Lecture: Shape Analysis Introduction

Guido Gerig

CS 7960, Spring 2010

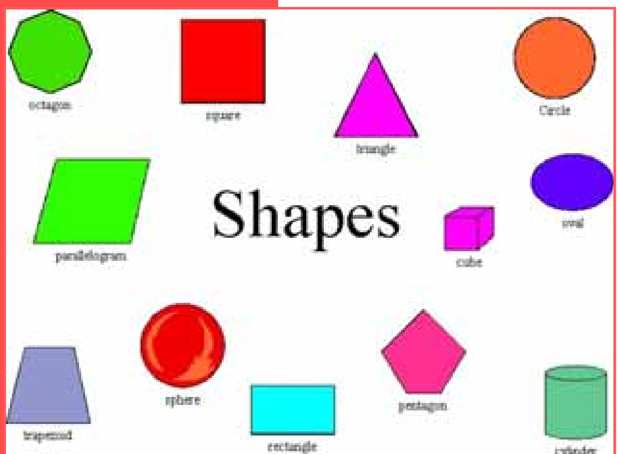
References

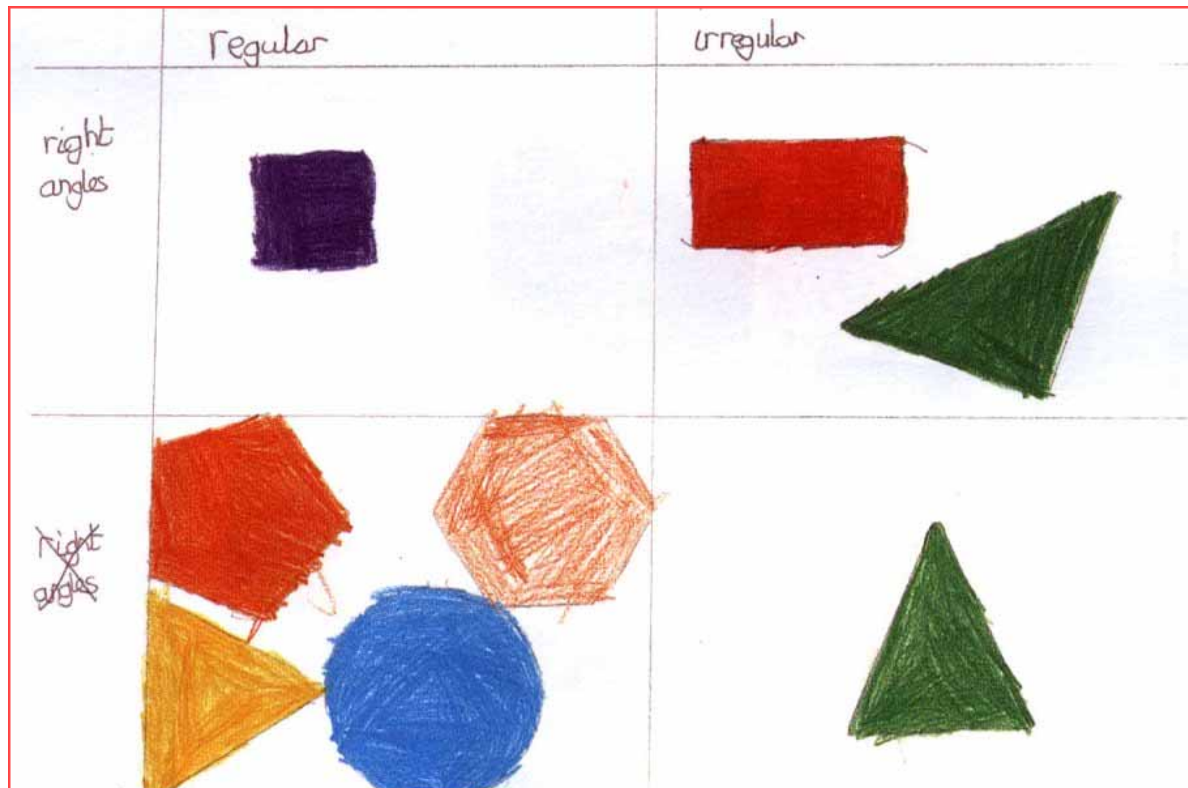
- **[Dryden&Mardia] Statistical Shape Analysis,** Wiley, Chichester, Dryden, I.L. and Mardia, K.V. (1998).
- DG Kendall (1984,Bull.Lond.Math.Soc)
- Bookstein (1986,Statistical Science)
- WS Kendall (1988,Adv.Appl.Probab.)
- Christopher G. Small, The Statistical Theory of Shape, Springer
- D’Arcy Thompson, 1917, On Growth and Form

Shape

The word “shape” is very commonly used in everyday language, usually referring to the appearance of an object.

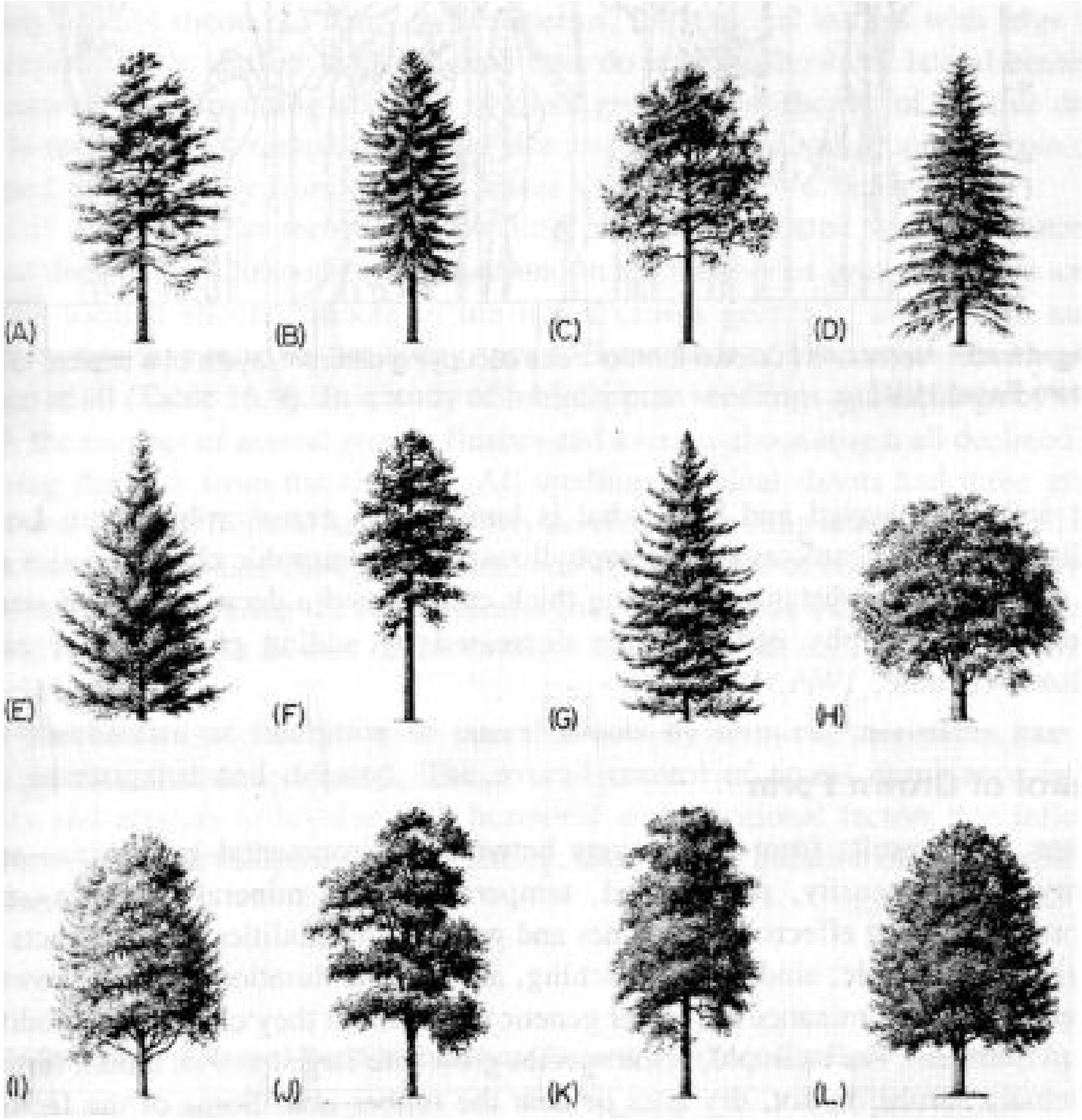
Shape Properties: School Performance Test



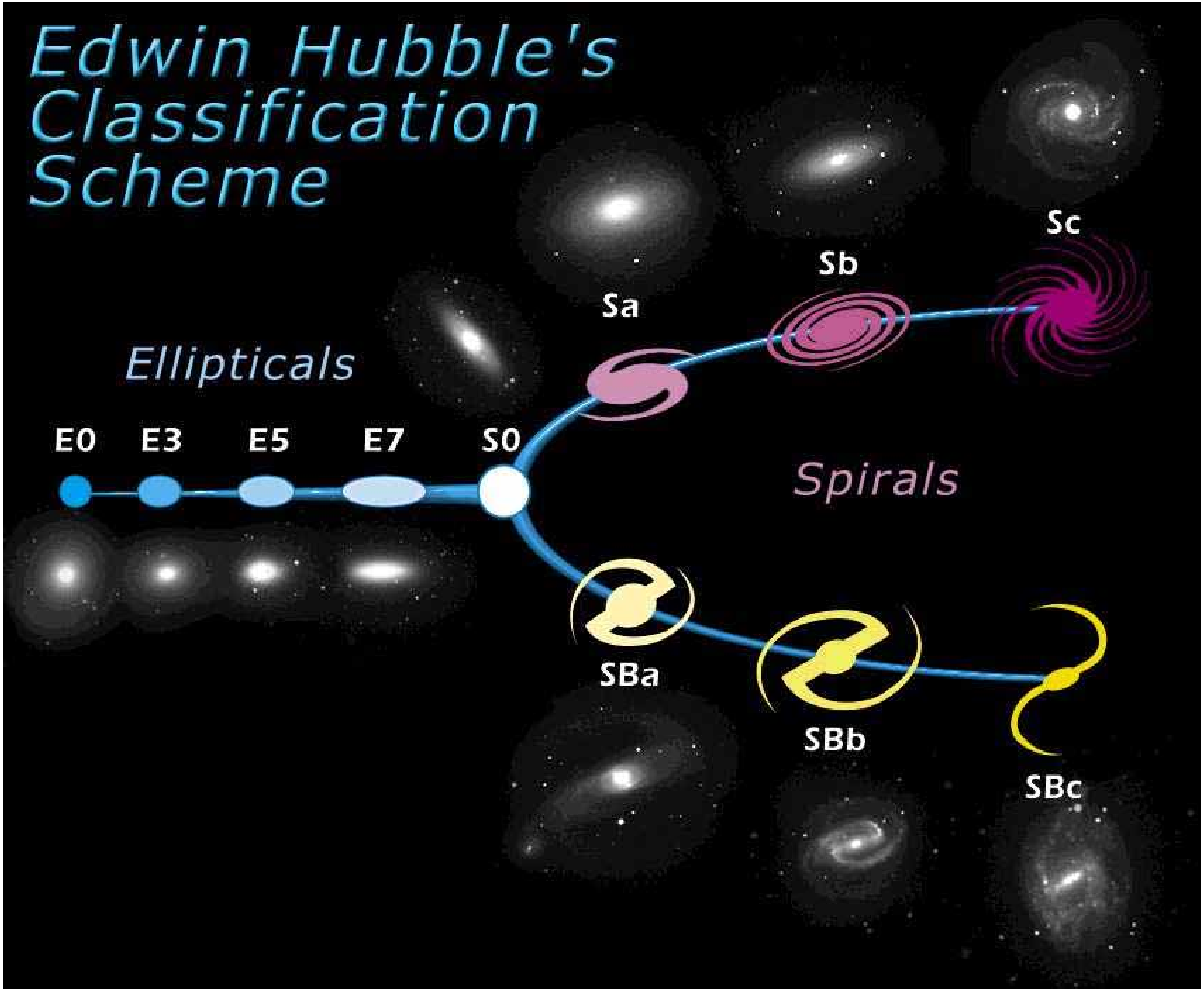


What is Shape?

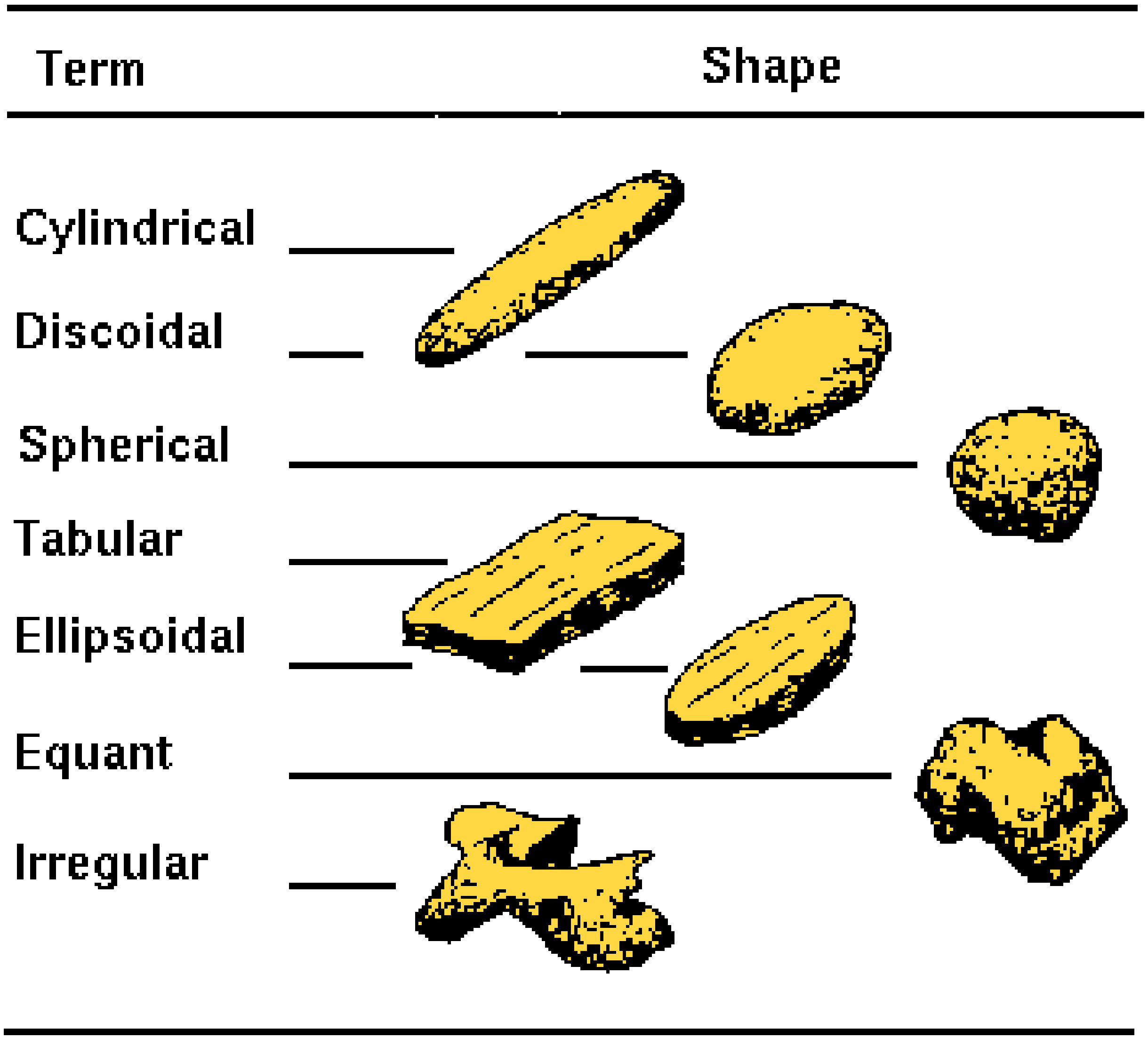
Example Biology



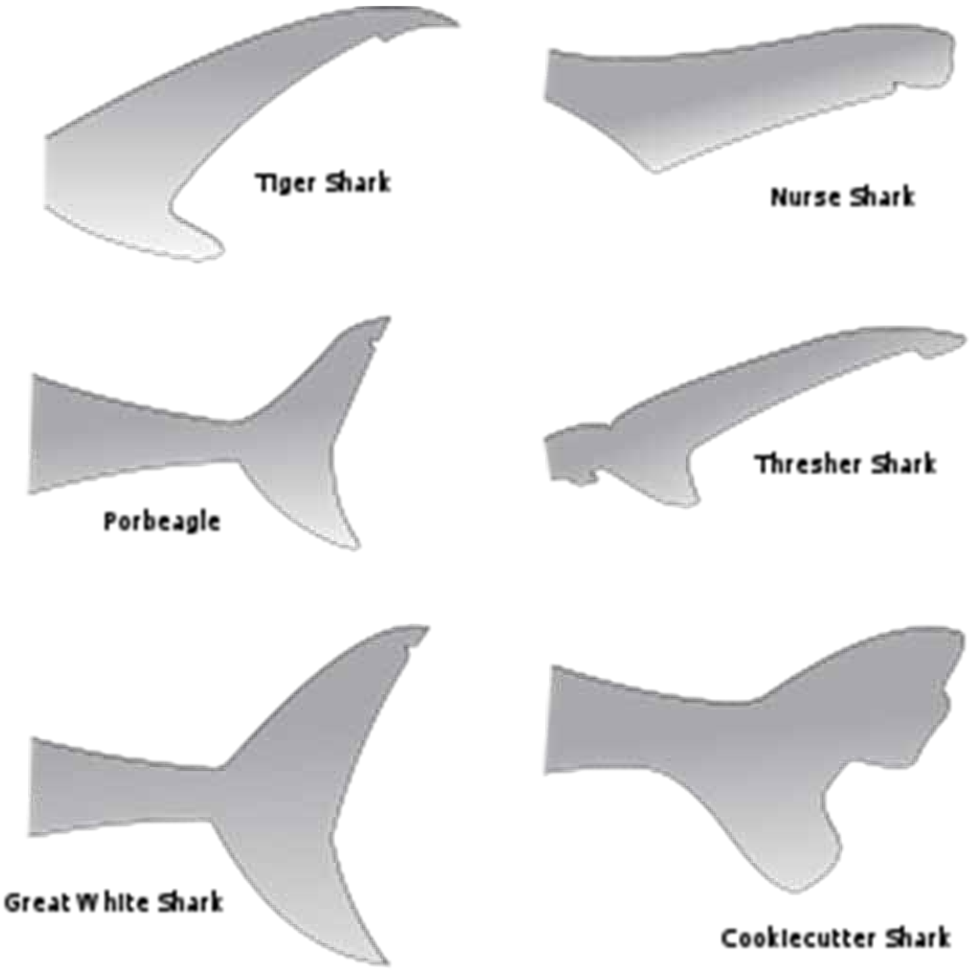
Example Astronomy



Example Geology



Example Biology



Shark Tails

The Diversity of Form and Function

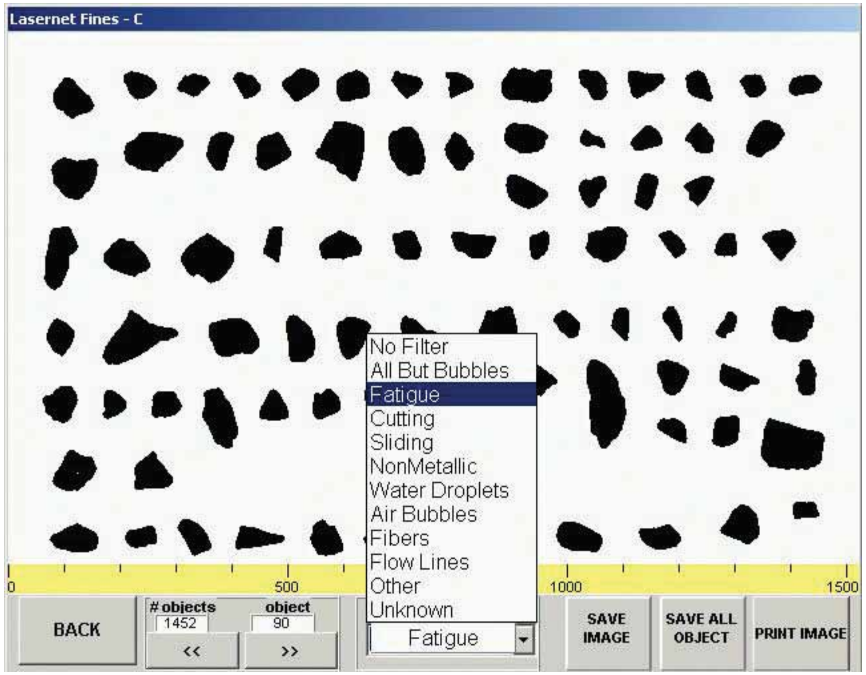
Example Biology

This picture clearly illustrates the typical shell shape differences between male (left) and female (right) eastern box turtles.

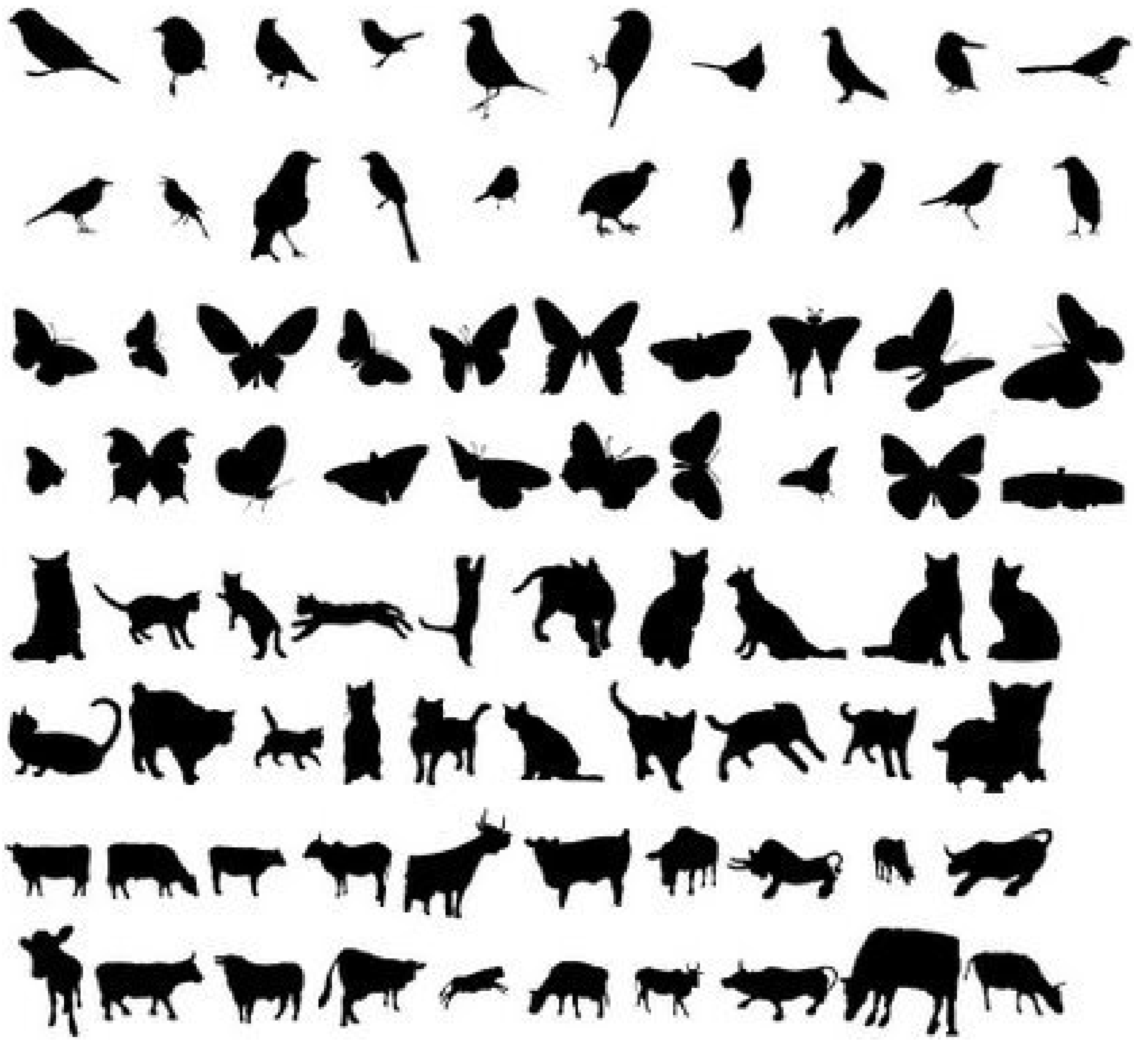
Industrial Example: Particle Analysis, Particle Size Testing, Shape Classification

In machine condition monitoring, the ability to determine the size and shape of contaminant particles is becoming a necessary, if not critical capability. Particles in an used oil sample may be due to the normal wear process, but an increase of particles, and their size and shape will assist the oil analysis laboratory in determining the source and severity of a potential malfunction.

http://www.spectroinc.com/products-lasernet-fines.htm

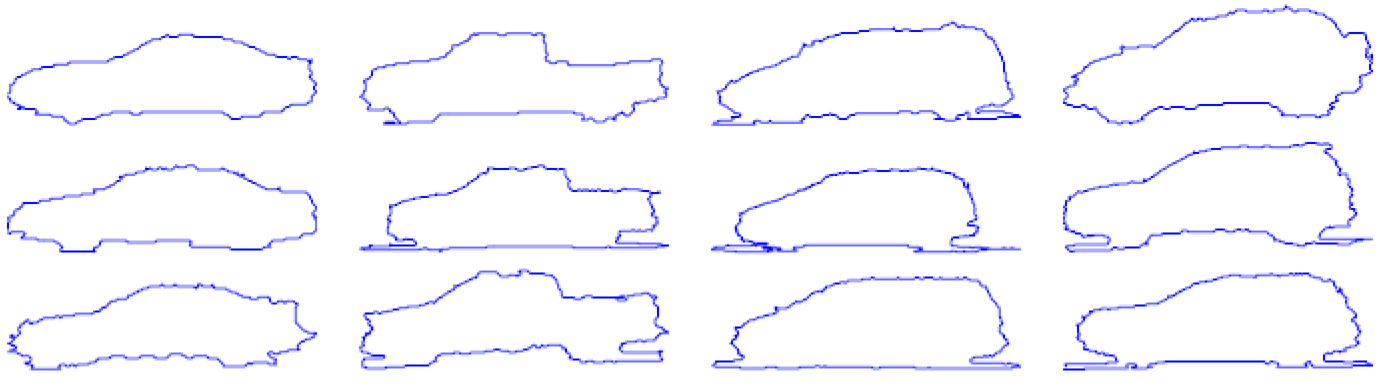


Shape Classes

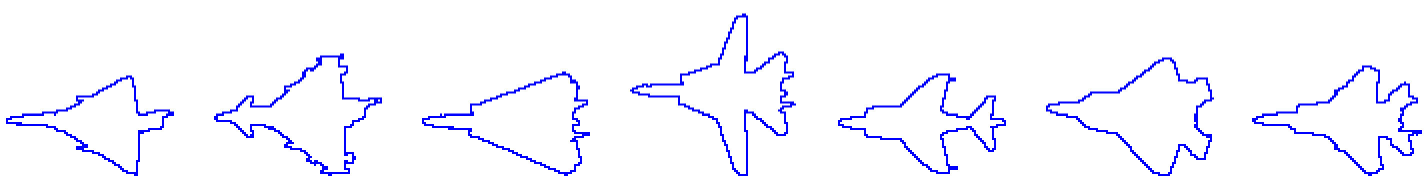


http://sites.google.com/site/xiangbai/try-large.jpg
http://sites.google.com/site/xiangbai/animaldataset

Computer Vision: MPEG-7



Vehicle shape classes: (a) Sedan, (b) Pickup, (c) Minivan, (d) SUV.

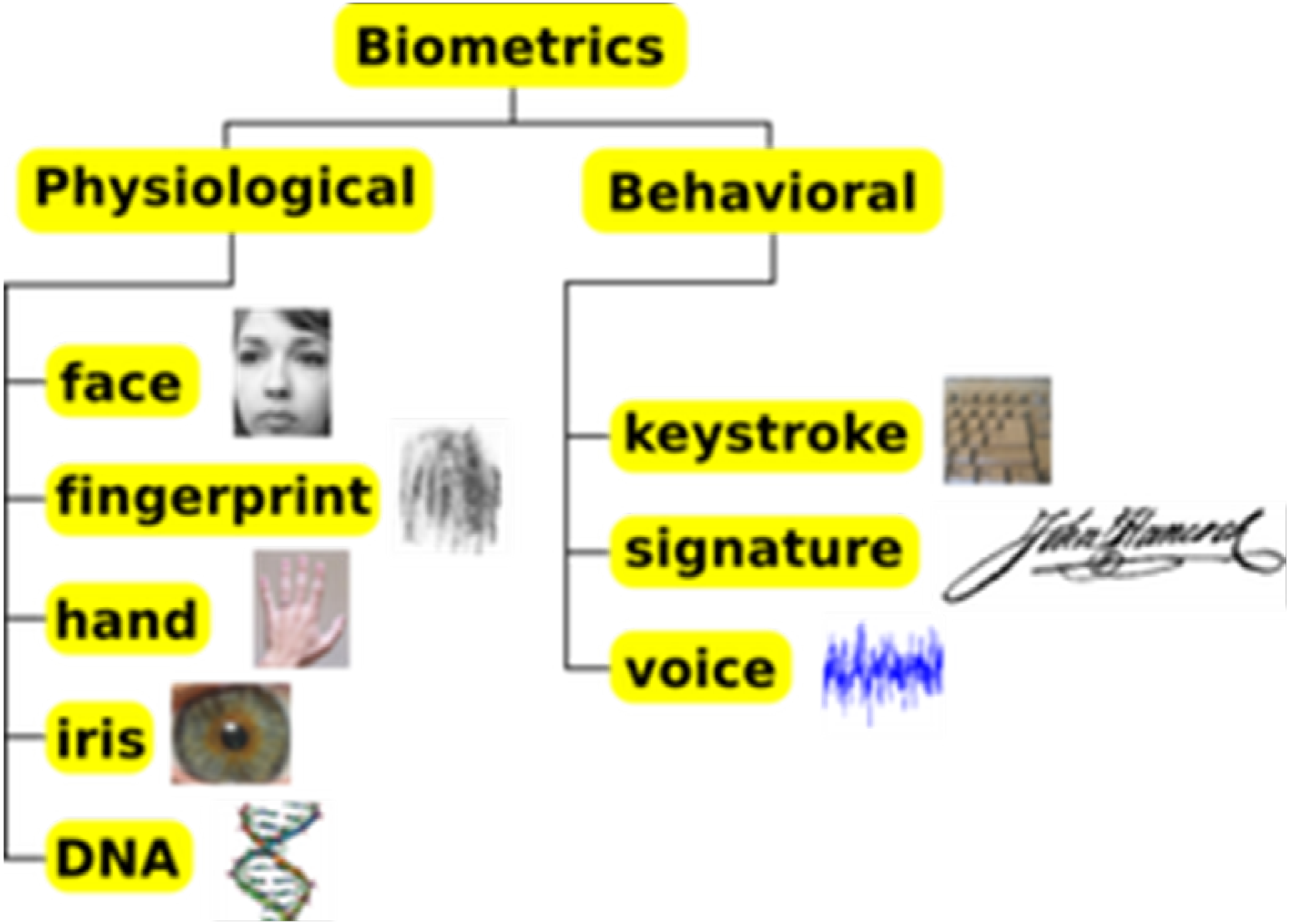


Airplane shape classes: (a) Mirage, (b) Eurofighter, (c) F-14 wings closed, (d) F-14 wings opened, (e) Harrier, (f) F-22, (g) F-15.

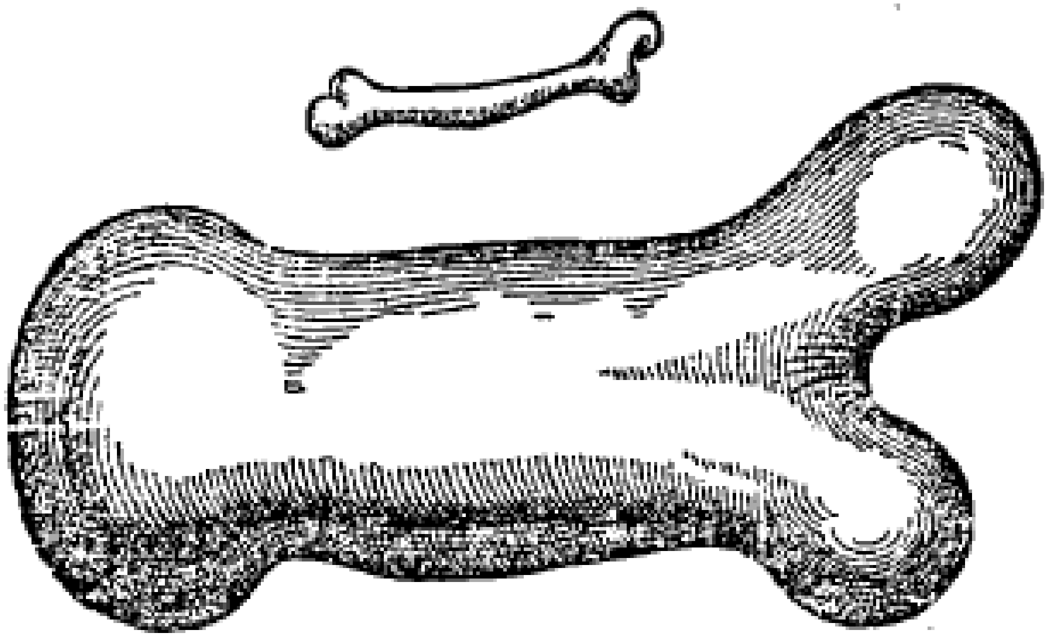
http://visionlab.uta.edu/shape_data.htm

http://www.cis.temple.edu/~latecki/TestData/mpeg7shapeB.tar.gz

Biometrics


Biometrics
Physiological
Behavioral
face
fingerprint
hand
iris
DNA
keystroke
signature
voice

Concept of Shape



From Galileo (1638) illustrating the differences in shapes of the bones of small and large animals.

Concept of Shape?

D.G. Kendall [7]:

Definition 1: **Shape** *is all the geometrical information that remains when location, scale and rotational effects are filtered out from an object.*

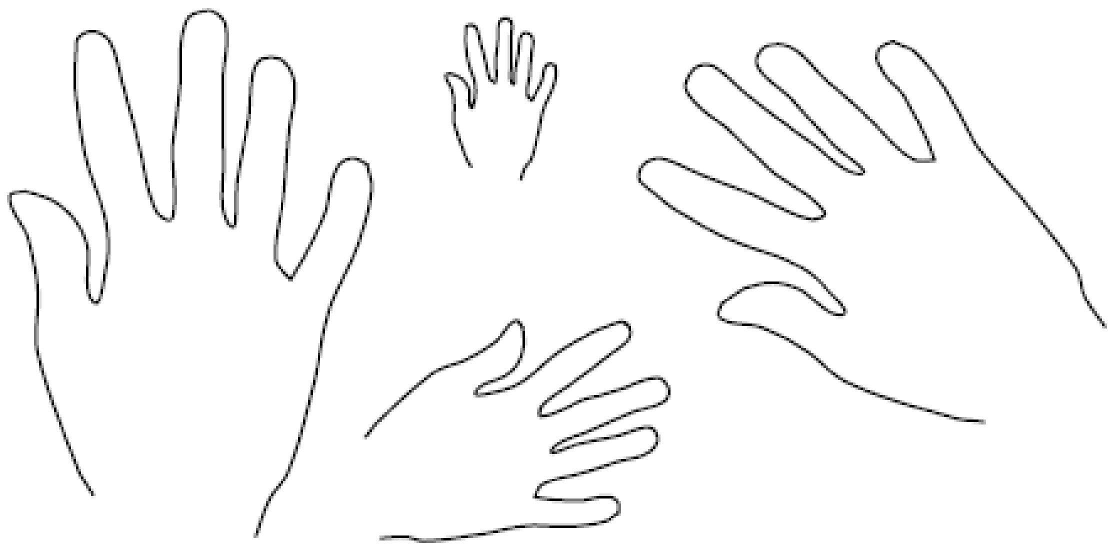
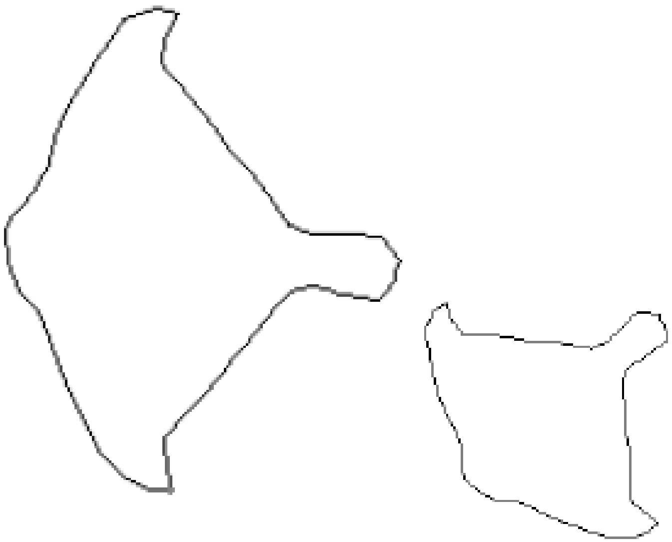


Figure 1: Four copies of the same shape, but under different Euclidean transformations.

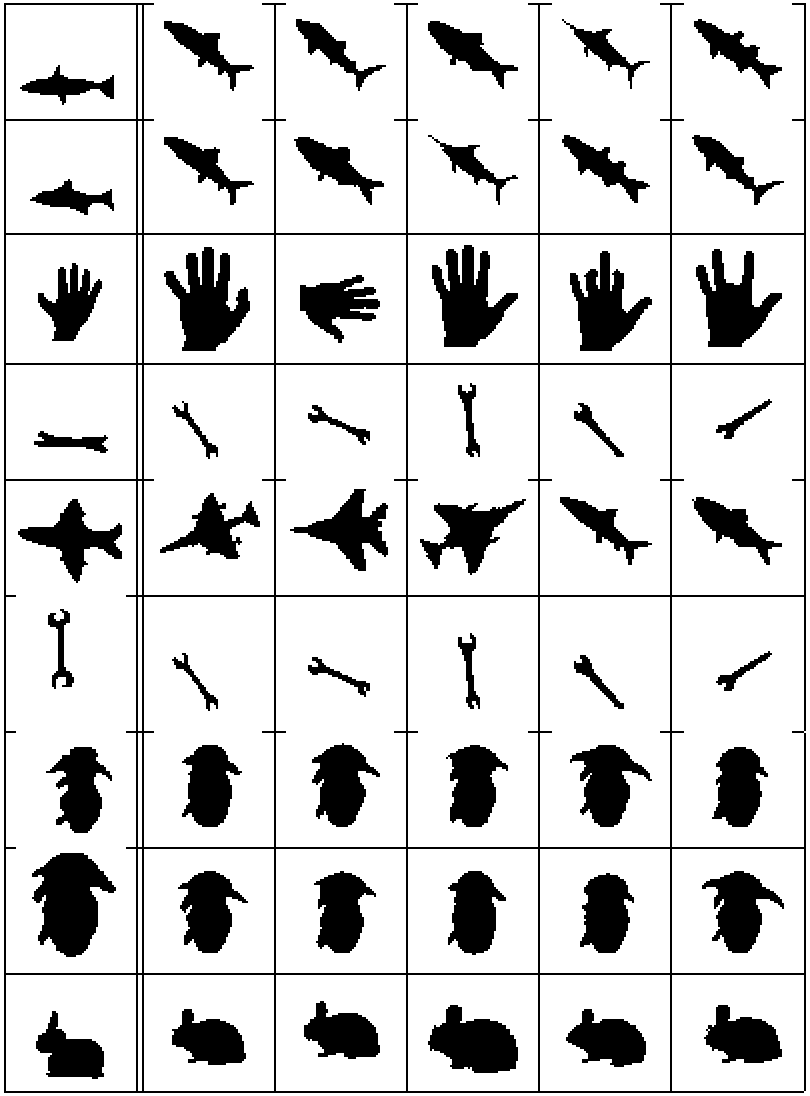
Shape: Definition

An object's shape is invariant under the similarity transformations of translation, scaling and rotation.



Two mouse second thoracic vertebra (T2 bone) outlines with the same shape.

Shape Definition

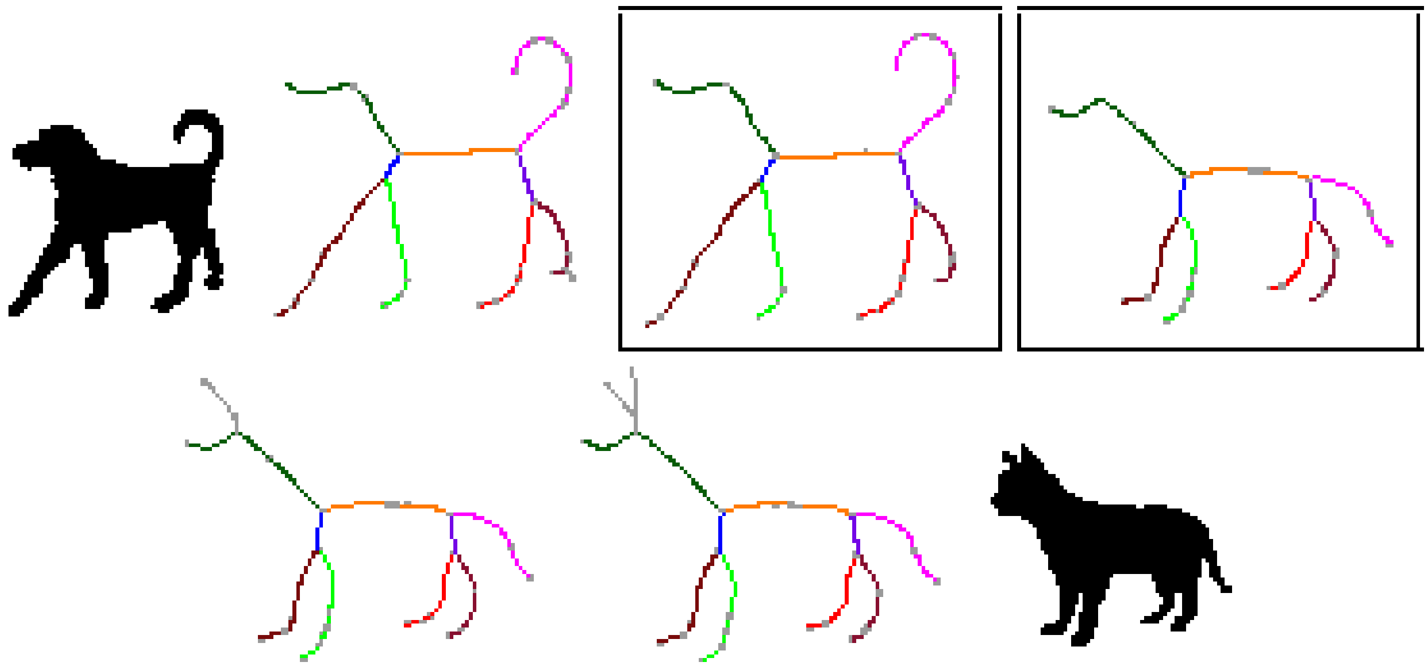


Dryden/Mardia, (Kendall 1977):

Shape is all the geometrical information that remains when location, scale and rotational effects are filtered out from an object.

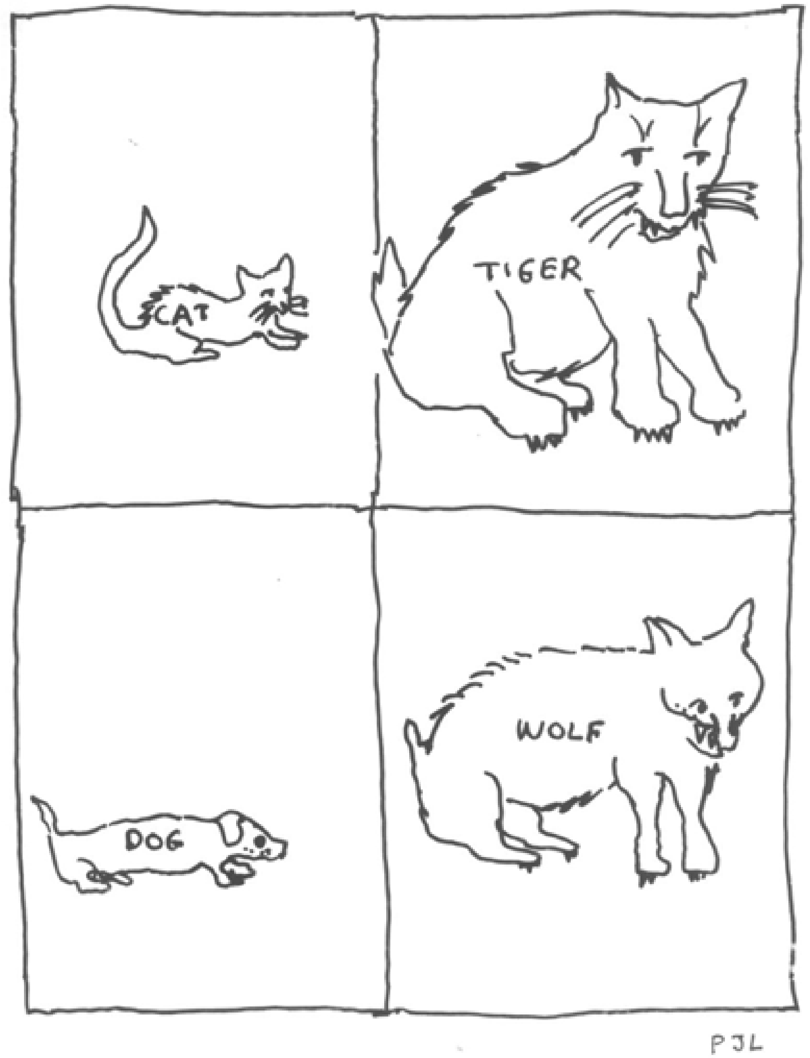
Shape Transformation

Dog to Cat



Symmetry Maps and Transforms
For Perceptual Grouping and Object Recognition,
Benjamin B. Kimia, Brown

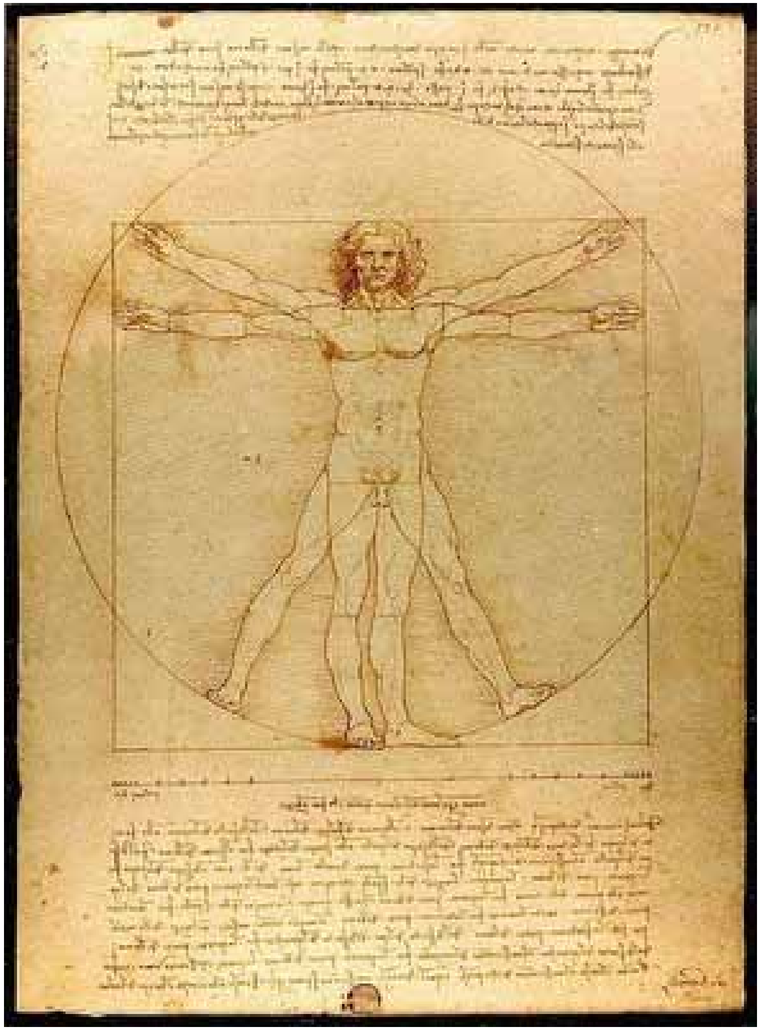
The Problem of Size and Shape

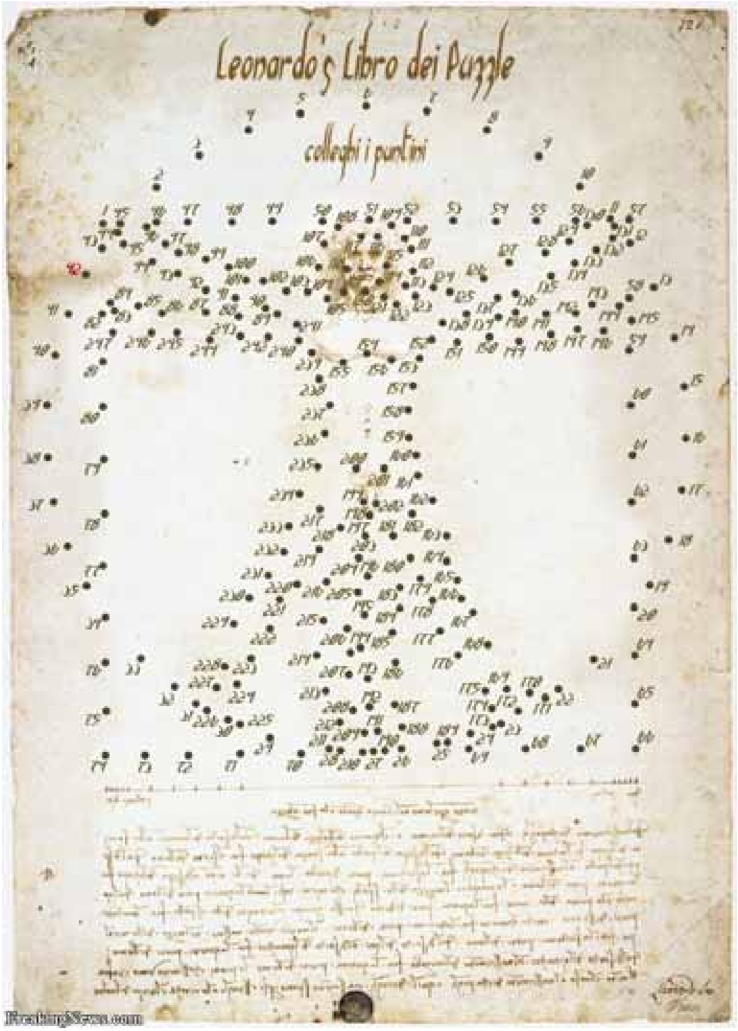


Dryden/Mardia (Kendall 1977): *(Sometimes we are also interested in retaining scale information as well as shape):*

Size-and-shape is all the geometrical information that remains when location and rotational effects are filtered out from an object.

Landmarks





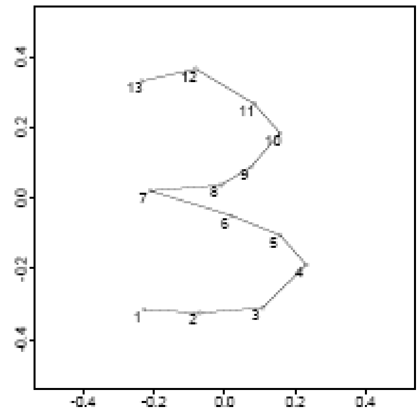
Google: Advanced Photoshop Pictures Contest

Landmarks (Dryden & Mardia)

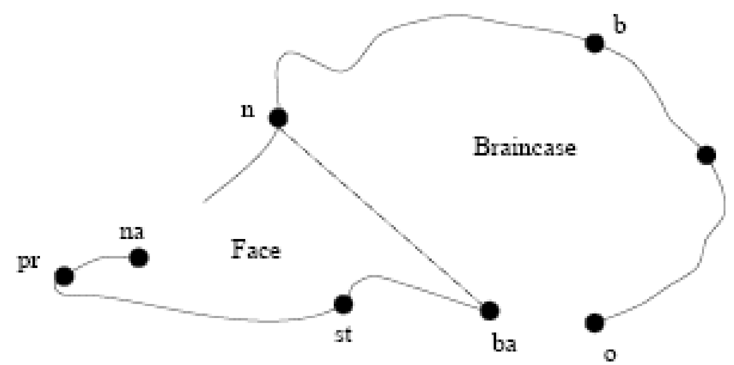
- A **landmark** is a point of correspondence on each object that matches between and within populations.
- An **anatomical landmark** is a point assigned by an expert that corresponds between objects of study in a way meaningful in the context of the disciplinary context.
- **Mathematical landmarks** are points located on an object according some mathematical or geometrical property of the figure.

[Dryden & Mardia]

Landmarks



Handwritten digit 3



Ape cranium

Landmarks

Three landmarks along a line for simple shape comparison

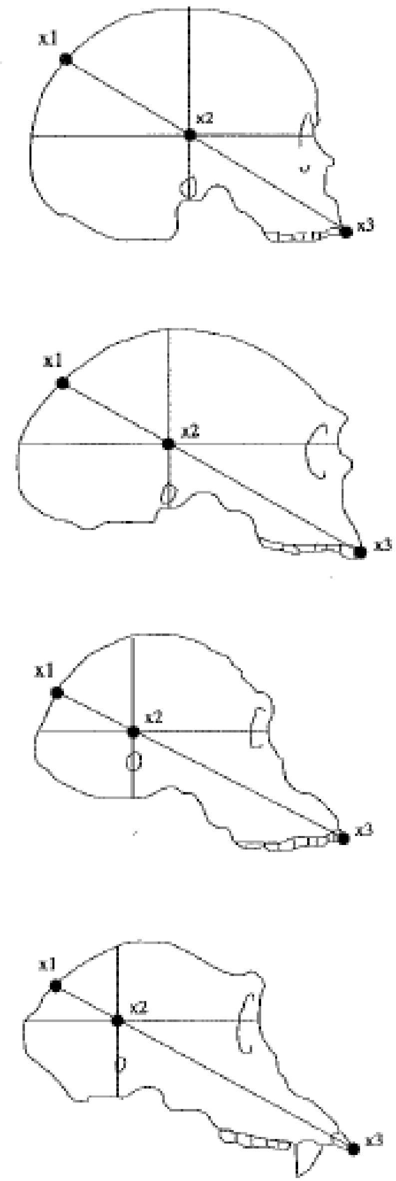


FIGURE 1.2. *Side view of skulls. From top to bottom: modern human, Neanderthal, australopithecine, chimpanzee. The skull profiles are redrawn from Figure 3.53 of [131].*

Ch. G. Small, The Statistical Theory of Shape

Shape and Registration

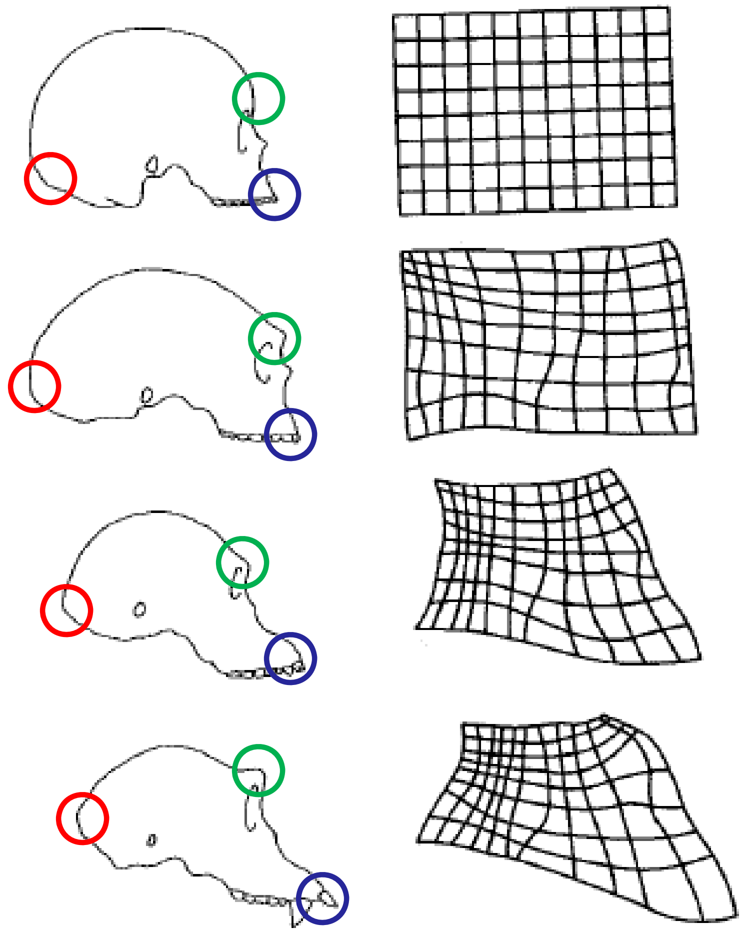


FIGURE 1.7. *Side view of skulls. From top to bottom: modern human, Neanderthal, australopithecine, chimpanzee. To the right of each skull is a coordinate grid determined with Thompson's method of coordinates, with the modern human skull as the base image. Reproduced from Figure 3.53 of [131] by kind permission of Hong Kong University Press.*

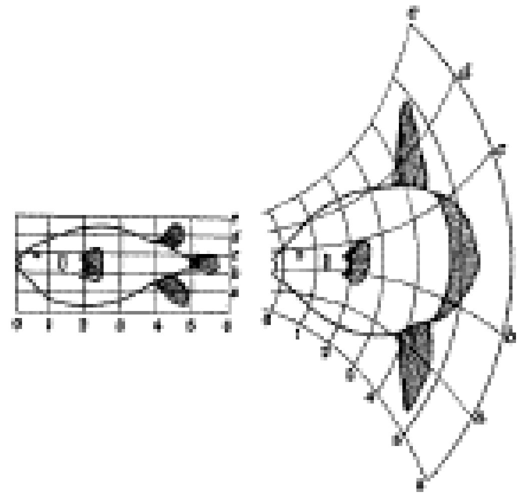
Homology:

Corresponding (homologous) features in all skull images.

Ch. G. Small, The Statistical Theory of Shape

Shape and Registration

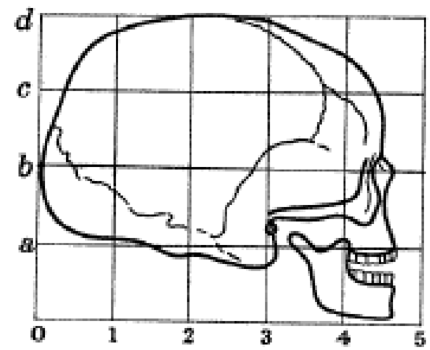
In the spirit of D'Arcy Thompson (1917) who considered the geometric transformations of one species to another

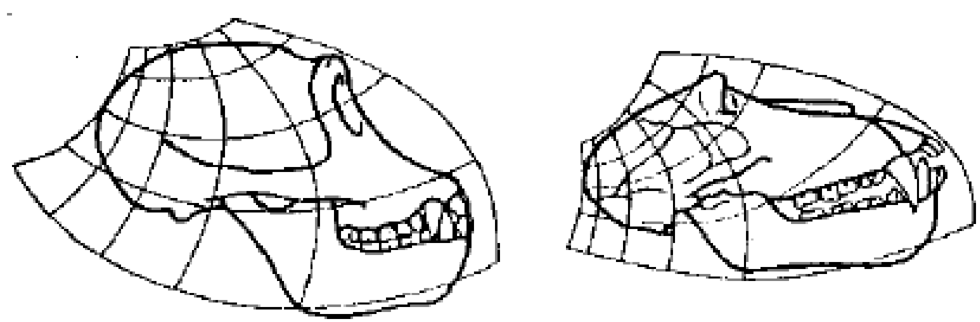


We conside a shape space obtained directly from the landmark coordinates, which retains the geometry of a point configuration at all stages.

Shape and Registration

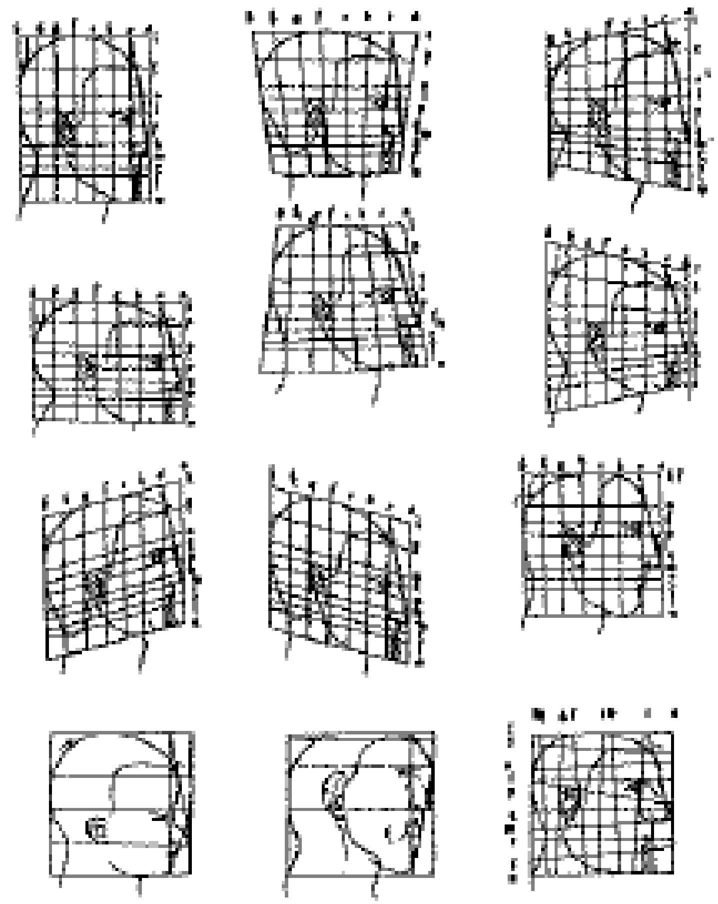
Following from the original ideas of D'Arcy Thompson (1917) we can produce similar transformation grids, using a pair of thin-plate splines for the deformation from configuration matrices T to Y.





(a) (b)

Shape and Registration



Early transformation grids of human profiles

Size of Configuration of Landmarks?

A **size measure** $g(X)$ is any positive real valued function of the configuration matrix such that $g(aX) = ag(X)$ for any positive scalar a.

The **centroid size** is given by $S(X) = \|CX\| = \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{m} (X_{ij} - \overline{X}_j)^2}$, $X \in \mathbb{R}^{km}$

where X_{ij} is the $(i,j)th$ entry of X, the arithmetic mean of the jth dimension is $\overline{X}_j = \frac{1}{k}\sum_{i=1}^{k} X_{ij}$, $C = I_k - \frac{1}{k}1_k 1_k^T$ is the centring matrix, $\|X\| = \sqrt{trace(X^T X)}$ is the Euclidean norm, I_k is the $k \times k$ identity matrix and 1_k is the $k \times 1$ vector of ones.

$S(X)$ is the square root of the sum of squared Euclidean distances from each landmark to the centroid:

$$S(X) = \sqrt{\sum_{i=1}^{k} \|(X)_j - \overline{X}\|^2},$$

where $(X)_j$ is the jth row of X ($j = 1, \ldots, k$) and $\overline{X} = (\overline{X}_1, \ldots \overline{X}_m)$ is the centroid.

Alternative: Baseline Size
A Shape Coordinate System

Bookstein coordinates - $(u_j^B, v_j^B)^T$, $j = 3, \ldots, k$ are the remaining coordinates of an object after translating, rotating, rescaling the baseline to $(-\frac{1}{2}, 0)$ and $(\frac{1}{2}, 0)$ so that

$$u_j^B = \{(x_2 - x_1)(x_j - x_1) + (y_2 - y_1)(y_j - y_1)\}/D_{12}^2 - \tfrac{1}{2},$$

$$v_j^B = \{(x_2 - x_1)(y_j - y_1) + (y_2 - y_1)(x_j - x_1)\}/D_{12}^2,$$

where $j = 3, \ldots, k$, $D_{12}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 > 0$ and $-\infty < u_j^B, v_j^B < \infty$

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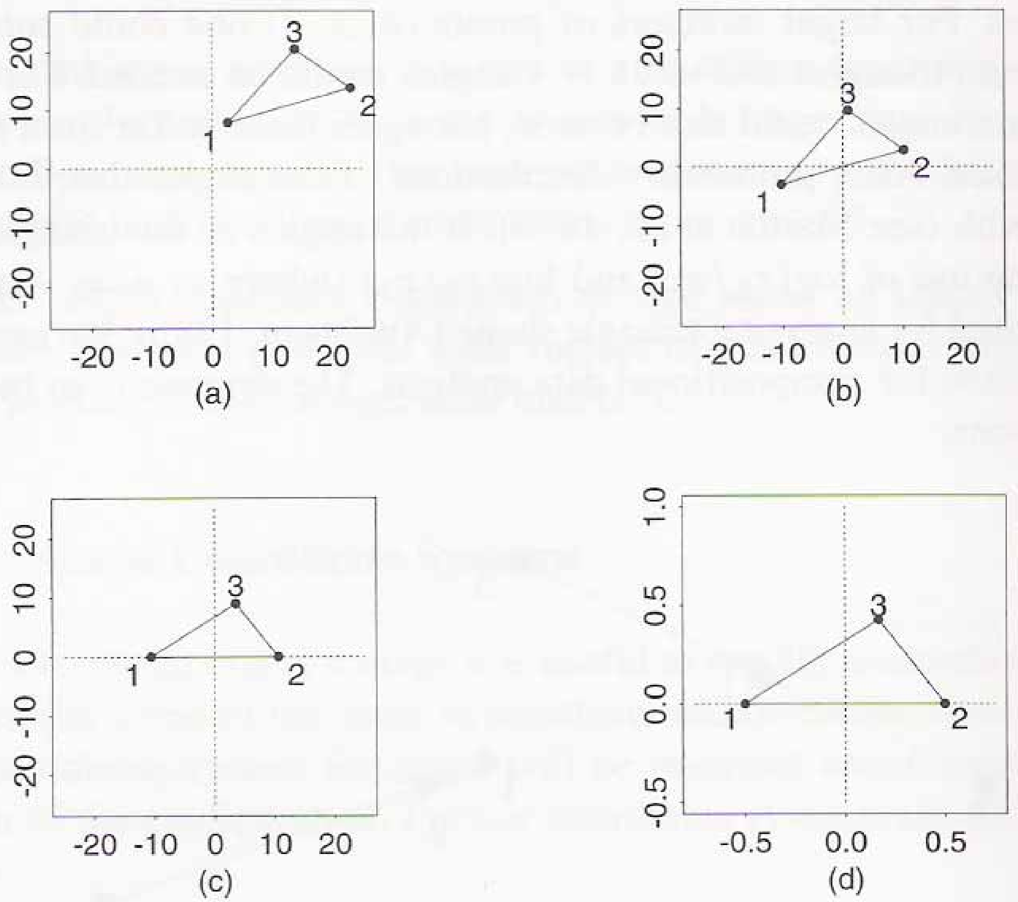
- Widely used in shape analysis for planar data.
- Most straightforward method for a newcomer to shape analysis.
- Experienced shape analysts often use Bookstein coordinates in the first stages of an analysis.

Bookstein Coordinates



More General: Equivalence Relationships

Two objects are equivalent (x1 ~ x2) if they can be transformed into each other by the following transformation:

$$\vec{x}_1 = \alpha * R(\varphi) * \vec{x}_2 + \vec{t}$$

α , R , $\vec{t}$ define equivalence relationships:

Properties:

- Reflexivity: $m \approx m$
- Symmetry $m \approx n \rightarrow n \approx m$
- Transitivity $m \approx n \wedge n \approx p \rightarrow m \approx p$