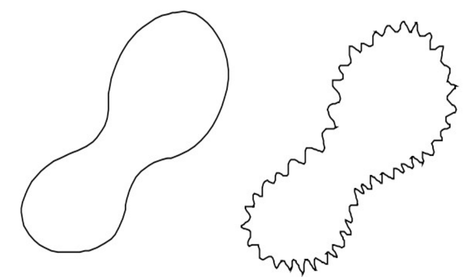
Continuous Medial Representations for Geometric Object Modeling

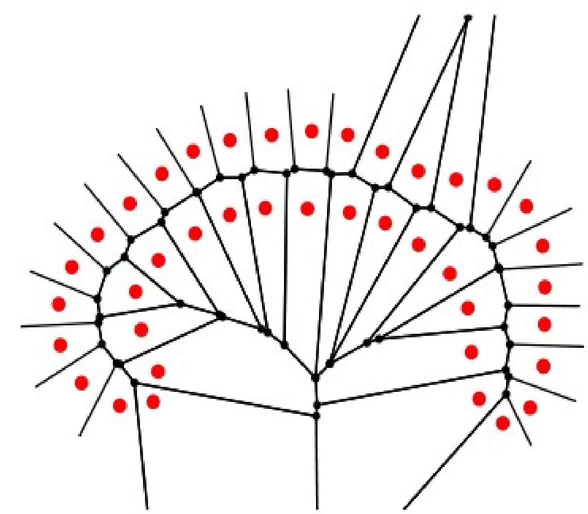
Wei Liu

Mar 18, 2010

Methods to define skeleton of object

- maximal inscribed ball
- Hierarchal Voronoi
- Shock of boundary evolution
- core tracking.
- m-rep
- cm-rep

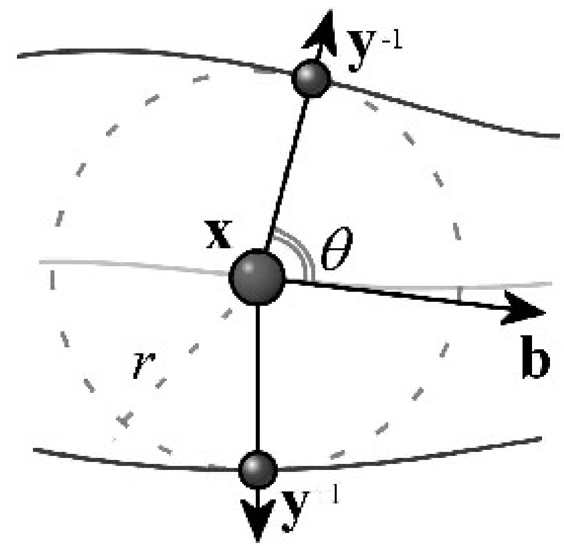




b.

M-rep: Inverse skeletonization

- Traditional: the intersection of the normal.
- Now: a medial point with two boundary point.



a.

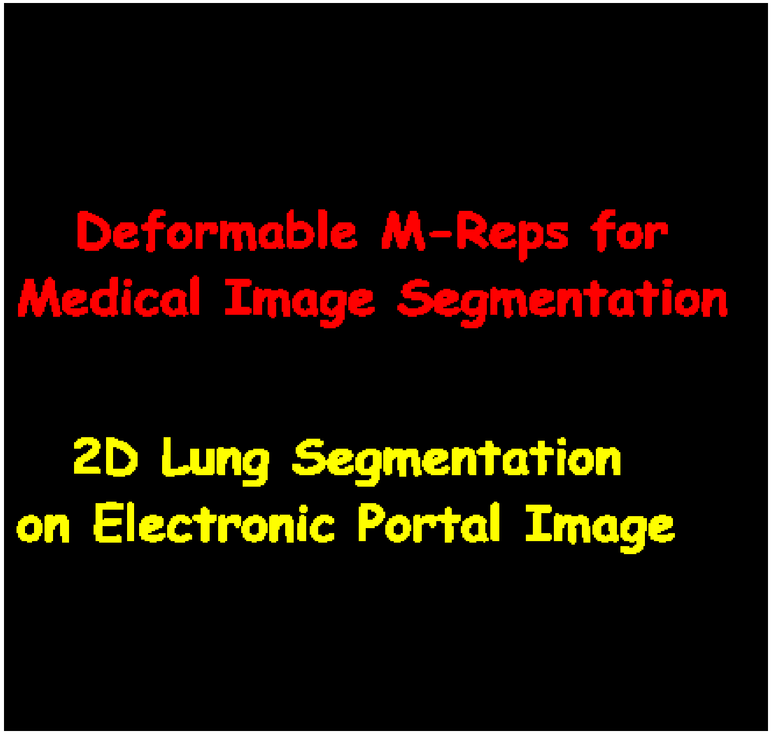
$$\mathbf{U}^{\pm 1} = \mathbf{R} \begin{bmatrix} \cos(\theta) \\ \pm \sin(\theta) \end{bmatrix},$$

$$m = \{\mathbf{x}, r, \mathbf{R}, \theta\}$$

$$\mathbf{y}^{\pm 1} = \mathbf{x} + r\mathbf{U}^{\pm 1},$$

M-rep: sampling

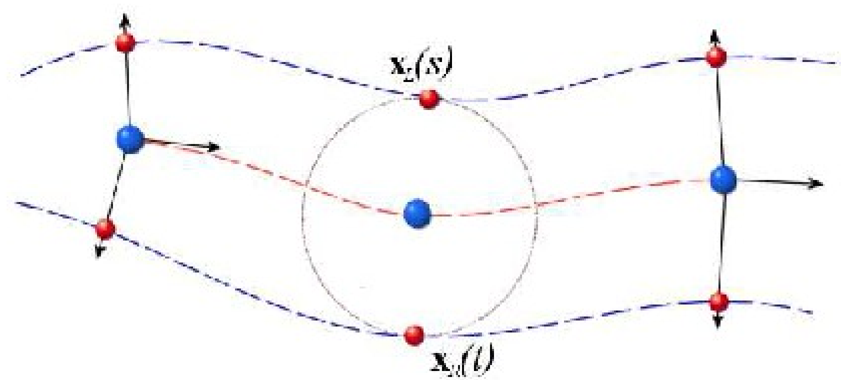
- Medial point are sampled on the axis.
- A sparse representation.
- Sampled medial points are fixed relative to the axis.

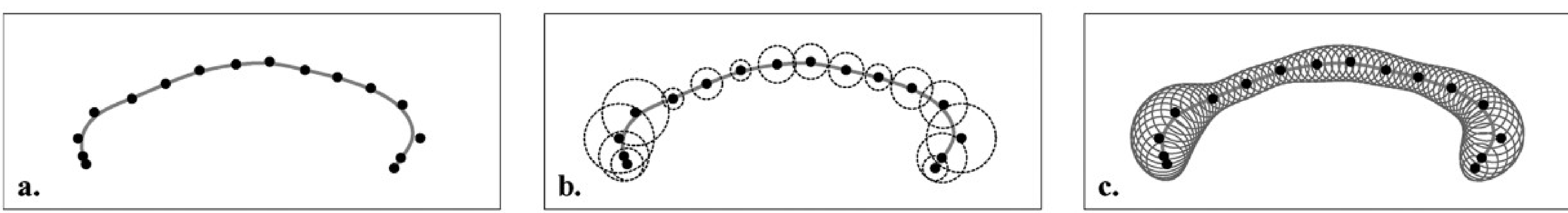


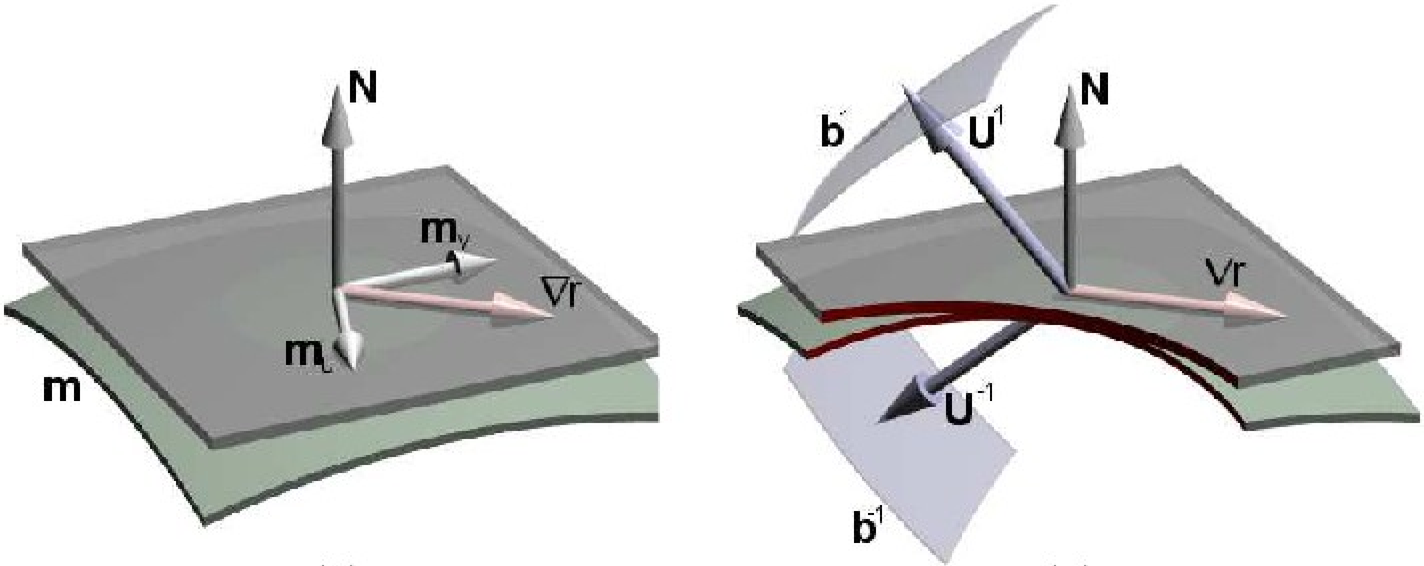
http://midag.cs.unc.edu/defmreps/jl3_movie2.gif

Continuous m-rep: cm-rep

- Can move the medial points along the axis.
- Generate a synthetic given set of b-spline control points.
- Maximize overlapping between this shape and template by adjusting control point.



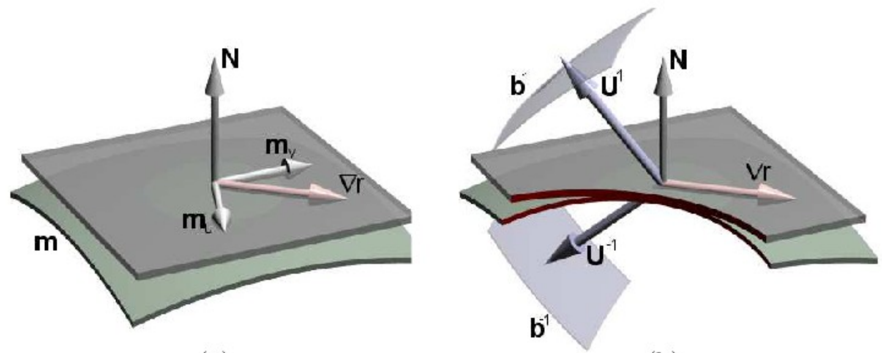




(a) (b)

$\mathbf{m}$	Medial surface
r	Radial scalar field
u, v	Parametrization of $(\mathbf{m}, r)$
$\mathbf{b}^t$	Boundary counterparts of $(\mathbf{m}, r)$
t	Indexes the two parts $(-1, 1)$ of the implied boundary.
$\mathbf{u}^t$	Unit normal to the boundary, also the direction from a point on $\mathbf{m}$ to its boundary counterpart.
$\mathbf{n}$	Unit normal to the medial surface.

$\mathbf{m}$	Medial surface
r	Radial scalar field
u, v	Parametrization of $(\mathbf{m}, r)$
$\mathbf{b}^t$	Boundary counterparts of $(\mathbf{m}, r)$
t	Indexes the two parts $(-1, 1)$ of the implied boundary.
$\mathbf{u}^t$	Unit normal to the boundary, also the direction from a point on $\mathbf{m}$ to its boundary counterpart.
$\mathbf{n}$	Unit normal to the medial surface.



(a) (b)

$$f(\mathbf{x}, u, v) = |\mathbf{x} - \mathbf{m}(u, v)|^2 - r(u, v)^2 = 0$$

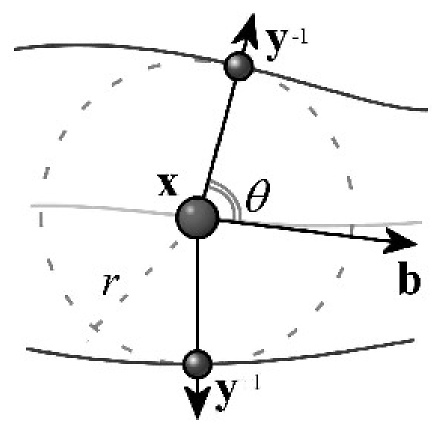
$$f = 0\ , \quad f_u = 0\ , \quad f_v = 0$$

$$\mathbf{b}^t \;=\; \mathbf{m} + r\mathbf{u}^t\ ,$$

$$\mathbf{u}^t \;=\; -\nabla r + t\sqrt{1 - \|\nabla r\|^2}\mathbf{n}$$

$$\nabla r = \left[\begin{array}{cc} \mathbf{m}_u & \mathbf{m}_v \end{array}\right] \mathbf{I}_{\mathbf{m}}^{-1} \left[\begin{array}{c} r_u \\ r_v \end{array}\right]$$

$$\|\nabla r\| \leq 1$$

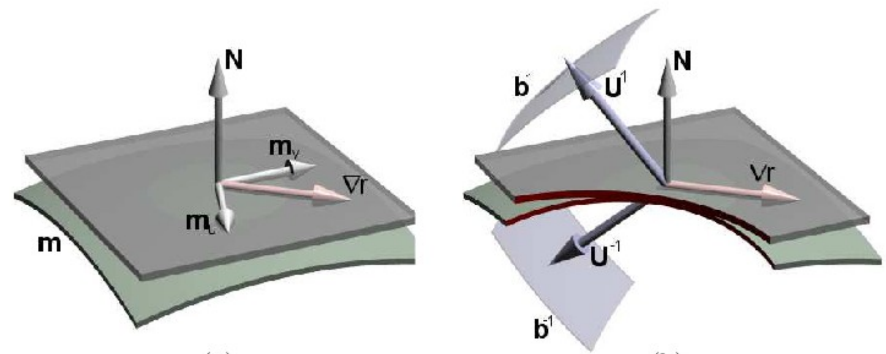


a.

•When ||del r|| = 0, meaning r does not change, U = N.

B-spline interpolation

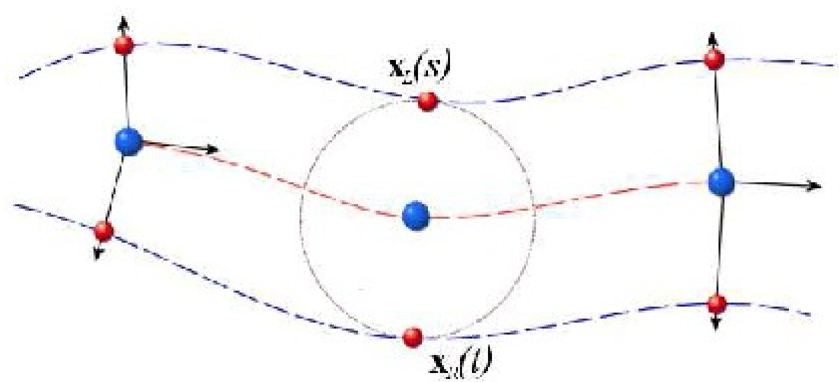
$\mathbf{m}$	Medial surface
r	Radial scalar field
u, v	Parametrization of $(\mathbf{m}, r)$
$\mathbf{b}^t$	Boundary counterparts of $(\mathbf{m}, r)$
t	Indexes the two parts $(-1, 1)$ of the implied boundary.
$\mathbf{u}^t$	Unit normal to the boundary, also the direction from a point on $\mathbf{m}$ to its boundary counterpart.
$\mathbf{n}$	Unit normal to the medial surface.



(a) (b)

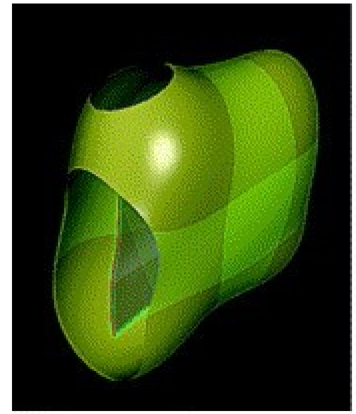
$$\mathbf{m}(u,v) = \sum_{i=0}^{d_1}\sum_{j=0}^{d_2} N_i^3(u)N_j^3(v)\bar{\mathbf{m}}_{ij}$$

$$r(u,v) = \sum_{i=0}^{d_1}\sum_{j=0}^{d_2} N_i^3(u)N_j^3(v)\bar{r}_{ij}$$

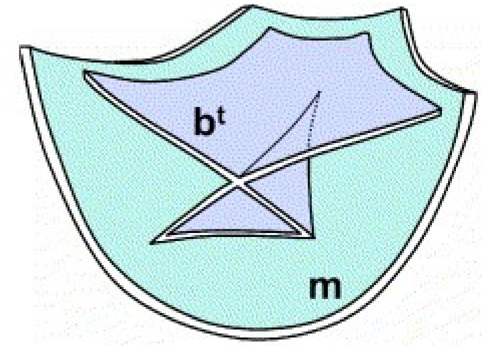


constraints

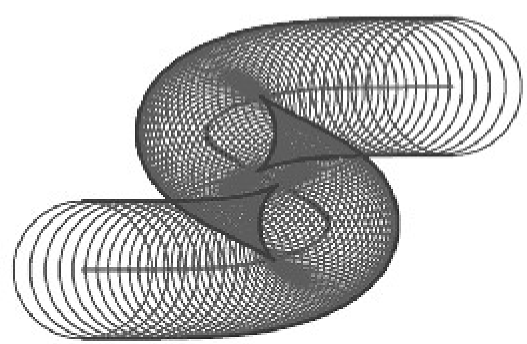
- Closed
- Connected
- non-singular



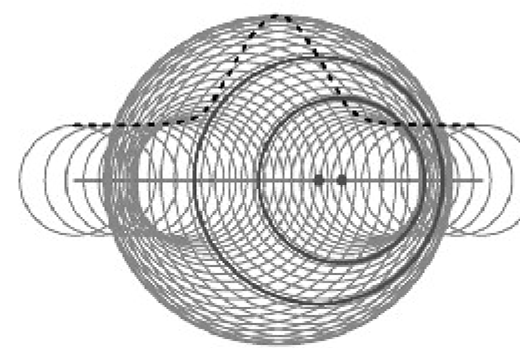
(a)



(b)

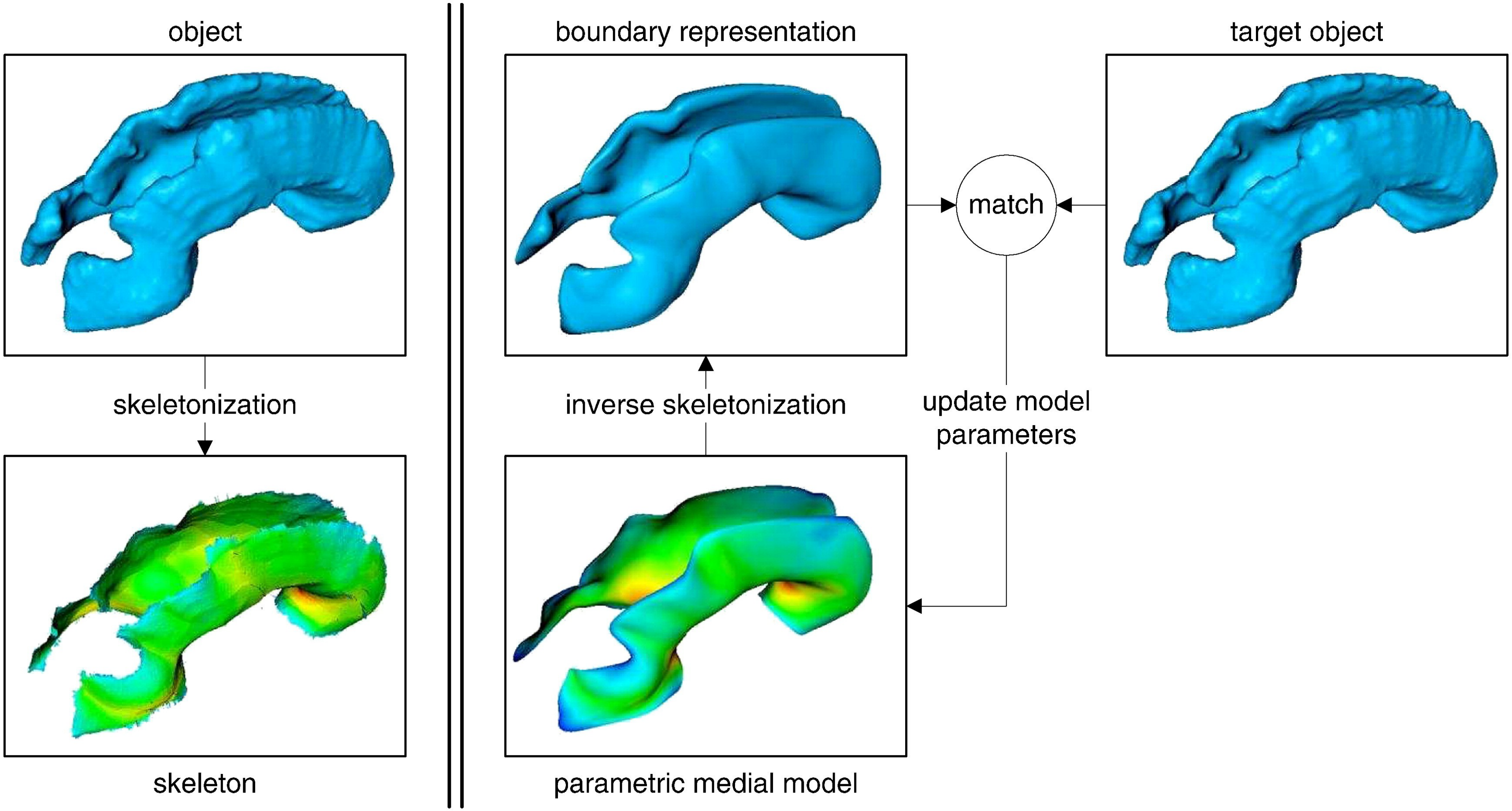


a.



b.

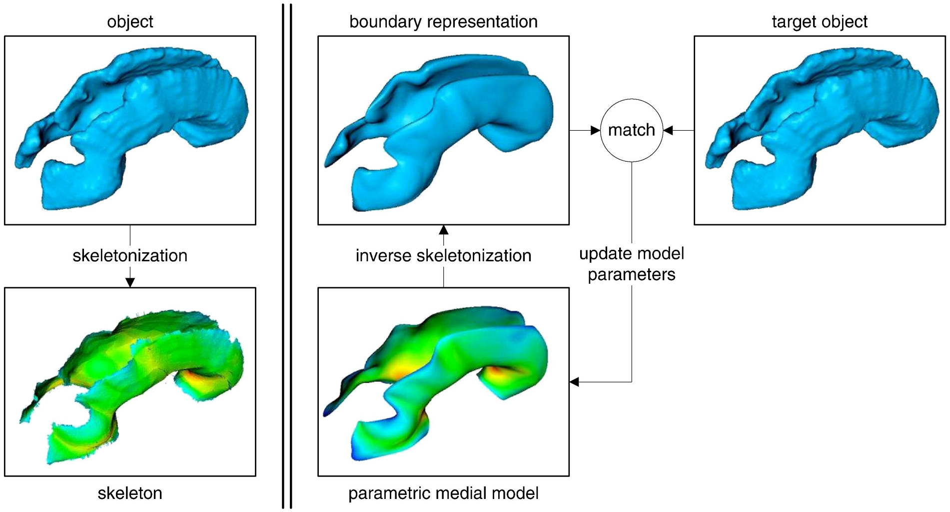
Parameter estimation



Parameter estimation

- Obj F = prior + Energy
- Prior: prefer low curvature.

$$(|\bar{\mathbf{m}}_{i+1} - \bar{\mathbf{m}}_i| - |\bar{\mathbf{m}}_i - \bar{\mathbf{m}}_{i-1}|)^2$$



Energy: mean square distance between interpolated boundary and target shape, summed over all sampled points.

backup(1)

$$\mathbf{II}_{\mathbf{b}^t} = \begin{bmatrix} \mathbf{b}^t_{uu}\cdot\mathbf{u}^t & \mathbf{b}^t_{uv}\cdot\mathbf{u}^t \\ \mathbf{b}^t_{vu}\cdot\mathbf{u}^t & \mathbf{b}^t_{vv}\cdot\mathbf{u}^t \end{bmatrix} = -\begin{bmatrix} \mathbf{b}^t_{u}\cdot\mathbf{u}^t_{u} & \mathbf{b}^t_{v}\cdot\mathbf{u}^t_{u} \\ \mathbf{b}^t_{u}\cdot\mathbf{u}^t_{v} & \mathbf{b}^t_{v}\cdot\mathbf{u}^t_{v} \end{bmatrix}$$

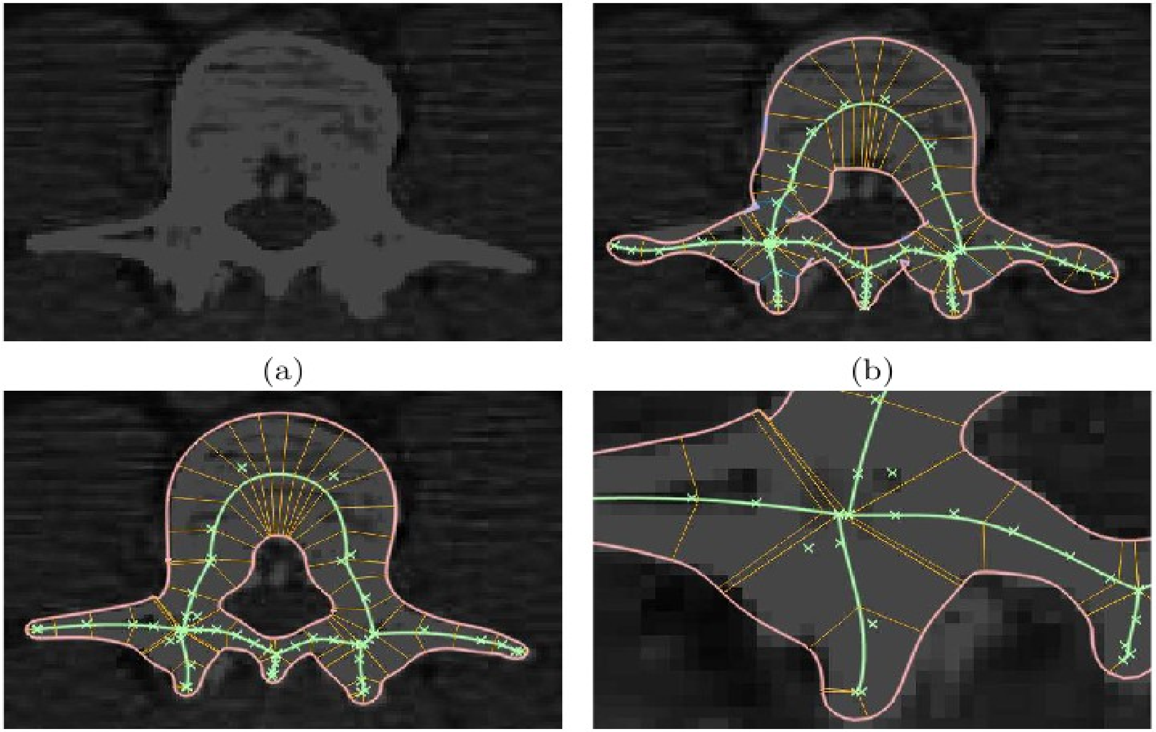
$$\mathbf{I}_{\mathbf{b}^t} = \begin{bmatrix} \mathbf{b}^t_{u}\cdot\mathbf{b}^t_{u} & \mathbf{b}^t_{u}\cdot\mathbf{b}^t_{v} \\ \mathbf{b}^t_{v}\cdot\mathbf{b}^t_{u} & \mathbf{b}^t_{v}\cdot\mathbf{b}^t_{v} \end{bmatrix}$$

$$\nabla r = \frac{dr}{ds}\mathbf{t} = \frac{r'}{\sqrt{x'^2 + y'^2}}\mathbf{t} \ ,$$

Narrated Quicktime movie about model construction in 3D (.mov with audio, 3 min, 11mb)

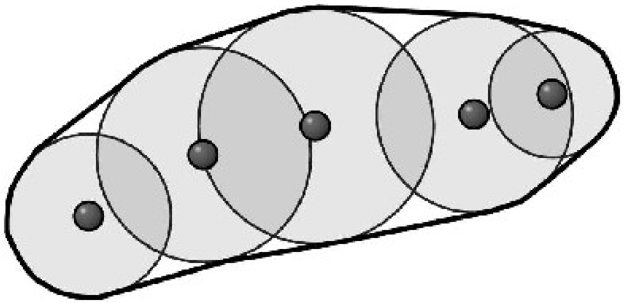
Narrated Quicktime movie
 about fitting models to images in 3D (.mov with audio, 90 sec, 18mb)

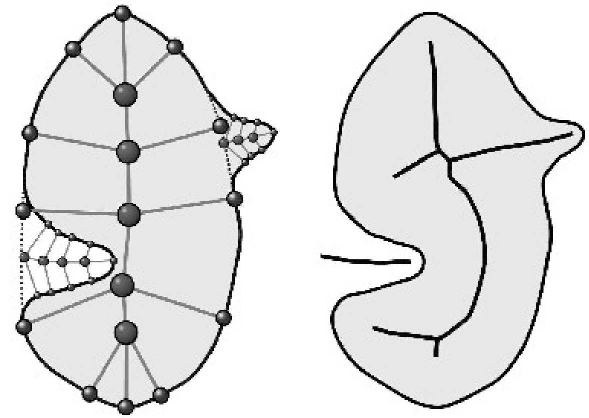
Movie of a spinning 3D model of the hippocampus (.avi loop, 200kb)

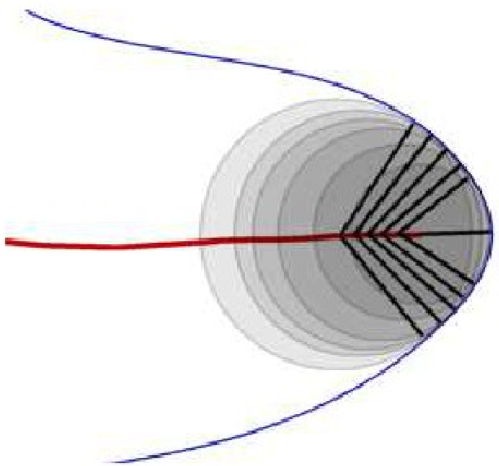


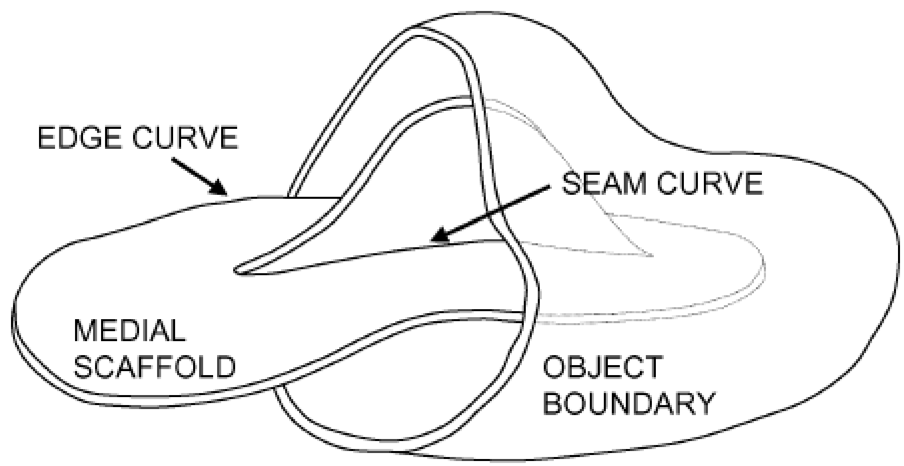
(a) (b) (c) (d)

backup(2)









backup(3)