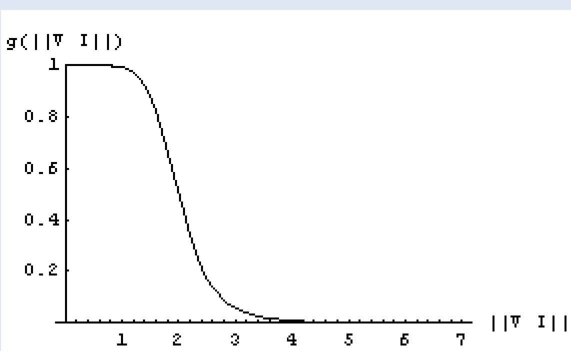
- Anisotropic Diffusion (Perona & Malik)
  - Use gradient magnitude for diffusion speed

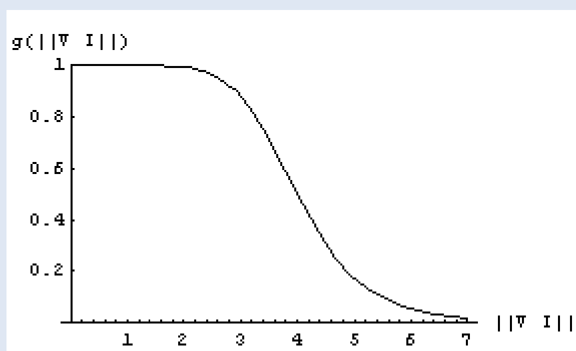
$$g(I) = \frac{1}{1 + \|\nabla \hat{I}\|^2}$$

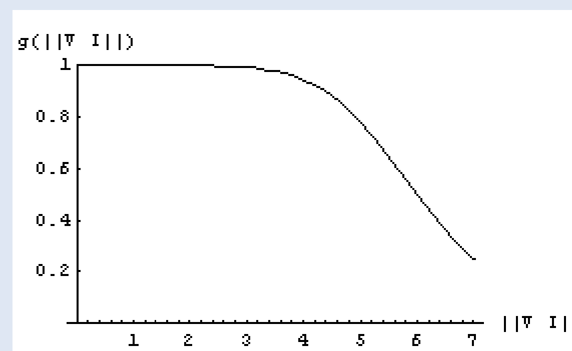
(Quadratic)

$$g(I) = e^{-\left(\|\nabla \hat{I}\|^2\right)}$$

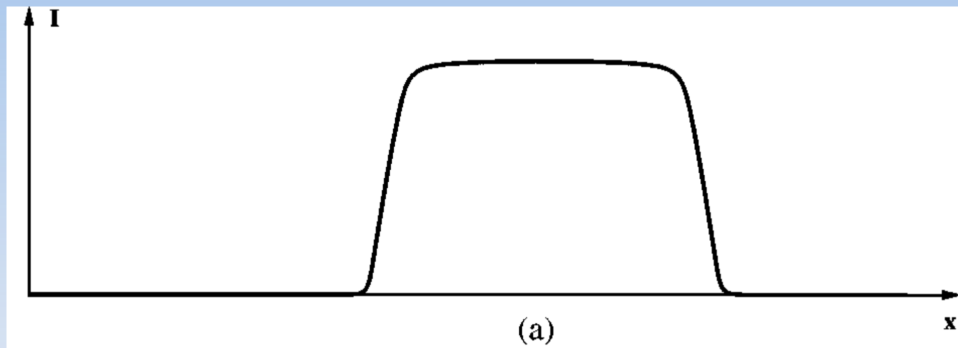
(Exponential)

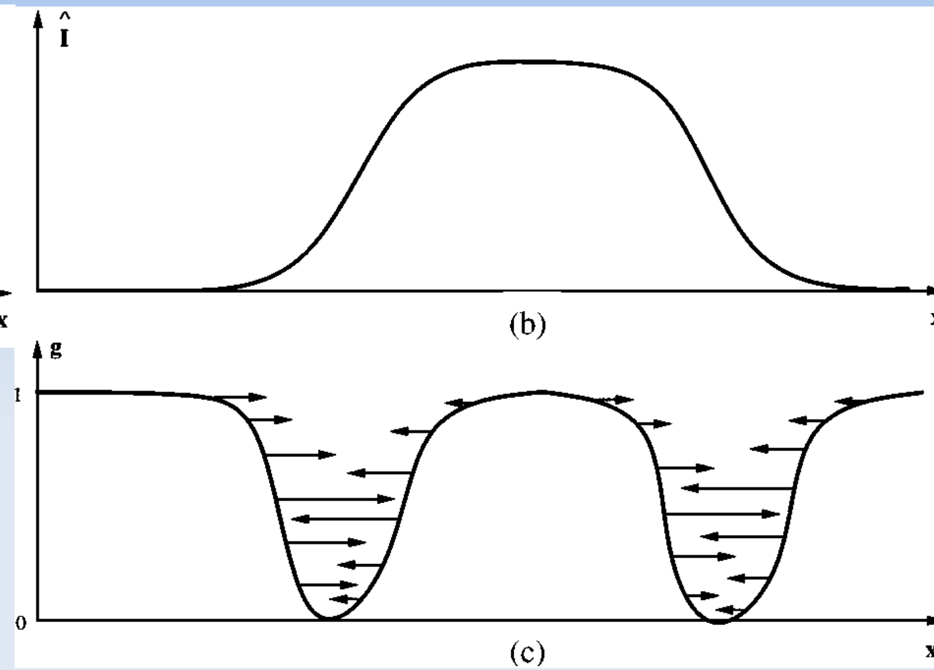






Mathematical Formulation





- What if we overshoot?
- Want to pull toward edges

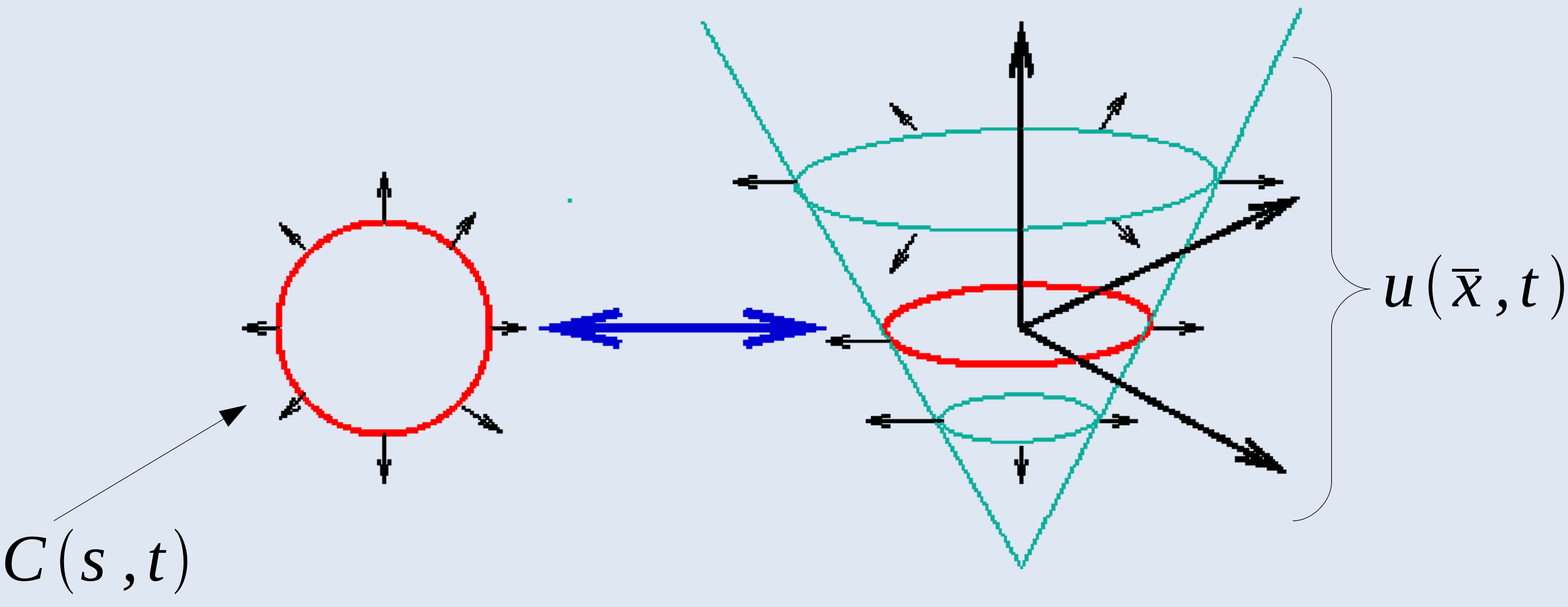
$$\frac{\partial C}{\partial t} = g(I)(\beta+\kappa)\vec{N} - \underbrace{\left(\nabla g(I)\cdot\vec{N}\right)\vec{N}}_{\text{Advection Term}}$$

Embedding Contour $C(s,t)$ into Surface $u(x,t)$

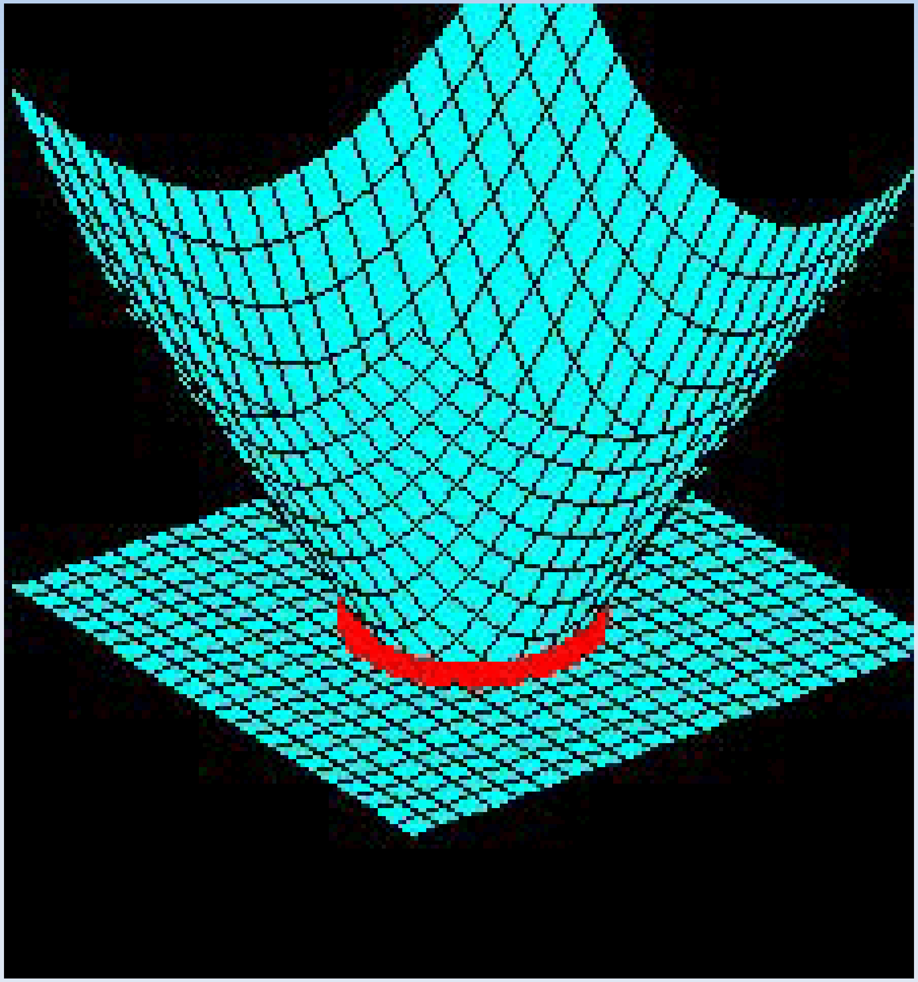
Embedding

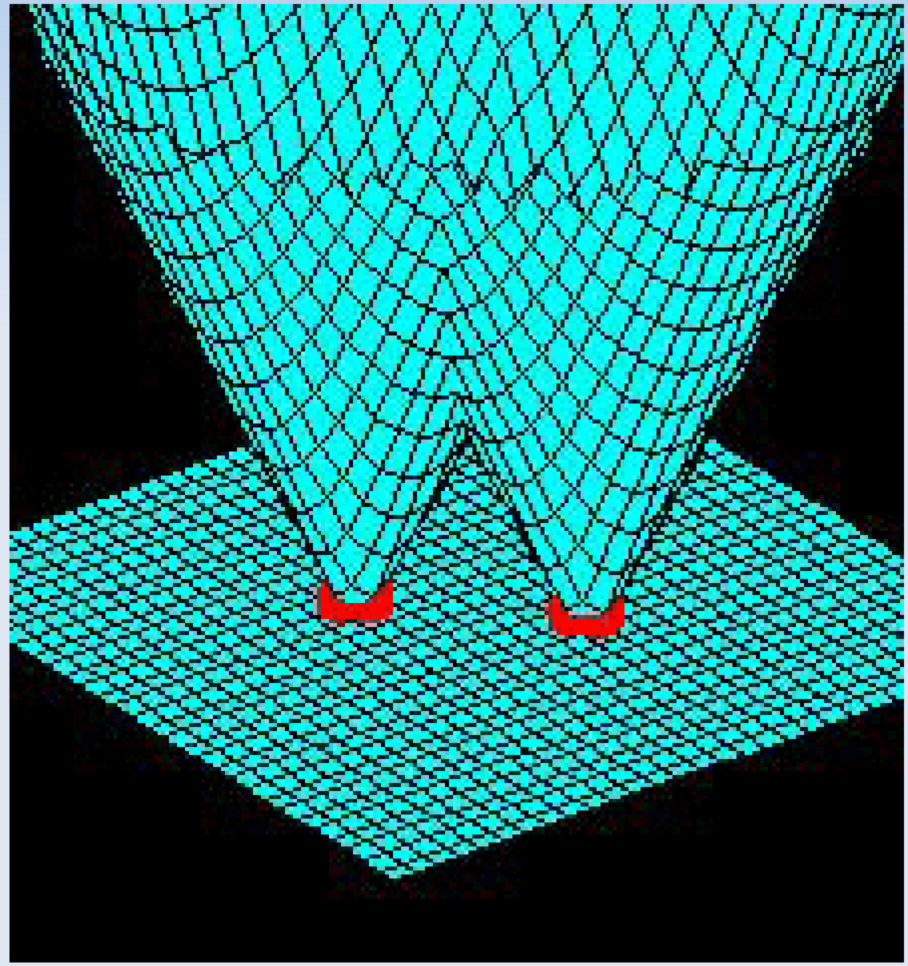
- Embedding function: $u(\bar{x}, t)$ $u \in \Re^3$
- Contour: $C(s, t) \longrightarrow u(C, t) = 0$ $C \in \Re^2$

(Zero level-set)



Embedding





Embedding Formulation

- How does surface vary over time? $u(C(t),t)=0$

$$\frac{d}{dt}u(C(t),t)=\frac{\partial u}{\partial t}+\left(\frac{\partial C}{\partial x}\frac{\partial x}{\partial t}\right)+\left(\frac{\partial C}{\partial y}\frac{\partial y}{\partial t}\right)+\left(\frac{\partial C}{\partial z}\frac{\partial z}{\partial t}\right)$$

Chain Rule

$$\frac{d}{dt}u(C(t),t)=\frac{\partial u}{\partial t}+\nabla u\cdot\frac{dC}{dt}$$

$$\frac{dC}{dt}=\beta\vec{N}=\beta\frac{-\nabla u}{|\nabla u|}$$

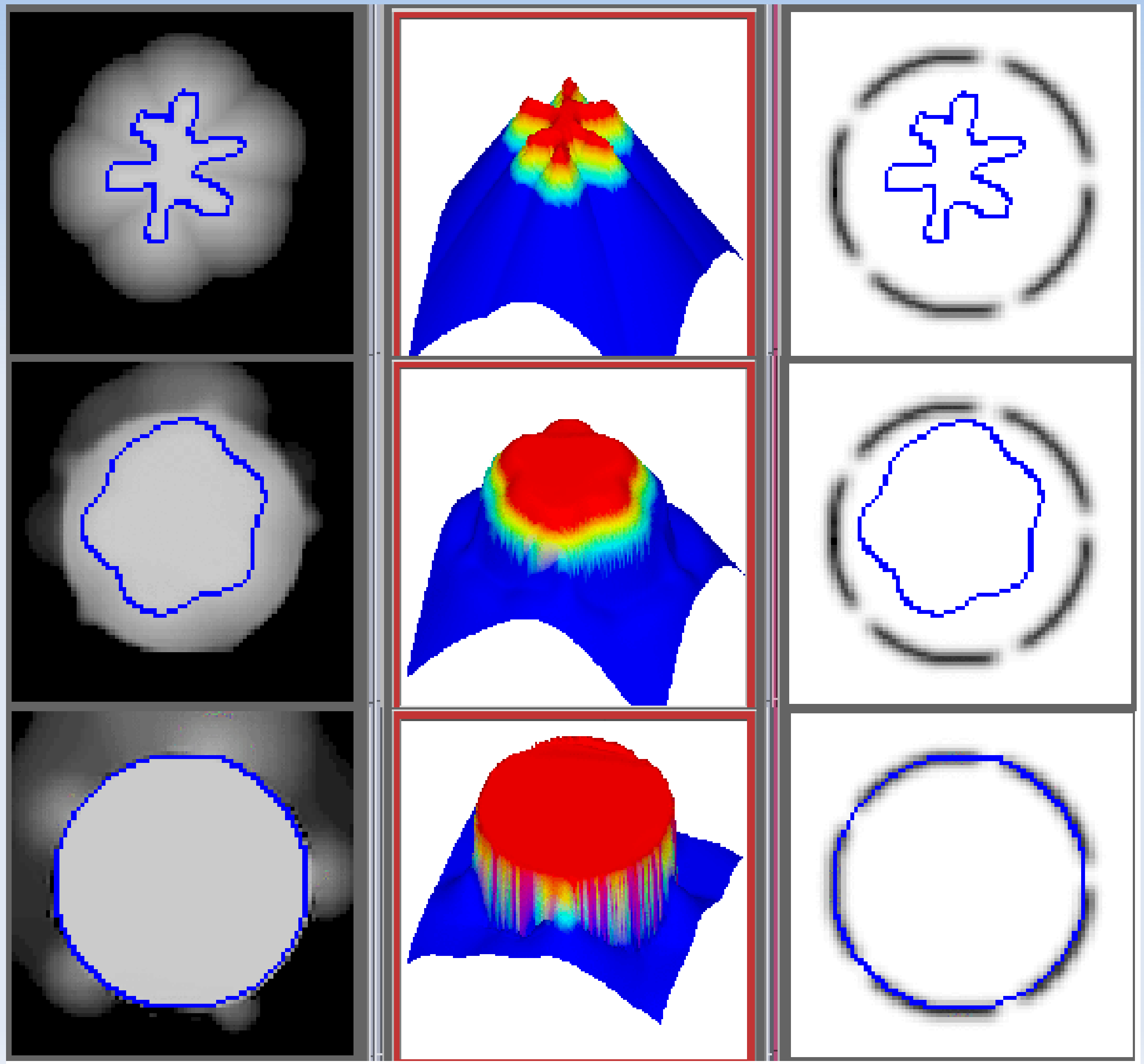
$$\frac{d}{dt}u(C(t),t)=\frac{\partial u}{\partial t}-\beta|\nabla u|=0$$

$$\frac{\nabla u\nabla u}{|\nabla u|}=|\nabla u|$$

$$\frac{\partial u}{\partial t}=\beta|\nabla u|$$

Hamilton-Jacobi Equation for certain speeds β

Interpretation

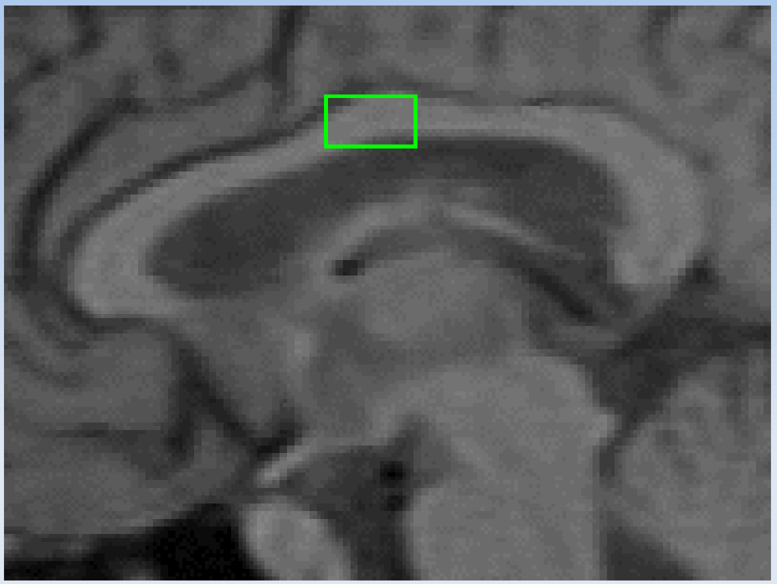


Summary

- Implicit Solution
- Solvable using PDE's (stable)
  Parameterization Free
- Seamlessly handles Topological Changes
- Extends to 3D in Straightfoward Manner

- Common Implementations
  - Fast Marching Method
  - Fast Iterative Method

Examples



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