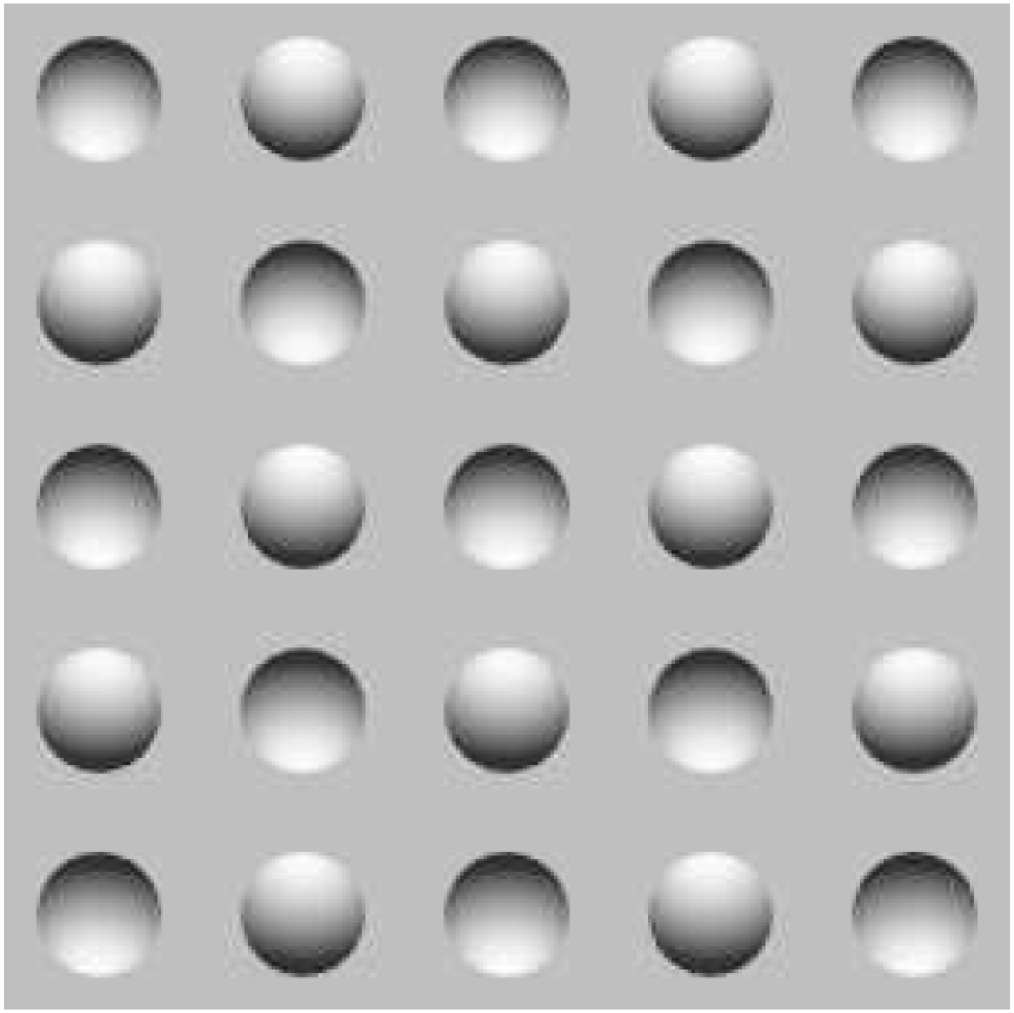
Sources, shading and photometric stereo F&P Ch 5 (old), Ch 2 (new)

Guido Gerig

CS 6320, Spring 2015

Credits: modified from original slides by David A. Forsyth plus modifications by Marc Pollefeys,
Materials from Ohad Ben-Shahar, CS 202-1-5261, http://www.cs.bgu.ac.il/~ben-shahar/

Shape from Shading



Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

Shape from Shading

Inverting the image formation process

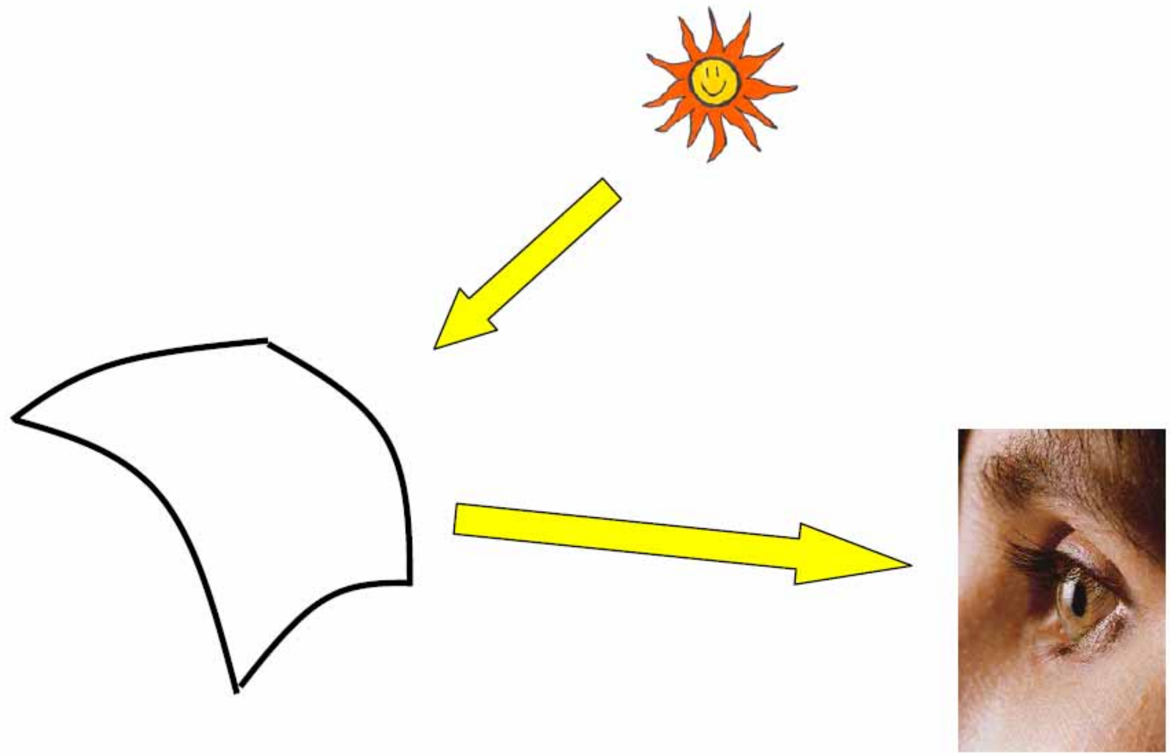
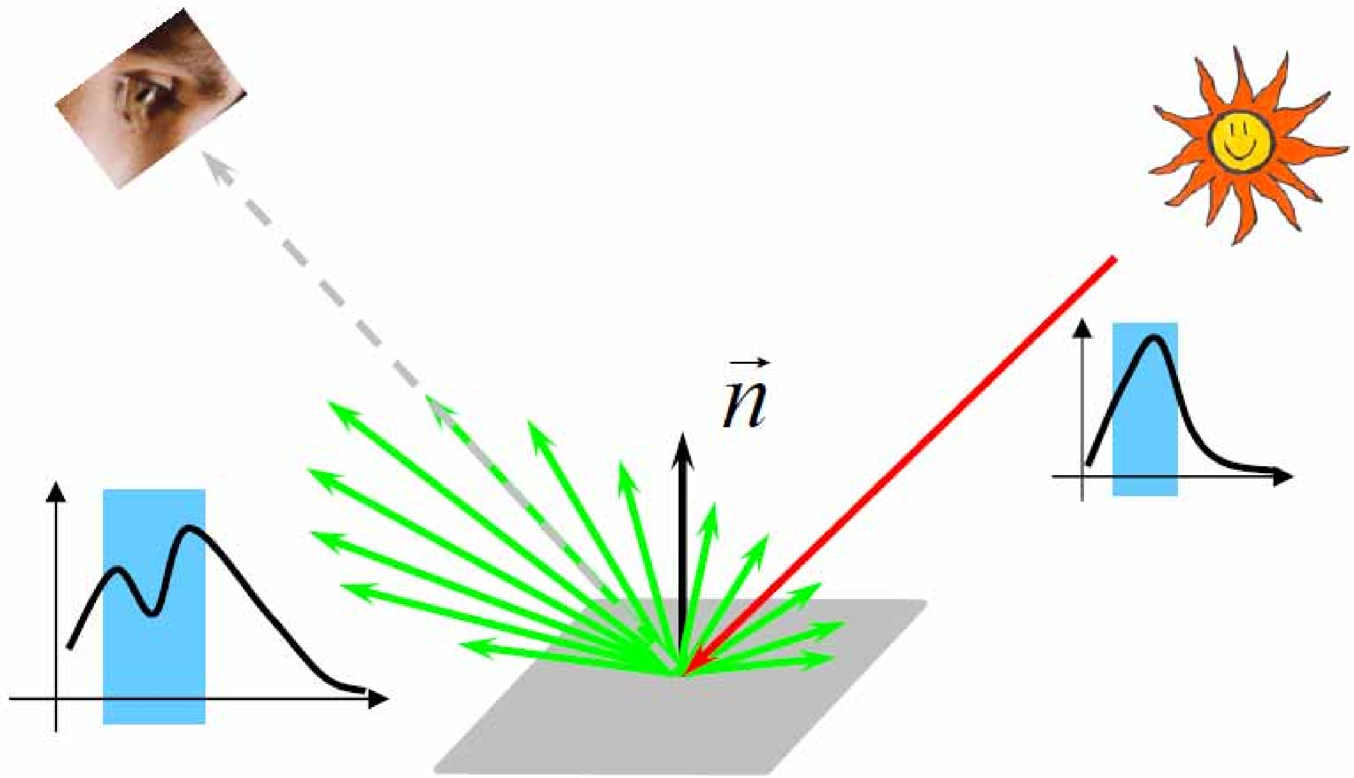


Image formation = "Shading from shape" (and light sources)

Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

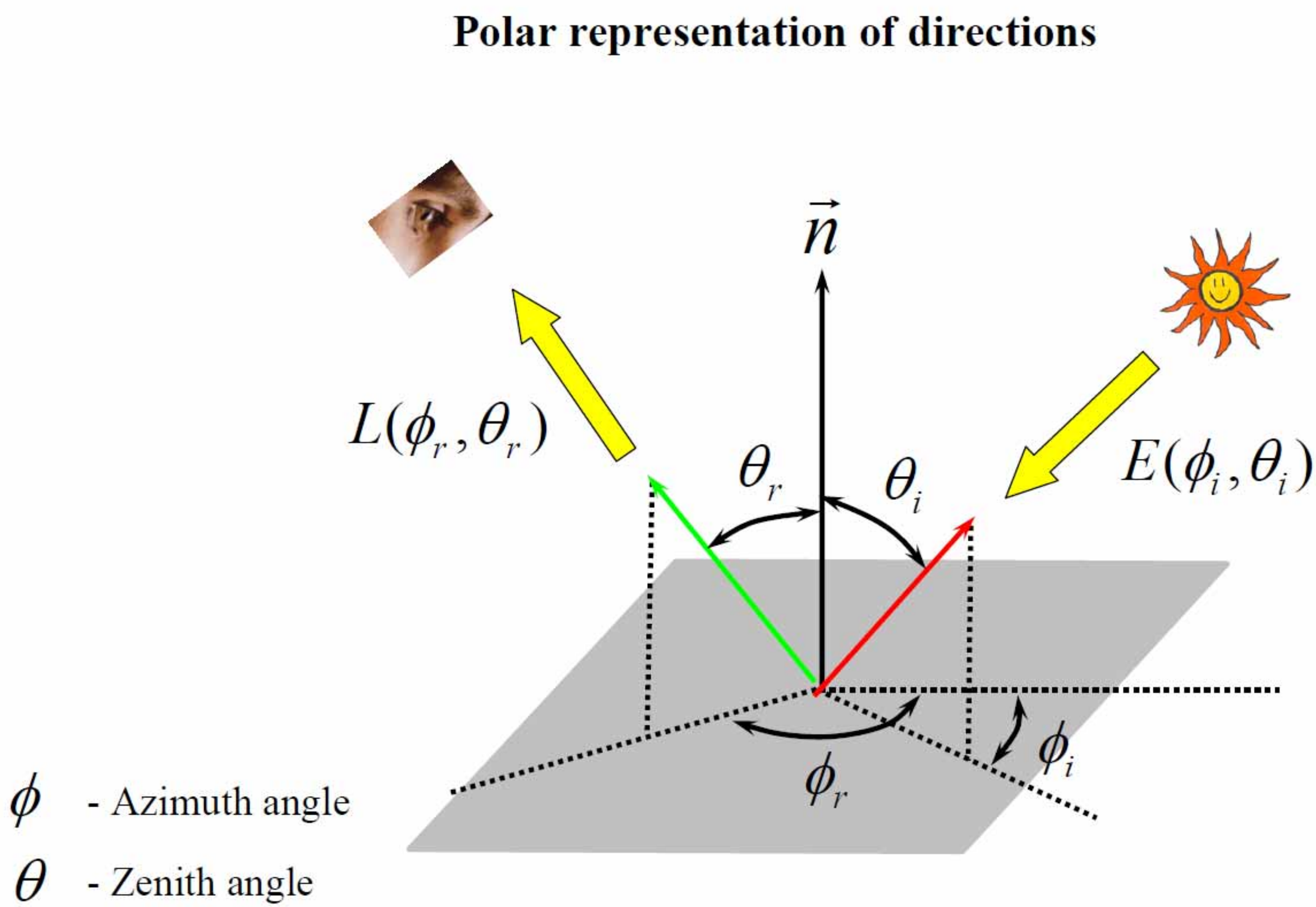
Shape from Shading

Image formation



Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

Shape from Shading

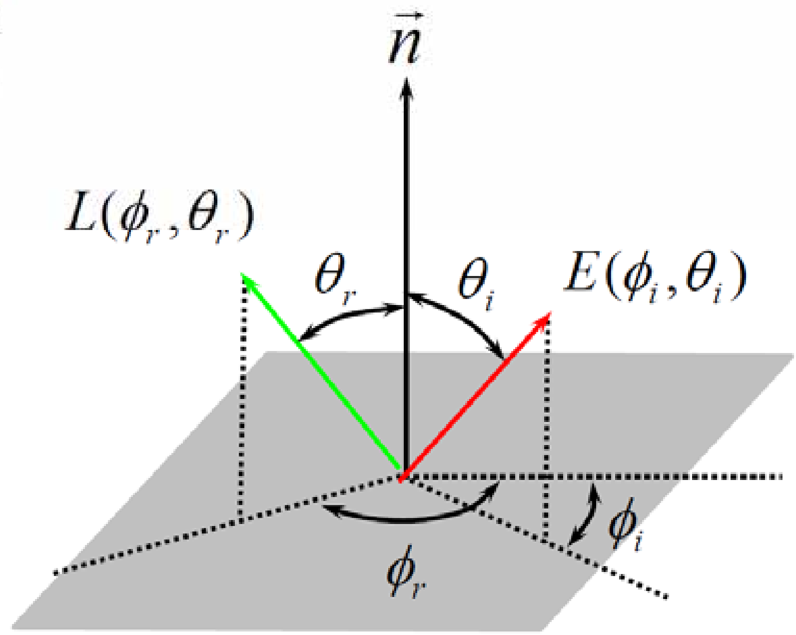


Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

Shape from Shading

The Bidirectional Reflectance Distribution Function (BRDF)

$$f_\lambda(\phi_i,\theta_i;\phi_r,\theta_r)=\frac{L_\lambda(\phi_r,\theta_r)}{E_\lambda(\phi_i,\theta_i)}$$



Helmholtz's reciprocity

$$f(\phi_i,\theta_i;\phi_r,\theta_r)=f(\phi_r,\theta_r;\phi_i,\theta_i)$$

Isotropic materials:

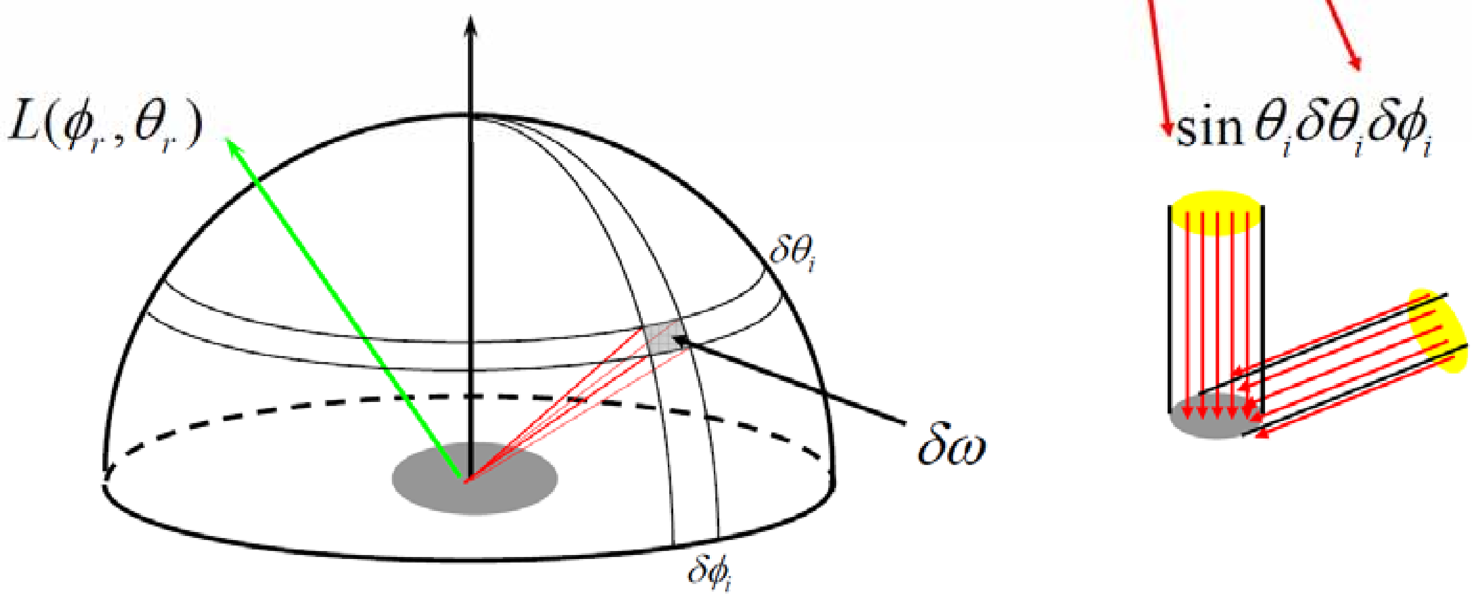
$$f(\phi_i,\theta_i;\phi_r,\theta_r)=f(\phi_i-\phi_r,\theta_i,\theta_r)$$

Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

Shape from Shading

Total surface reflection

$$L(\phi_r,\theta_r)=\int_{\omega} f(\phi_i,\theta_i;\phi_r,\theta_r)\cdot E(\phi_i,\theta_i)\cdot\cos\theta_i\, d\omega$$

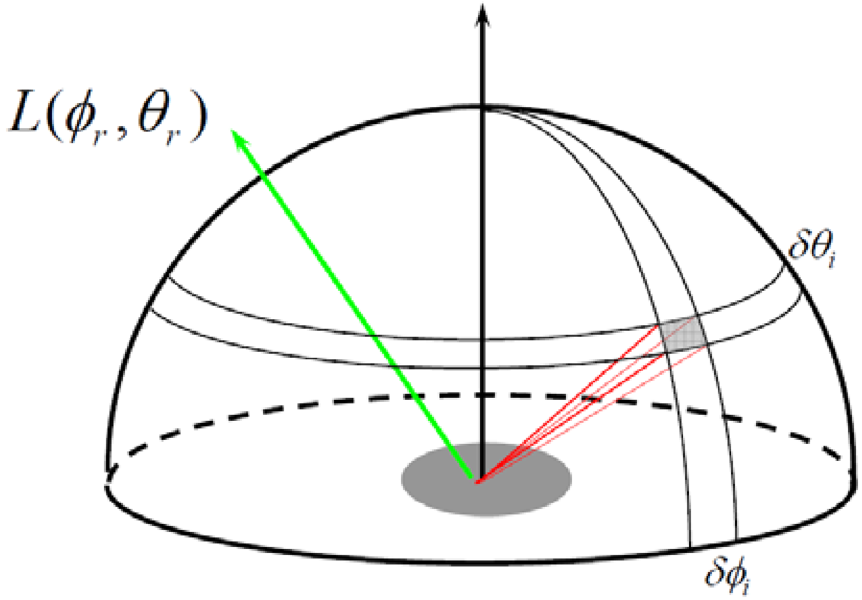


Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

Shape from Shading

Total surface reflection

$$L(\phi_r,\theta_r)=\int_{-\pi}^{\pi}\int_{0}^{\pi/2} f(\phi_i,\theta_i;\phi_r,\theta_r)\cdot E(\phi_i,\theta_i)\cdot \sin\theta_i \cdot \cos\theta_i \cdot \delta\theta_i\delta\phi_i$$



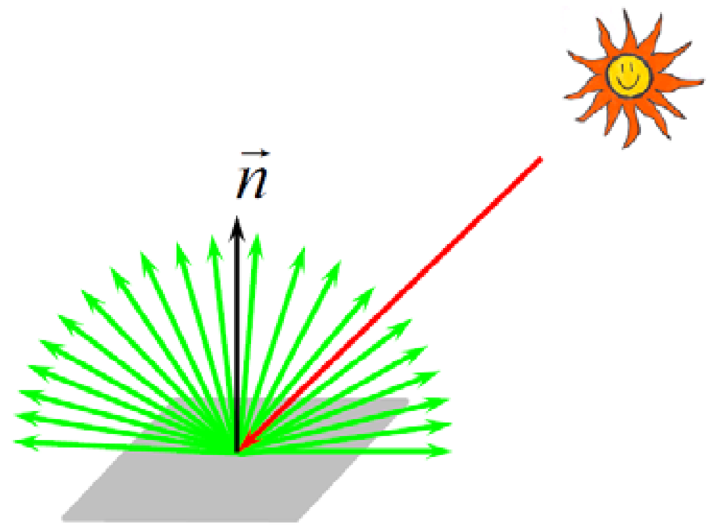
Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

Shape from Shading

Lambertian (perfectly diffused) surfaces

$$f_L(\phi_i, \theta_i; \phi_r, \theta_r) = const = \bar{f} = \rho \frac{1}{\pi}$$

Albedo

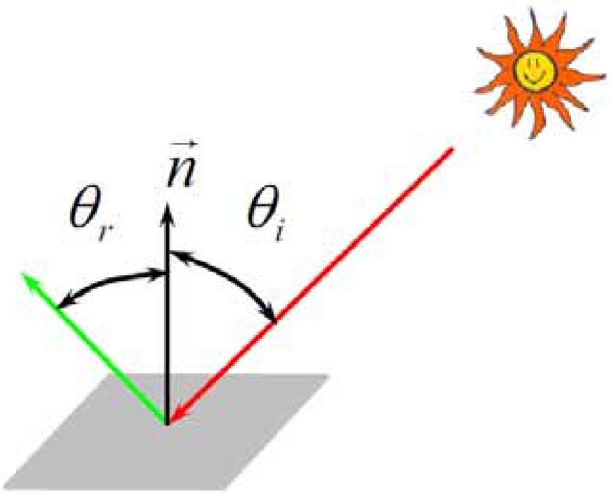


Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

Shape from Shading

Mirrored (perfectly secular) surfaces

$$f_S(\phi_i,\theta_i;\phi_r,\theta_r)=\frac{\delta(\theta_r-\theta_i)\delta(\phi_r-\phi_i-\pi)}{\sin\theta_i\cos\theta_i}$$



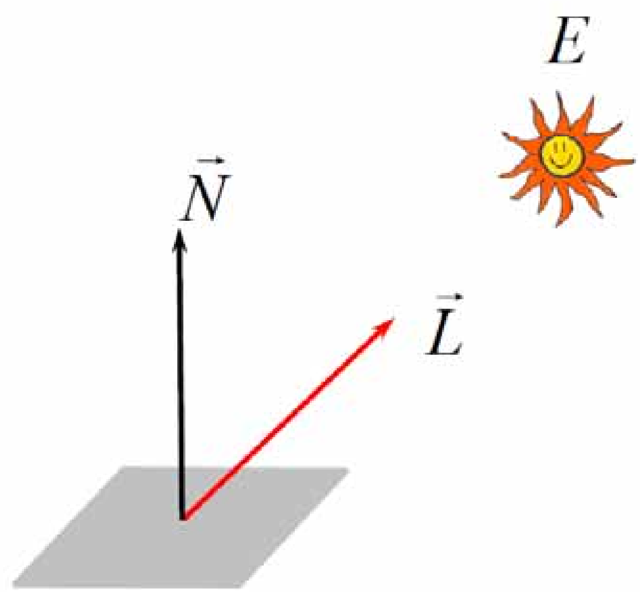
Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

Shape from Shading

Point light source from direction (ϕ_L, θ_L)

$$E(\phi_i, \theta_i) = E \cdot \frac{\delta(\theta_L - \theta_i) \cdot \delta(\phi_L - \phi_i)}{\sin \theta_L}$$

$$\int_{-\pi}^{\pi} \int_{0}^{\pi/2} E(\phi_i, \theta_i) \cdot \sin \theta_i \cdot \delta\theta_i \delta\phi_i = E$$



Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/

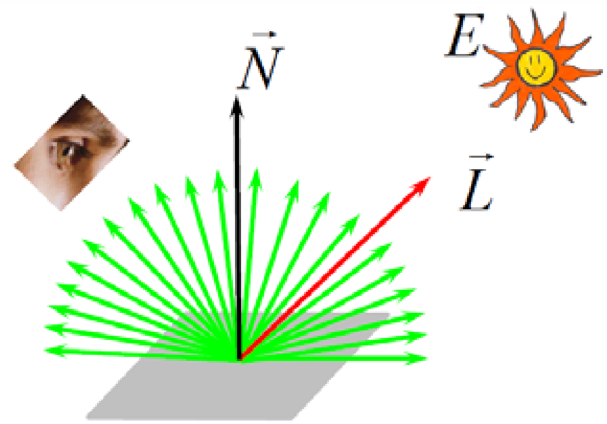
Shape from Shading

Surface brightness – appearance in the Lambertian case and point light source

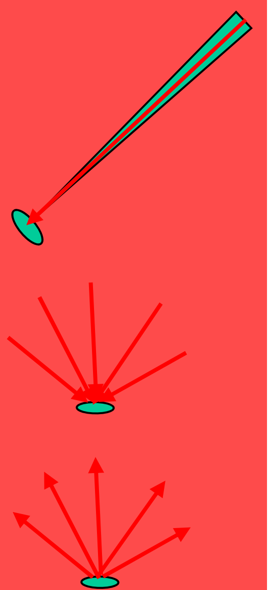
$$f_L(\phi_i,\theta_i;\phi_r,\theta_r) = \rho\frac{1}{\pi} \qquad E(\phi_i,\theta_i) = \frac{\delta(\theta_L-\theta_i)\delta(\phi_L-\phi_i)}{\sin\theta_L}$$

$$I(x,y) \propto L(\phi_r,\theta_r) = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} f(\phi_i,\theta_i;\phi_r,\theta_r)\cdot E(\phi_i,\theta_i)\cdot\sin\theta_i\cdot\cos\theta_i\cdot\delta\theta_i\delta\phi_i$$

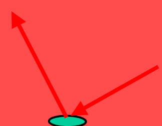
$$L = \rho\frac{1}{\pi}E\cos\theta_L \propto \rho(\hat{N}\cdot\hat{L})$$

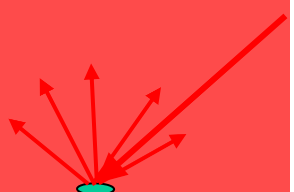


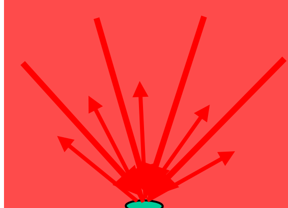
Courtesy Ohad Ben-Shahar, BGU, http://www.cs.bgu.ac.il/~ben-shahar/



Term	Definition	Units	Application
Radiance	the quantity of energy travelling at some point in a specified direction, per unit time, per unit area *perpendicular to the direction of travel*, per unit solid angle.	wm^2sr^{-1}	representing light travelling in free space; representing light reflected from a surface when the amount reflected depends strongly on direction
Irradiance	total incident power per unit surface area	wm^{-2}	representing light arriving at a surface
Radiosity	the total power leaving a point on a surface per unit area on the surface	wm^{-2}	representing light leaving a diffuse surface



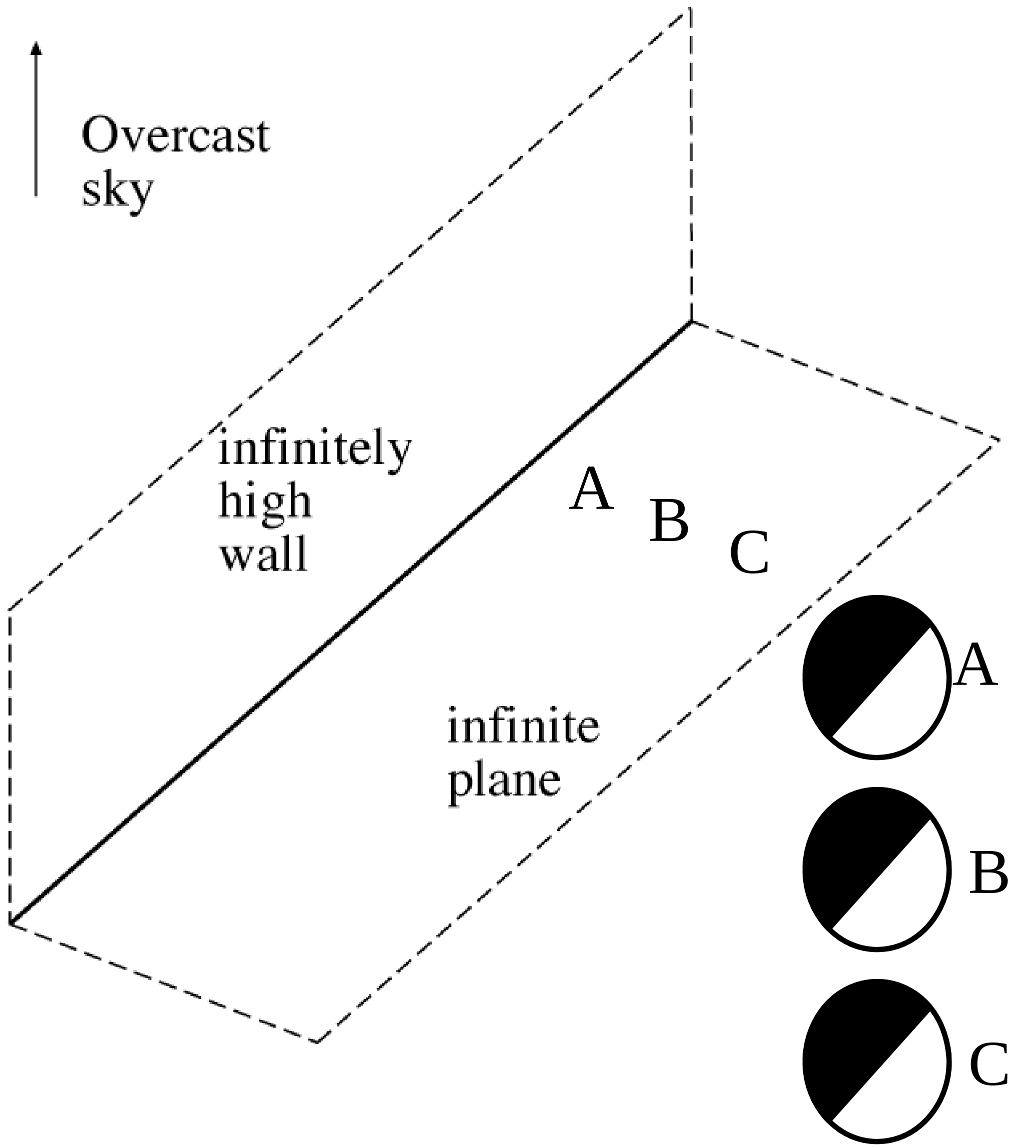




Term	Definition	Units	Application
BRDF (Bidirectional Reflectance Distribution Function)	the ratio of the radiance in the outgoing direction to the incident irradiance	sr^{-1}	representing reflection off general surfaces where reflection depends strongly on direction
Directional Hemispheric Reflectance	the fraction of the incident irradiance in a given direction that is reflected by the surface, whatever the direction of reflection	unitless	representing reflection off a surface where direction is unimportant
Albedo	Directional hemispheric reflectance of a diffuse surface	unitless	representing a diffuse surface

Term	Definition	Examples
Diffuse surface; Lambertian surface	A surface whose BRDF is constant	Cotton cloth; many rough surfaces; many paints and papers; surfaces whose apparent brightness doesn't change with viewing direction
Specular surface	A surface that behaves like a mirror	Mirrors; polished metal
Specularity	Small bright patches on a surface that result from specular components of the BRDF	

Lambert’s wall

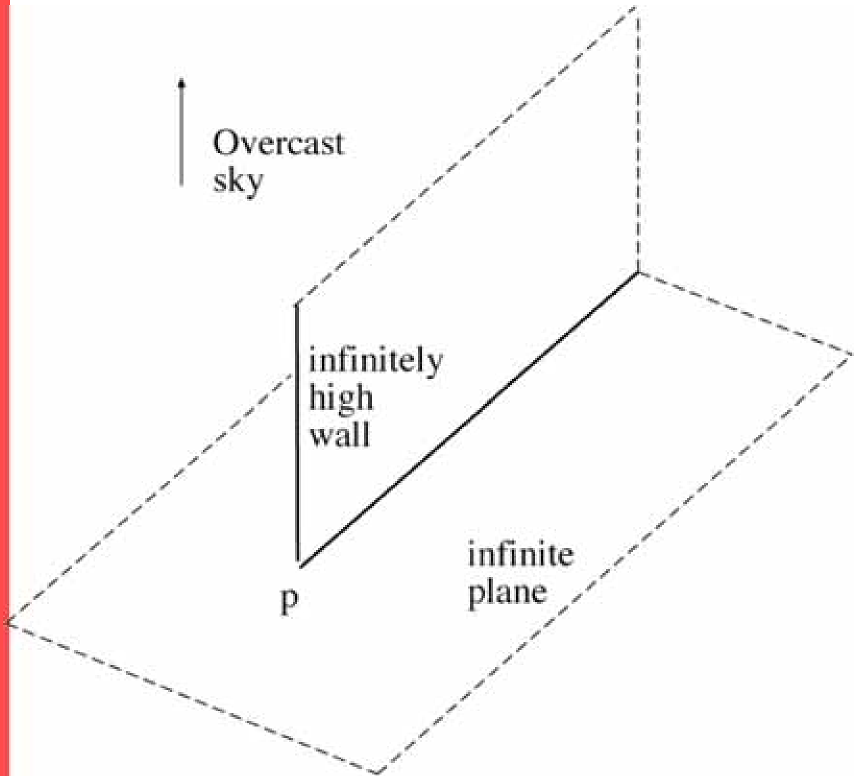


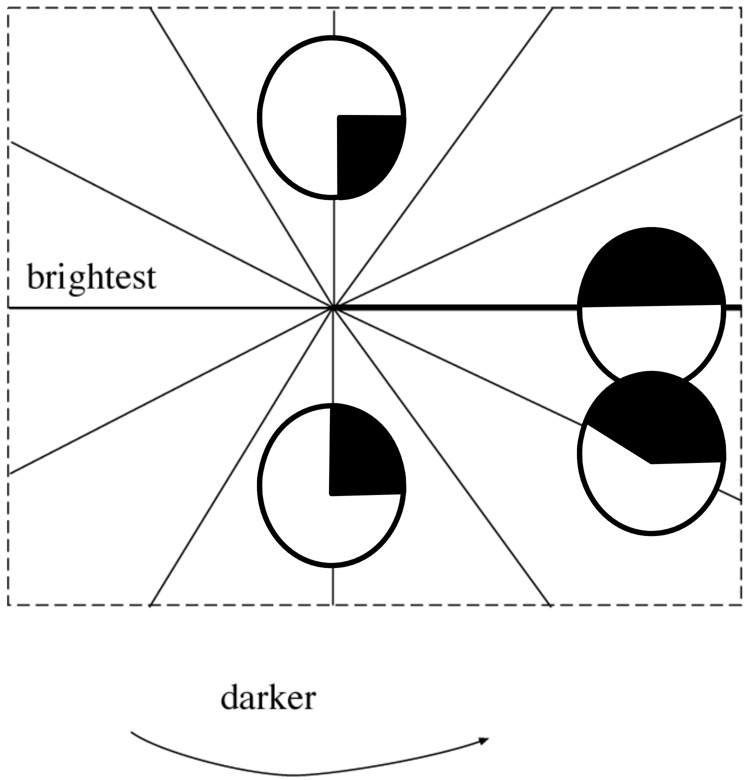
Vertical wall: black
Horizontal plane: uniform

What is distribution of brightness on the ground?

Answer: every point sees the same input hemisphere -> each point must be the same.

More complex wall





Rays are isophotes

Sources and shading

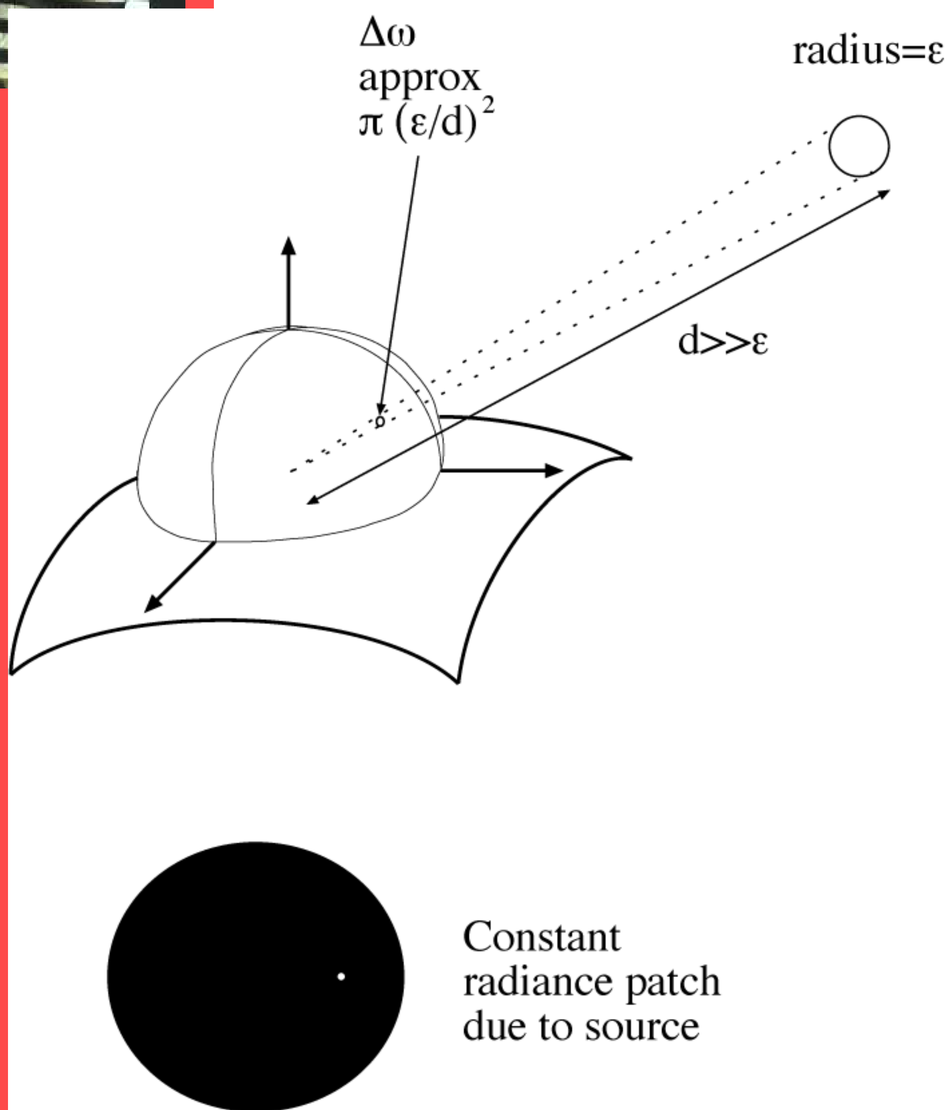
- How bright (or what colour) are objects?
- One more definition: Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- similar to radiosity: a source can have both
  - radiosity, because it reflects
  - exitance, because it emits

- General idea:

$$B(x) = E(x) + \int_{\Omega} \left\{ \begin{array}{l} \text{radiosity due to} \\ \text{incoming radiance} \end{array} \right\} d\omega$$

- But what aspects of the incoming radiance will we model?

Radiosity due to a point sources



- small, distant sphere radius ε and exitance E, which is far away subtends solid angle of about

$$\pi\left(\frac{\varepsilon}{d}\right)^2$$

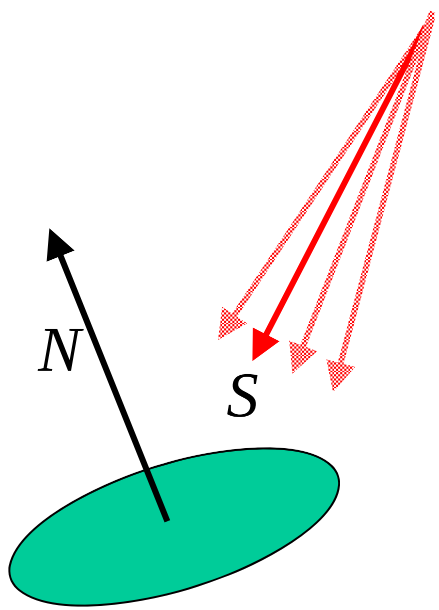
Radiosity due to a point source

- Radiosity is

$$
\begin{aligned}
B(x) &= \pi L_o(x) \\
&= \rho_d(x) \int_{\Omega} L_i(x,\omega) \cos\theta_i d\omega \\
&= \rho_d(x) \int_{D} L_i(x,\omega) \cos\theta_i d\omega \\
&\approx \rho_d(x)(\text{solid angle})(\text{Exitance term}) \cos\theta_i \\
&= \frac{\rho_d(x) \cos\theta_i}{r(x)^2} (\text{Exitance term and some constants})
\end{aligned}
$$

Standard nearby point source model

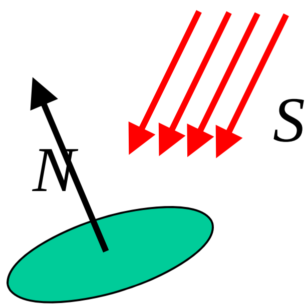
$$\rho_d(x)\left(\frac{N(x).S(x)}{r(x)^2}\right)$$



- N is the surface normal
- rho is diffuse albedo
- S is source vector - a vector from x to the source, whose length is the intensity term
  - works because a dot-product is basically a cosine

Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer
  - the sun doesn't for any reasonable binding of closer
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn't vary much, and the distance doesn't vary much either, and we can roll the constants together to get:



$$\rho_d(x)\left(N(x).S_d(x)\right)$$

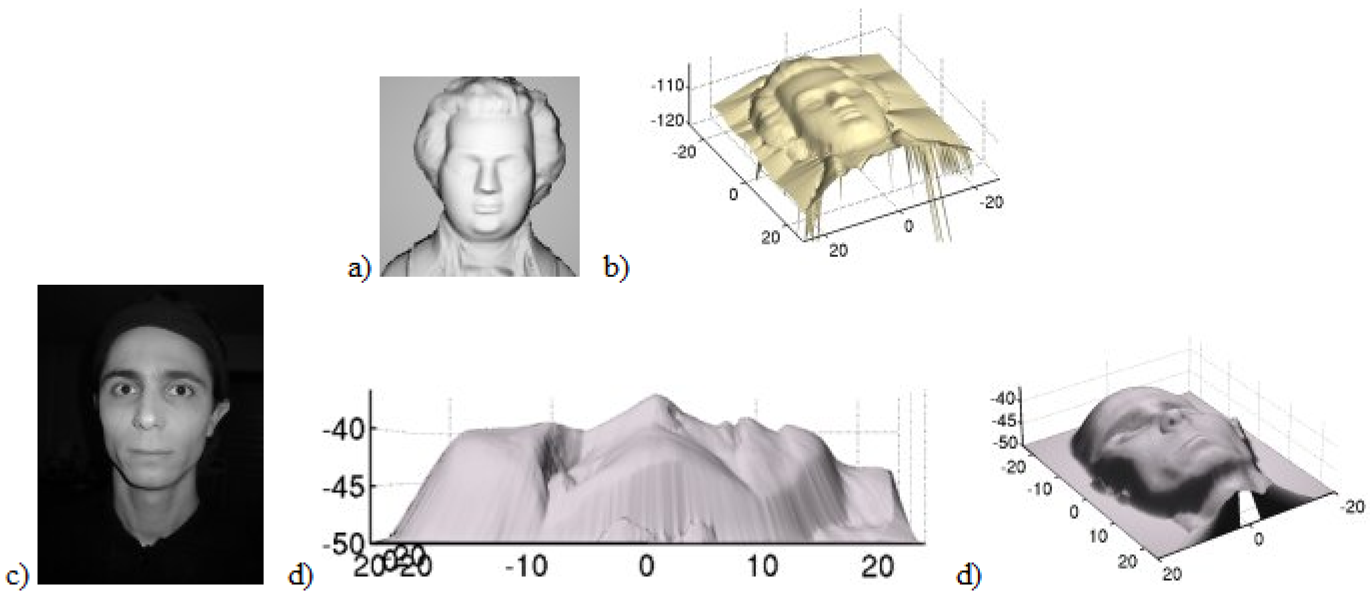
Shading models

- Local shading model
  - Surface has radiosity due only to sources visible at each point
  - Advantages:
    - often easy to manipulate, expressions easy
    - supports quite simple theories of how shape information can be extracted from shading
- Global shading model
  - surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
  - Advantages:
    - usually very accurate
  - Disadvantage:
    - extremely difficult to infer anything from shading values

Shape from Shading

Authors: Emmanuel Prados and Olivier Faugeras

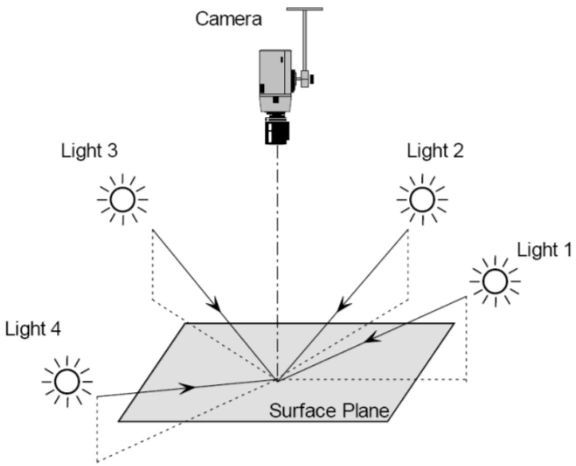
CVPR'2005, International Conference on Computer Vision and Pattern Recognition, San Diego, CA, USA, June 2005.



a) Synthetic image generated from the classical Mozart's face [Zhang-Tsai-etal:99]; b) reconstructed surface from a) by new algorithm; c) real image of a face; d)-e) reconstructed surface from c) by new algorithm.

Photometric stereo

- Assume:
  - a local shading model
  - a set of point sources that are infinitely distant
  - a set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
  - A Lambertian object (or the specular component has been identified and removed)



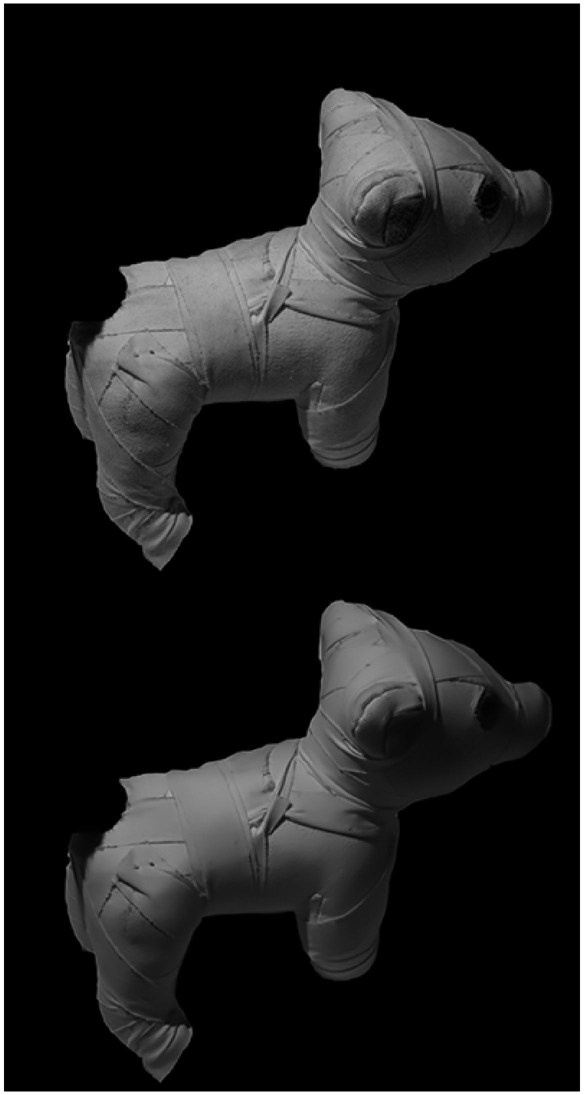
Photometric Stereo
Christopher Bireley

Bandage Dog

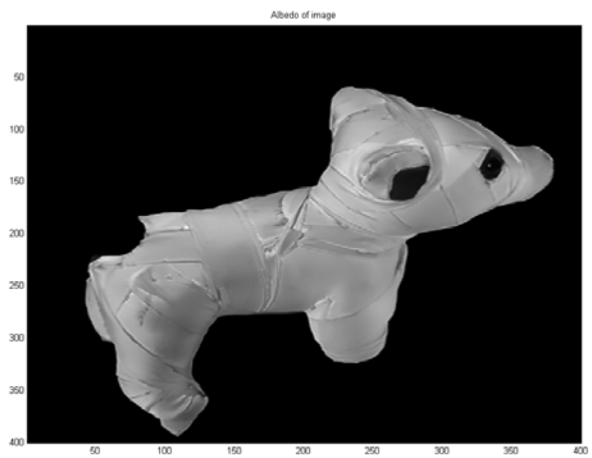
Imaging Setup

Preprocessing

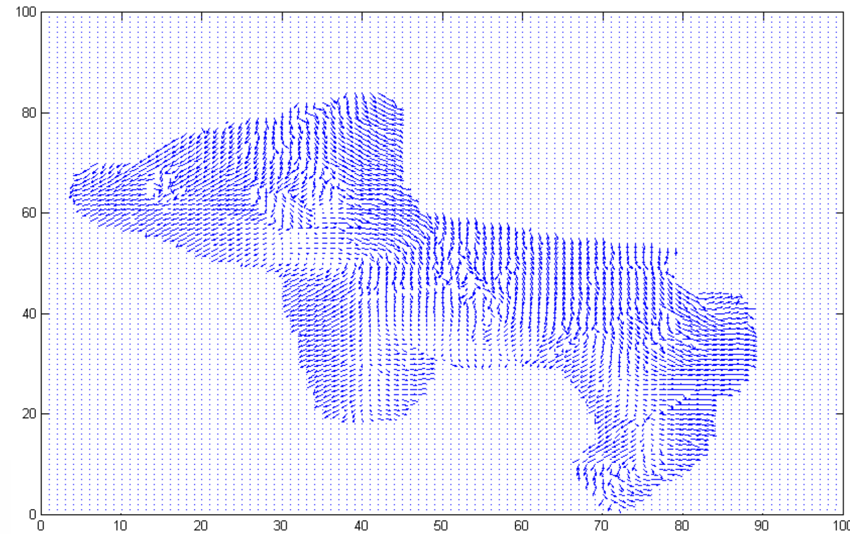
- Remove background isolate dog
- Filter with NL Means



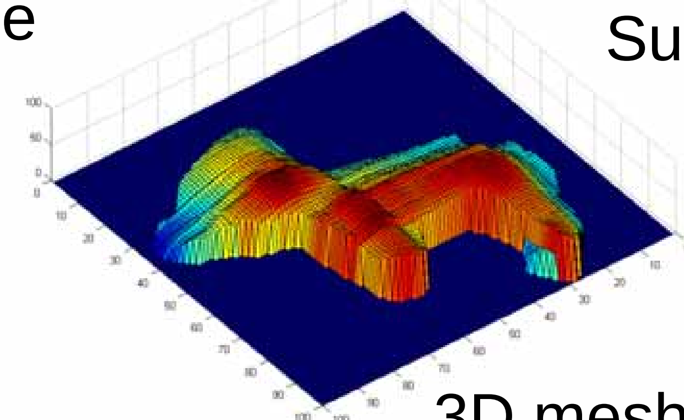
Photometric Stereo
Christopher Bireley



Albedo image



Surface Normals



3D mesh