# THE REFLECTANCE MAP AND SHAPE-FROM-SHADING

http://www.cs.jhu.edu/~wolff/course600.461/week9.3/index.htm

November 4, 1998

# REFLECTANCE MODELS

## LAMBERTIAN MODEL

$$E = L \rho \cos \theta$$

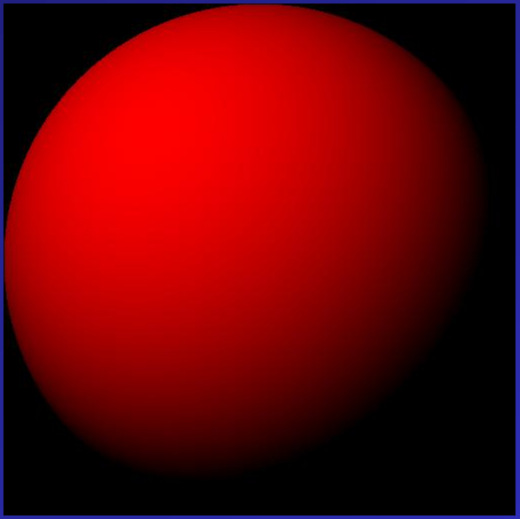
albedo

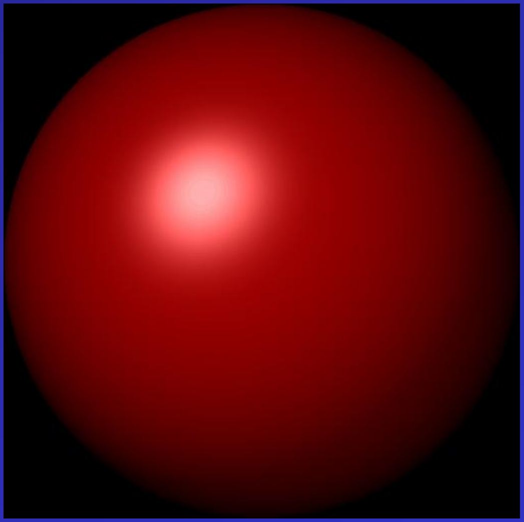
## PHONG MODEL

$$E = L (a \cos \theta + b \cos^n \alpha)$$

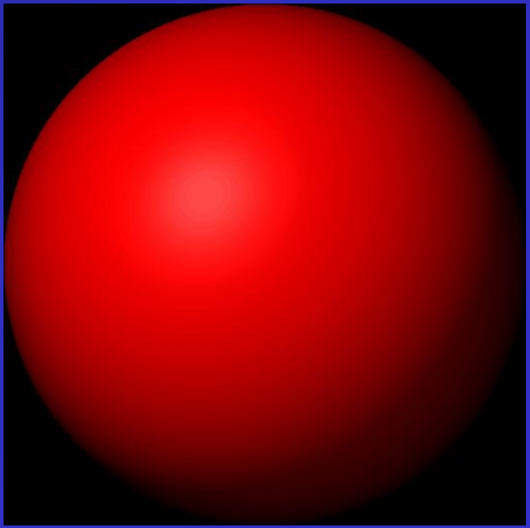
Diffuse albedo

Specular albedo





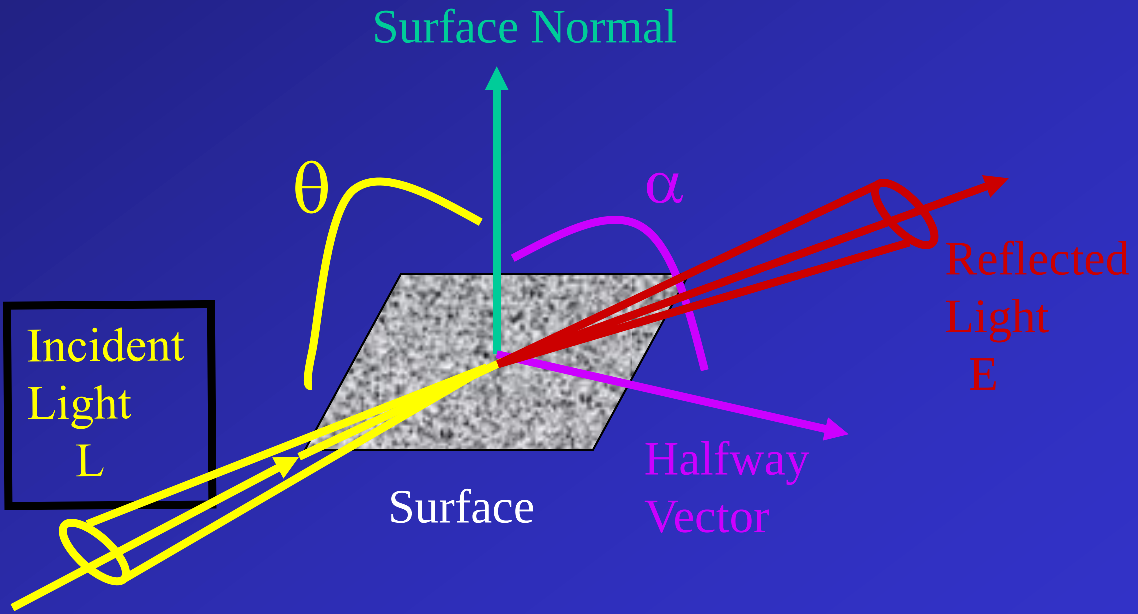
a=0.3, b=0.7, n=2



a=0.7, b=0.3, n=0.5

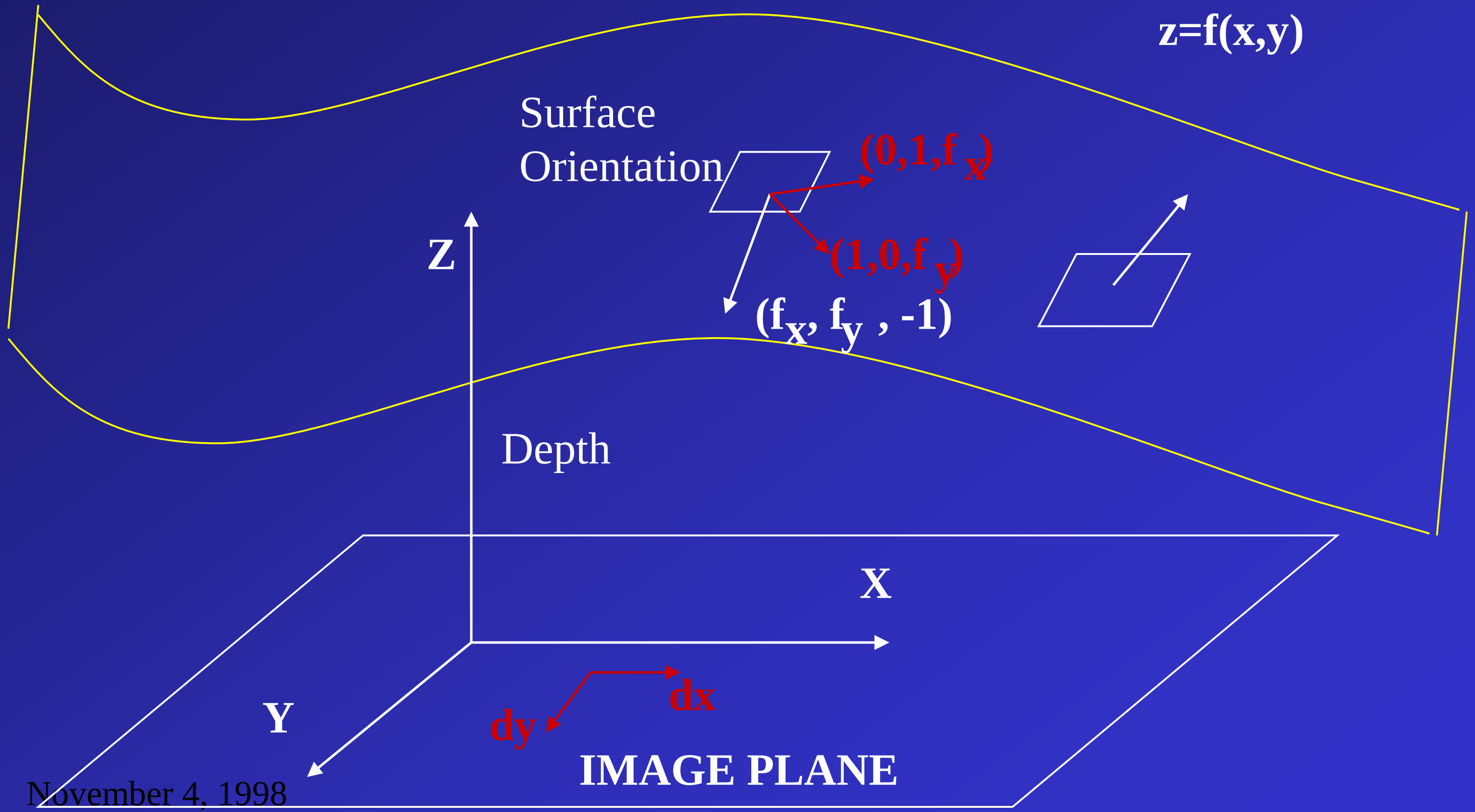
# REFLECTANCE MODELS

- Description of how light energy incident on an object is transferred from the object to the camera sensor



# REFLECTANCE MAP IS A VIEWER-CENTERED REPRESENTATION OF REFLECTANCE

$(f_x, f_y, -1) = (0,1,f_x) \times (1,0,f_y)$



# REFLECTANCE MAP IS A VIEWER-CENTERED REPRESENTATION OF REFLECTANCE

$$(f_x, f_y, -1) = (p, q, -1)$$

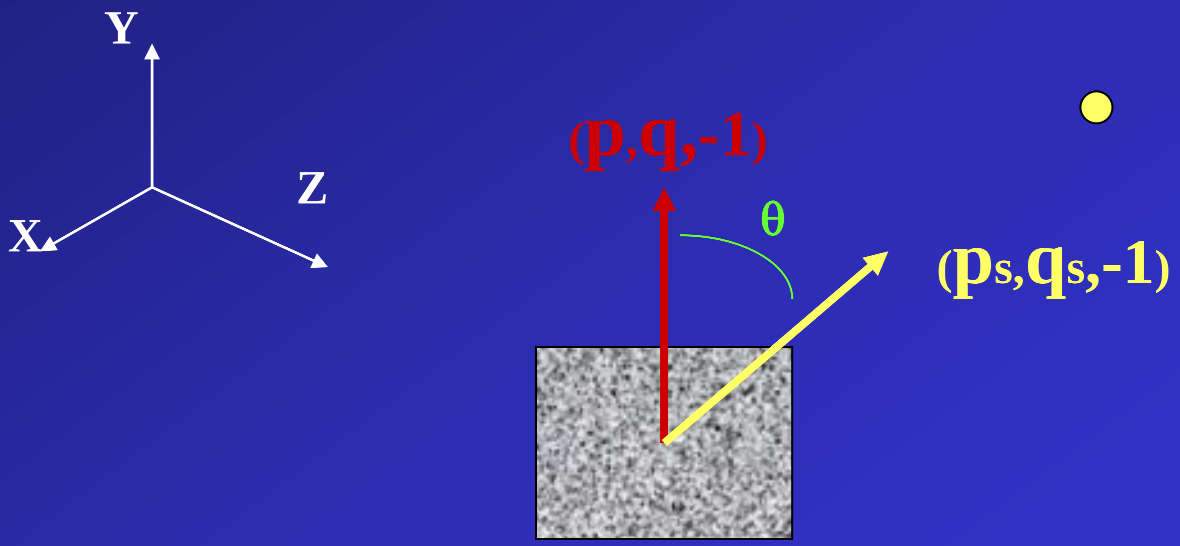
**p, q comprise a gradient or gradient space representation for local surface orientation.**

**Reflectance map expresses the reflectance of a material directly in terms of viewer-centered representation of local surface orientation.**

# LAMBERTIAN REFLECTANCE MAP

## LAMBERTIAN MODEL

$$E = L \rho \, COS \, \theta$$



$$COS\theta = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\,\sqrt{1 + p_s^{\;2} + q_s^{\;2}}}$$

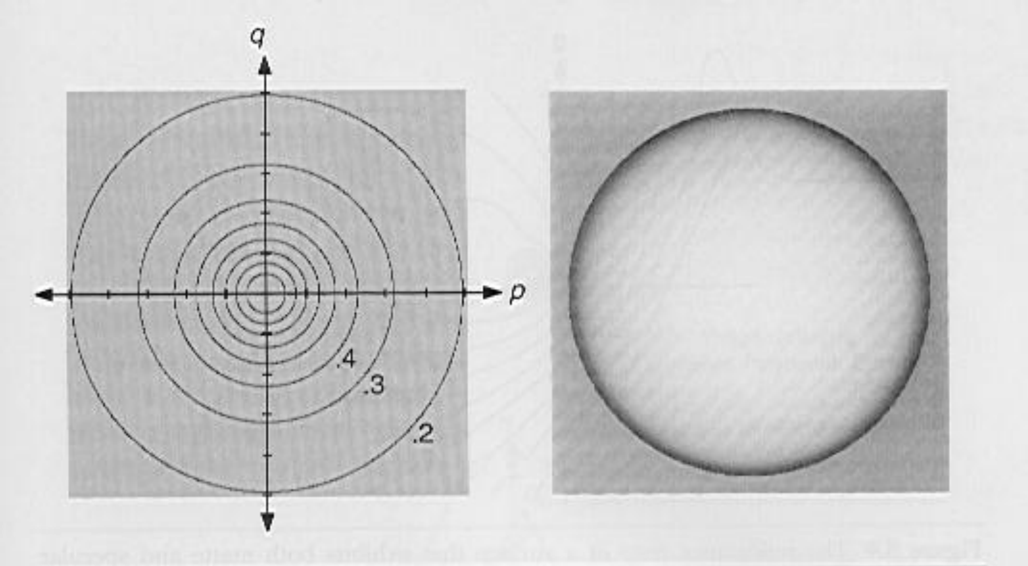
# LAMBERTIAN REFLECTANCE MAP

$$E = L\rho \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s{}^2 + q_s{}^2}}$$

Grouping L and ρ as a constant , local surface orientations that produce equivalent intensities under the Lambertian reflectance map are quadratic conic section contours in gradient space.

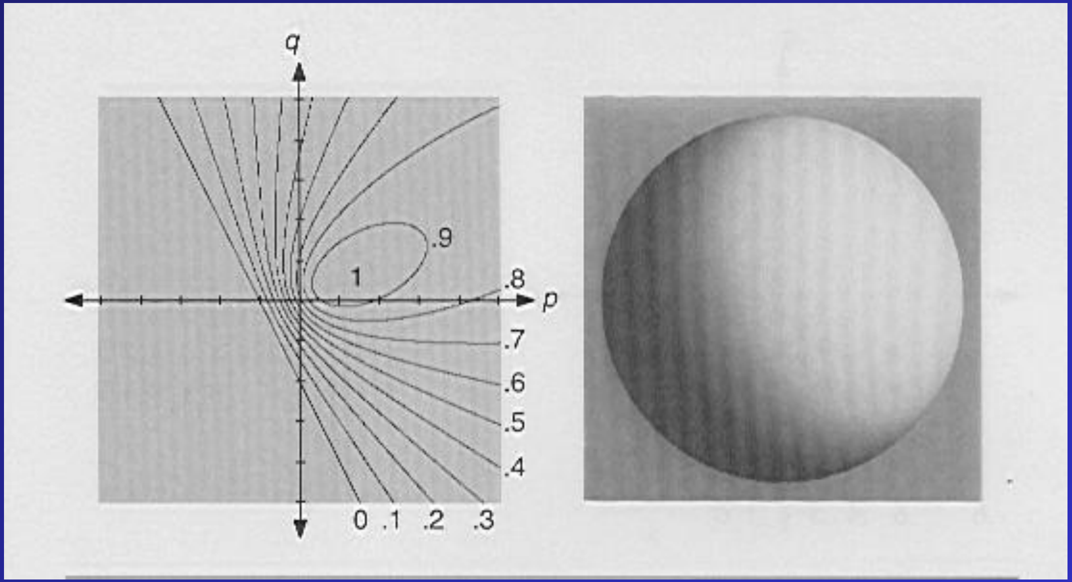
$$I = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\sqrt{1 + p_s{}^2 + q_s{}^2}}$$

# LAMBERTIAN REFLECTANCE MAP



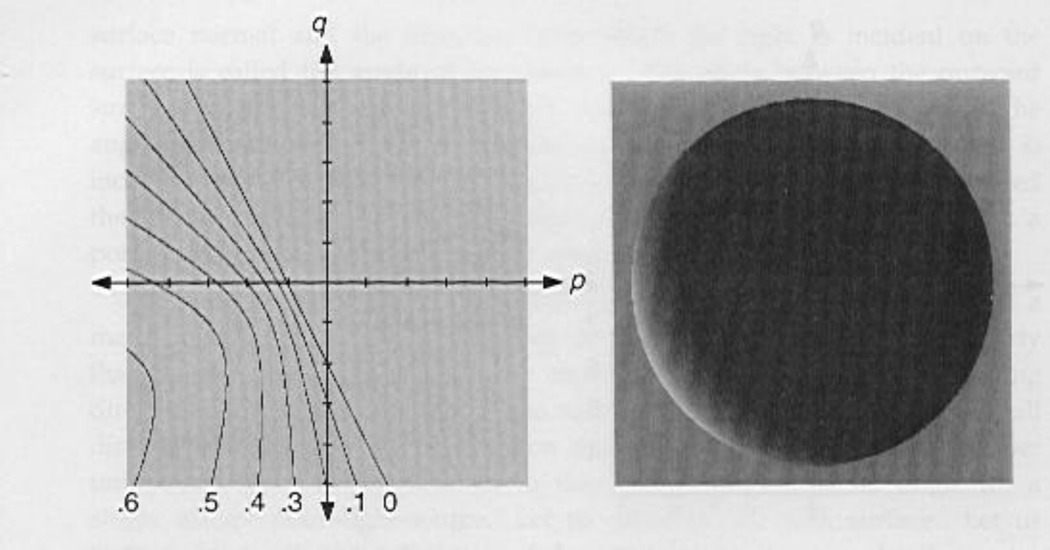
$p_{s=0}$   $q_{s=0}$

# LAMBERTIAN REFLECTANCE MAP



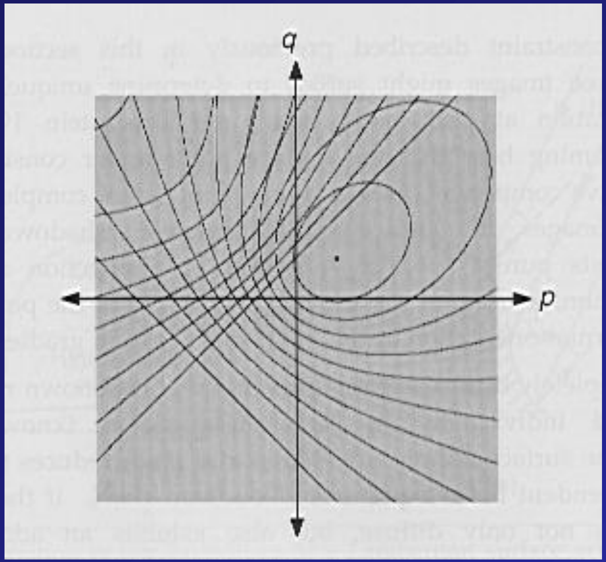
$p_s$=0.7 $q_s$=0.3

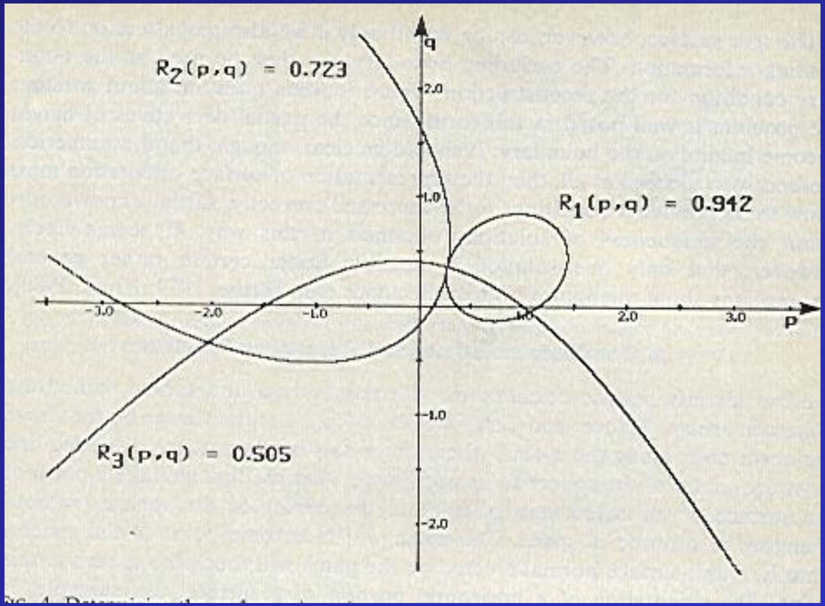
# LAMBERTIAN REFLECTANCE MAP



$p_s = -2$ $q_s = -1$

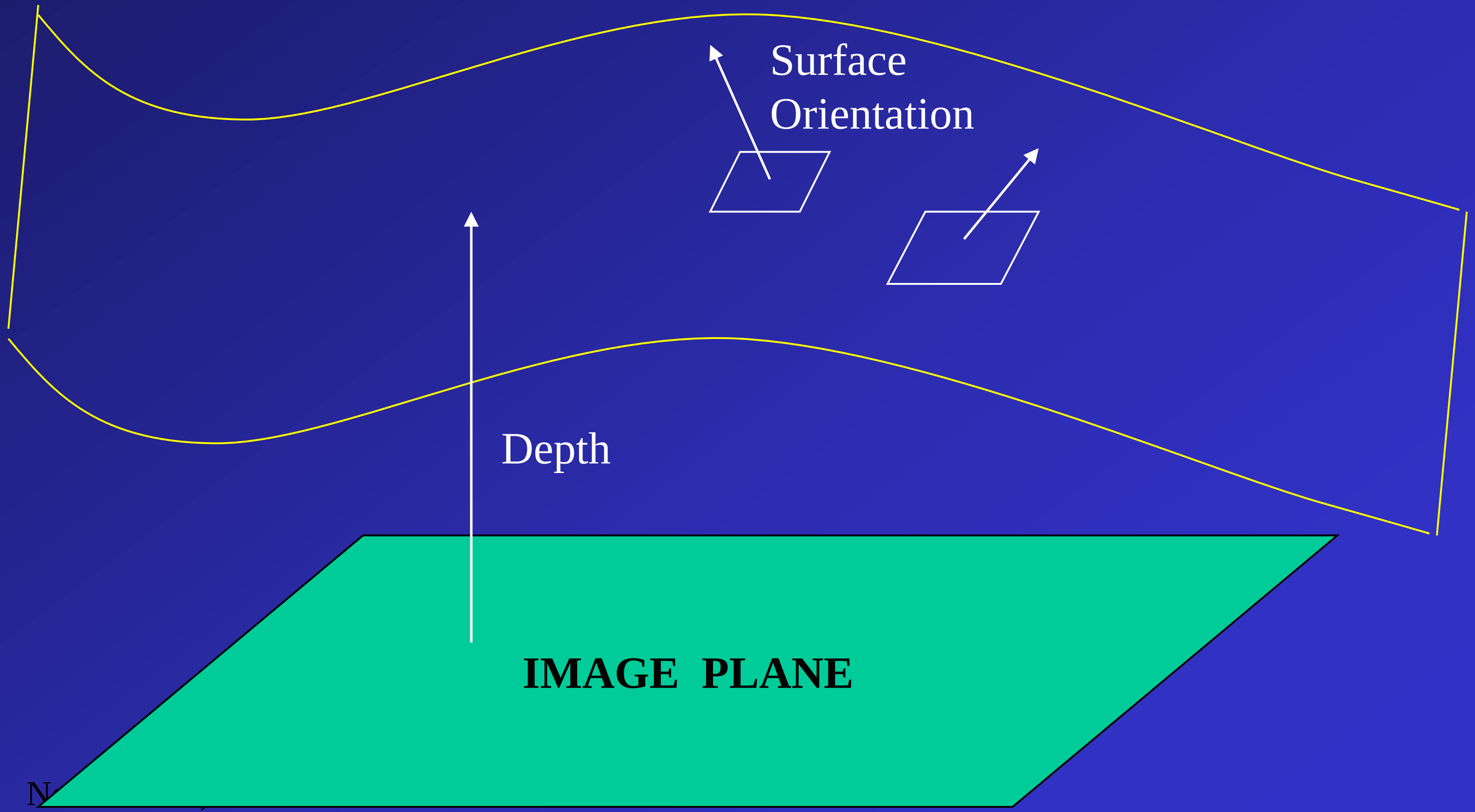
# PHOTOMETRIC STEREO





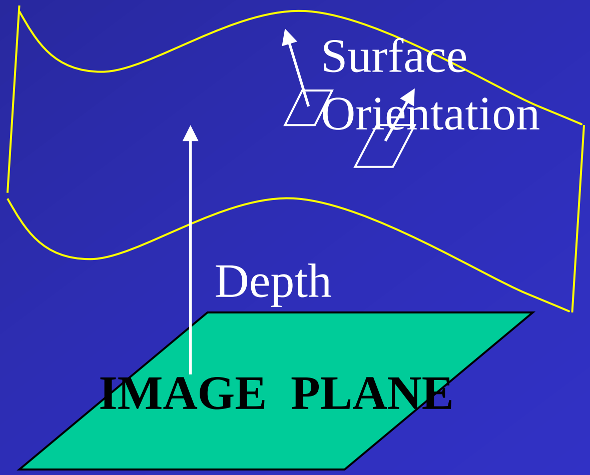
**Derivation of local surface normal at each pixel creates the derived normal map.**

# NORMAL MAP vs. DEPTH MAP



# NORMAL MAP vs. DEPTH MAP

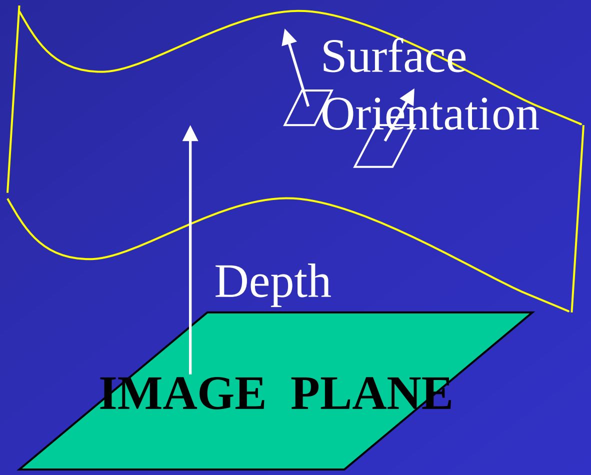
- Can determine Depth Map from Normal Map by integrating over gradients p,q across the image.
- Not all Normal Maps have a unique Depth Map. This happens when Depth Map produces different results depending upon image plane direction used to sum over gradients.
- Particularly a problem when there are errors in the Normal Map.



# NORMAL MAP vs. DEPTH MAP

- A Normal Map that produces a unique Depth Map independent of image plane direction used to sum over gradients is called integrable.
- Integrability is enforced when the following condition holds:

$$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$$



# NORMAL MAP vs. DEPTH MAP

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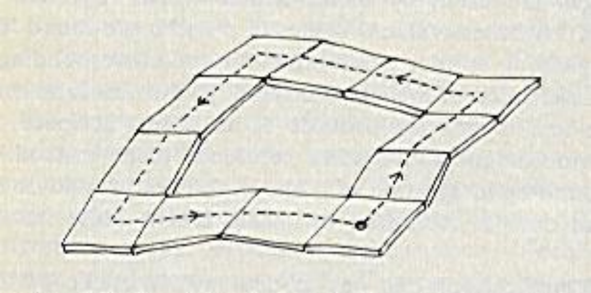
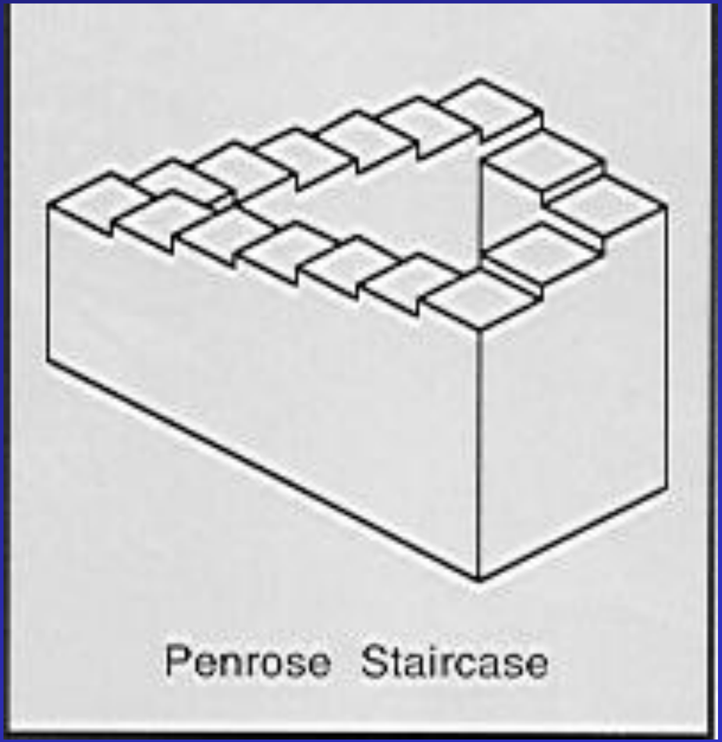
**GREEN'S THEOREM**

$$\iint (\partial p / \partial y - \partial q / \partial x) dxdy = \oint (pdx + qdy)$$



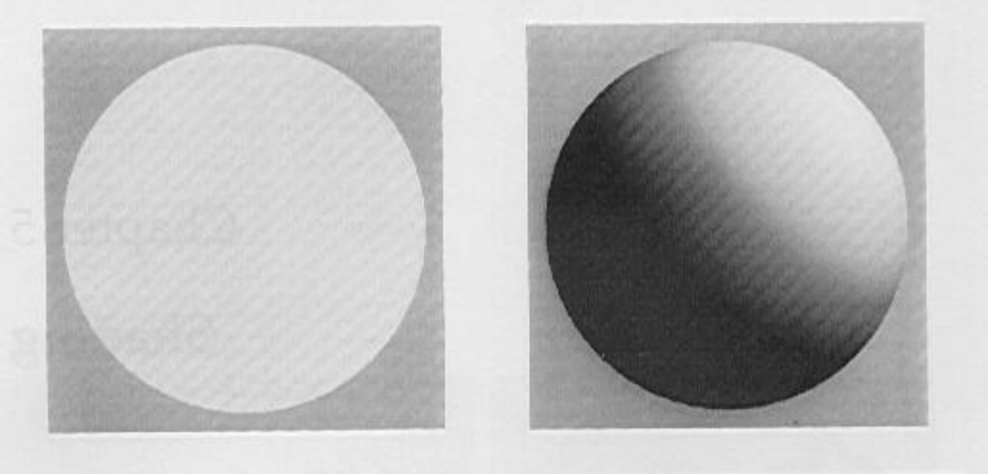
# NORMAL MAP vs. DEPTH MAP

## VIOLATION OF INTEGRABILITY



Penrose Staircase

# SHAPE FROM SHADING



**From a monocular view with a single distant light source of known incident orientation upon an object with known reflectance map, solve for the normal map.**

# SHAPE FROM SHADING

- Formulate as solving the Image Irradiance equation for surface orientation variables p,q:

$$\mathbf{I(x,y) = R(p,q)}$$

- Since this is underconstrained we can’t solve this equation directly
- What do we do ??.

# SHAPE FROM SHADING (Calculus of Variations Approach)

- First Attempt: Minimize error in agreement with Image Irradiance Equation over the region of interest:

$$\iint_{object} (I(x,y) - R(p,q))^2 \, dxdy$$

# SHAPE FROM SHADING
# (Calculus of Variations Approach)

- Better Attempt: Regularize the Minimization of error in agreement with Image Irradiance Equation over the region of interest:

$$\iint_{object} p_x{}^2 + p_y{}^2 + q^2{}_x + q^2{}_y + \lambda(I(x,y) - R(p,q))^2\, dxdy$$